Lisbon School of Economics & Management
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PROJECT

VALUATION AND SENSITIVITY ANALYSIS: FIXED COUPON EXPRESS CERTIFICATE LINKED TO THE EURO STOXX 50 INDEX

CAROLINA TORRES FURTADO

JUNE 2023

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Gostaria de dedicar este Trabalho Final de Mestrado aos meus pais por me ensinarem a ser uma pessoa sonhadora, lutadora e dedicada. Ensinaram-me que, apesar de todas as dificuldades que possamos encontrar, se trabalharmos arduamente, podemos alcançar o que quisermos. Pai e mãe, muito obrigada por serem os meus pilares, sem vocês nada disto seria possível.

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Carolina Torres Furtado

ABSTRACT

The main objective of this project is to carry out a valuation and sensitive analysis of the structured product called Fixed Coupon Express Certificate linked to EURO STOXX 50 Index. This product is classified as an autocallable product that has specific characteristics. In particular, this Express Certificate is automatically redeemed from the market at predefined dates if the value of its underlying asset is equal to or higher than the predefined autocall value. To perform this valuation, we consider the Monte Carlo simulation model, in order to determine the future cash-flow of the product. We consider the historical data of the underlying asset from five years prior to the issuance date of the product, which corresponds to the period from 4 February 2015 to 4 February 2020. From this period, we calculate the minimum, maximum and one-day historical volatility prior to the issue date of the product, which are then applied to Monte Carlo. Similarly, the future cash flows allow us to estimate the unconditional and conditional probabilities that the product will survive each year, as well as its expected price. However, for a detailed analysis of the Express Certificate under different scenarios, a sensitive analysis is performed. This aims to examine the behaviour of the product and its performance under different levels of volatility, strikes and barriers in isolation. For this exercise, volatilities are created from 0%, 10%, 20% up to 100. Likewise, strike levels are produced starting at 70% of the value of the underlying at issuance up to 140% and barrier levels from 30% up to 100% of the value of the underlying asset at issuance. Consequently, the various unconditional and conditional probabilities, as well as the expected price of the product under different scenarios are estimated.

KEYWORDS: Structured product, Express Certificate, autocallable product, Monte Carlo, underlying asset, index, EURO STOXX 50.

RESUMO

O objetivo deste projeto é avaliar e realizar uma análise sensitiva do produto estruturado denominado de Fixed Coupon Express Certificate linked to EURO STOXX 50 Index. Este produto é classificado como um produto autocallable que apresenta caraterísticas peculiares. Em particular, este Express Certificate em datas previamente definidas será removido automaticamente do mercado, caso o valor do seu produto subjacente seja igual ou superior ao valor predefinido de chamada automática. Desta forma, para realizar esta avaliação o modelo de análise Monte Carlo é utilizado, de modo a obter os futuros cash-flow do produto. Para tal, são considerados os dados históricos do produto subjacente de cinco anos antes da data de emissão do produto, correspondendo ao período de 4 de fevereiro de 2015 a 4 de fevereiro de 2020. A partir deste período, calcula-se a volatilidade histórica mínima, máxima e de um dia antes da emissão do produto que, posteriormente, são aplicadas ao Monte Carlo. De igual forma, os futuros cash-flows permitem estimar as probabilidades não condicionadas e condicionadas de o produto sobreviver a cada ano, assim como a estimação do seu preço. Todavia, para uma análise detalhada do Express Certificate perante diversos cenários é realizada uma análise sensitiva. Esta tem como objetivo examinar o comportamento do produto e a sua performance perante diferentes níveis de volatilidade, strikes e barreiras, isoladamente. Para tal exercício, são criadas volatilidades de 0%, 10%, 20% até 100%. Da mesma forma, são criados níveis de strike a começar em 70% do valor do produto subjacente na data de emissão do produto até aos 140%, assim como são produzidas barreiras de 30% até 100% do valor inicial do produto subjacente na data de emissão do produto. Por conseguinte, estimam-se as diversas probabilidades não condicionadas e condicionadas, assim como preço esperado do produto perante diversos cenários.

PALAVRAS-CHAVES: Produto estruturado, Express Certificate, produto autocallable, Monte Carlo, produto subjacente, índice, EURO STOXX 50.

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1. INTRODUCTION

The aim of this project is to do a detailed valuation of the Fixed Coupon Express Certificate linked to the EURO STOXX 50 Index, as well as a sensitivity analysis, in order to describe how the product behaves with different strike and barrier levels, as well as volatility. This product is a structured product issued by the Deutsche Bank AG in Frankfurt.

Financial intuitions have created structured products with the purpose of making it easier for companies to issue debt instruments. Most structured products contain a bond component and a derivative component which are usually linked to the performance of an index or a single security such as stocks, commodities, mutual funds, and exchange-traded funds. Investors' desire for an alternative investment strategy and for a product with risk-return characteristics has made them popular in Europe, making them accessible to retail investors in the same way that stocks, bonds, and mutual funds are traded.

An Express Certificate is a type of autocallable product. According to Deng and McCann (2011), the first autocallable product was registered in August 2003 by BNP Paribas in the United States. According to Deng and McCann (2011) and Alm *et al.* (2013), autocallable products have an autocall feature which automatically redeems them if the value of the reference asset reaches a specified barrier condition on a predefined autocall date. If the condition is met, the product is immediately terminated, shortening its expected maturity. In the case where the condition is not met, the product continues to exist until the next autocall date and so on until maturity.

Meanwhile, valuating the Express Certificate is challenging because it is extremely complex and there are many payoff features to consider. One particularity is the existence of a barrier at maturity that affects the product's payoff. For example, if the final reference value of the underlying asset is at or above the barrier at maturity, the investor receives the payoff of the principal notional amount. However, if the reference value is below the barrier, the payoff is directly linked to the performance of the underlying. With that, the product under study is also characterized by its strike level and barrier levels.

For the valuation of the Express Certificate, we employ the Monte Carlo simulation as the pricing tool, as it can be used to determine the expected payoff in a risk-neutral world. During this analysis, we draw upon the historical data of the index as dataset, as the performance of the product depends on the performance of the EURO STOXX 50 index.

The project is organized as follows. Chapter 2 contains the state of the art of the autocallable products, Chapter 3 describes the Express Certificate based on the KID, Chapter 4 displays the selection of the dataset and explains the methods used in the valuation of the Express Certificate and in the sensitivity analysis, and Chapter 5 presents the results of the analysis. Finally, Chapter 6 establishes the conclusions of this project and suggestions for further development.

2. LITERATURE REVIEW

An Express Certificate is a type of autocallable product, and, therefore, a modern structured product. For this reason, this chapter provides an introductory overview of the state of the art of structured products in general and of autocallable products in particular.

According to Linnainmaa (2014), an autocallable product consists of a structured bond that has no predetermined maturity date. This is due to the triggers of the barrier conditions for autocall on the predefined dates. When the trigger is activated, the product is automatically redeemed, paying to the investor the principal notional amount and coupon. The name "autocallable" comes from the callable bond. However, in callable bonds the issuer has the right, although not the obligation, to call the bond back on predefined dates. In the case of autocallable products, the autocall event is automatic and depends on the performance of the underlying.

Linnainmaa (2014) also states that autocallable products do not provide capital protection, i.e., there is no guarantee that the investor will get back their original investment. Generally, in this type of products, there is a protective barrier that prevents the investor from losing all of their capital. This barrier usually comes into existence when the underlying is below it at maturity, as result of that the holder usually sells a put option with maturity T.

In their study, Deng and McCann (2011) arrange autocallable products into discrete autocall dates and continuous autocall dates. According to the authors, a discrete autocallable verifies whether the value of the underlying asset is above or below the barrier condition on predefined observation dates. Moreover, it is demonstrated that the value of a discrete autocallable product can be calculated from the probability of the product being called. From that, the authors conclude that assets that possess an autocall feature are worth less than the products that do not have it. This results from the fact that a non-autocallable product guarantees the payment of the coupon until maturity, while the product with the autocall feature may pay the coupon corresponding to a shorter period

of time or not at all. The study concludes that a discrete autocallable product is less likely to be called and it is the most common type of autocallable being traded in the market.

Alm *et al.* (2013) find that using the standard Monte Carlo method for pricing autocallable products with one or more underlying assets, does not provide stable results in terms of numerical differentiation. For this reason, the authors aimed to develop a Monte Carlo valuation strategy that provides stable differentiation using simple finite differences. For the autocallable products with an underlying asset, they begin by using the Black-Scholes method to obtain constant parameters. To extend it to stochastic parameters, they adapted the Monte Carlo simulation to predict values with a minimum number of paths. For that, they exclude the values that are above the barrier. However, by using a different approach the authors notice that it is possible to calculate the sensitivity of the products using the Likelihood Ratio Method through the discontinuous payoffs. This method consists of weighting the payoffs in each Monte Carlo path. In turn, they conclude that using the adapted Monte Carlo offers better results than the Likelihood Ratio Method.

Our proposed method can be defined as a different valuation scheme. We use the Monte Carlo simulation as the pricing model, assuming that the underlying asset follows the geometric Brownian motion. As for the choice of parameters, we have as baseline the methods described by Hull (2022). In the meantime, to determine the value of the product, we consider the probabilities described by Martelinni *et al.* (2003) and the pricing methods from Adams and Smith (2019).

3. EXPRESS CERTIFICATE

3.1. The Product

According to the Key Information Document (KID), present in Figure A.1 in appendix, the structured product under study is a Fixed Coupon Express Certificate linked to the Euro Stoxx 50 Index. The Express Certificate is a German law governed certificate with an issue date on February 4, 2020 and a maturity date on February 6, 2025, i.e., it has a maturity of 5 years.

This type of product, according to KID, is aimed at private investors with a medium-term investment horizon and with an interest in capital formation or asset optimization. Likewise, investors are advised to have experience with this type of product, as there is no protection for the invested capital. In the meantime, this product aims to offer investors a return in the form of a fixed annual coupon of €2,65 and a payoff at the end of the product's life equal to the product notional amount or principal of €100,00.

The Express Certificate is composed by autocall observation dates, autocall barrier levels and autocall and coupon payment dates. Regarding the autocall observation dates, if the reference level of the underlying asset is at or above the autocall barrier level, the product can be early redeemed. In that case, the investor receives the coupon and principal, and no further payments are issued. In spite of this, as long as the product survives on each autocall observation date, it continues to receive the annual coupon.

The product has a strike level of 3.732,28, which is identical to the value of the index at issuance. Furthermore, it also contains a barrier level of 2.425,982, corresponding to 65% of the value of the index at issuance. Due to this barrier at the time of the maturity date, if the value of the index is below the barrier, the investor receives a payment linked to the performance of the underlying asset. Otherwise, if the final reference level is at or above the barrier level, the investor collects the final coupon and the principal.

3.2. The Issuer

The issuer of the Express Certificate is Deutsche Bank AG, Frankfurt. Founded in Berlin in 1870, it was in danger of collapsing after the Second World War. In 1957, Deutsche Bank was re-established in Frankfurt am Main through the merging of three banks. Since 1970, Deutsche Bank has been an international company listed on both the Frankfurt and New York Stock Exchanges. Through acquisitions and mergers of other financial institutions, it is now one of the largest multinational investment banks and financial services providers in the world. According to its official website[1], Deutsche Bank offers products and services in commercial and investment banking, retail banking, transaction banking, and asset and wealth management. According to the 2019 Annual Report, its mission is to "connect worlds and deliver a positive impact – for their clients, employees, investors, the economy and society as a whole". The structured products are part of the investment banking and can be found on the X-Markets investment platform, where it is also possible to find all legal documents relating to the products, as well as the new offerings.

3.3. The Underlying Asset

As mentioned above, the Express Certificate is linked to an underlying asset, the Euro Stoxx 50 Index. This index was created in 1998 and is characterized by being similar to the Dow Jones 30 Index. Euro Stoxx 50 is one of the main blue-chip indices in the eurozone and includes the 50 largest companies by market capitalization. According to the Capital.com website[2], the term "blue chip" describes companies that attain a good performance on the stock market, offer investors high returns and are financially stable. Meanwhile, to be included in the index, the company must be headquartered in a member country of the eurozone and share the same common currency, the Euro, such as Belgium, Finland, France, Germany and Portugal. Finally, STOXX Limited is the institution that manages the index and is owned by Deutsche Börse AG.

[1] Can be consulted at: https://www.db.com/files/documents/Deutsche-Bank-History--Chronicle-from-1870-until-today.pdf.
[2] Can be consulted at: https://capital.com/euro-stoxx-50-index-definition.

3.4. PRIIP-KID: Risk Indicator

According to Graf (2019), since January 1, 2018, the manufacturer of Packaged Retail and Insurance-Based Investment Products (PRIIPs) must accompany all products traded in the European Union with a document called KID, which complies with the European Commission Regulation EU 1286/2014. According to the regulation in force at the time of issuance of the product, European Commission Delegated Regulation (2019), the structure of KID must be standardized to allow investors to easily compare products. The document sets out the product description, risk assessment, performance assessment, producer insolvency case, cost composition, recommended holding period, complaint options, and other relevant information.

Graf (2019) criticizes the European Commission for providing formulas to calculate the risk indicator without specifying its assumptions and methods used. He further criticizes the fact that the VEV (value-at-risk equivalent volatility) formula is only provided for single premium products, and when applied to regular premium payments, overestimates the "true" risk of the product.

According to the European Commission Delegated Regulation (2019), both the market risk measure (MRM) and the credit risk measure (CRM) should be considered to assess the summary risk indicator. The MRM represents the market risk of the product, while the CRM represents the creditworthiness of the product provider. Together, both measures result in the summary risk indicator, which is a measure of the risk level of the products on a scale of 1 to 7. Deutsche Bank AG indicates that the Express Certificate is rated in the lowest risk category, which is 1 out of 7.

4. DATA AND METHODOLOGY

Firstly, this chapter describes the data used for the valuation of the Express Certificate and for the sensitivity analysis. After that, we present the methodology applied in the analysis.

Since the Euro Stoxx 50 Index is the underlying asset linked to the product, its historical data is used for the valuation and sensitivity analysis. This historical data is composed by the daily series of adjusted closing quotes obtained from the Investing.com website[3]. According to the European Commission Delegated Regulation (2019), when assessing a PRIIP it is necessary to consider a period of five years for the historical analysis. Therefore, in this project we use a historical data that covers a total of 5,557 trading days, corresponding to the period from February 4, 2015 to February 4th 2020, the product's issue date, given by the Figure 1.

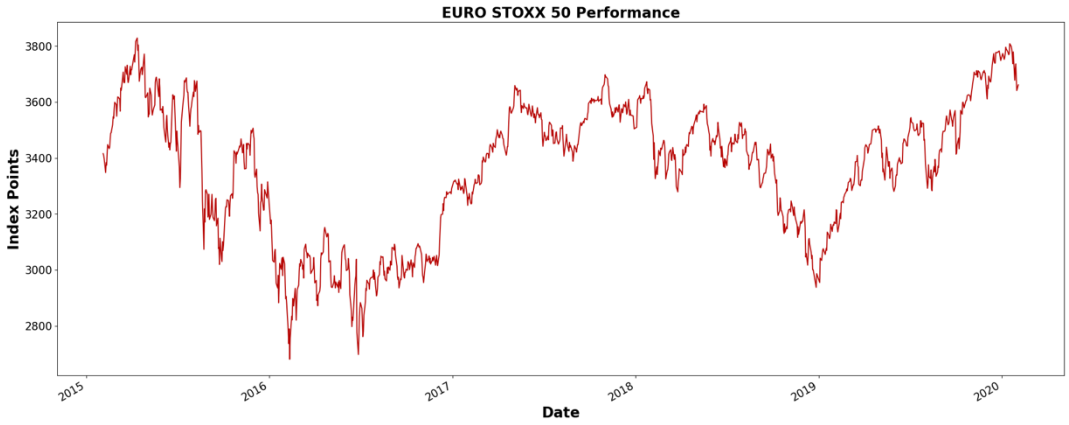

FIGURE 1: EURO STOXX 50 Index Historical Performance[4].

Due to an autocall event, the Express Certificate terminated in February 2022, and all information about the product is no longer available in the X-Markets. In order to compare the real value of the product with the theoretical value, both the Frankfurt Stock Exchange

[3]Can be consulted at: https://www.investing.com/indices/eu-stoxx50-historical-data.
[4] We have used python software Jupyter Notebook, version 6.4.5 for the figure above.

and Deutsche Bank X-Markets were contacted by email, but stated that they do not provide data about products that are no longer being traded.

Regarding to the methods used in the valuation part, first the volatility is analyzed, followed by the Monte Carlo simulation, product decomposition, Express Certificate valuation, and finally, unconditional and conditional probabilities. For the sensitivity analysis, we explain the different volatility levels, strikes and barriers chosen, which can alter the value of product and its unconditional and conditional probability of surviving an autocall event.

4.1. Volatility Analysis

A key feature of any pricing model is the volatility, σ, of the underlying asset. Volatility can be defined as a measure of uncertainty regarding the returns provided by the index. For this analysis we considered the method described by Hull (2022), which consists of computing historical volatilities from historical data.
First, we calculate the daily logarithm returns, u_i from the daily series of adjusted closing quotes, S_i, for the period between 2015 and 2020, according to Equation 1:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right), \quad (1)$$

for i = 1, 2, ..., n.

After that, we estimate the standard deviation, s, according to Equation 2, present below for seven periods. These periods correspond to 3 months, 6 months, 1 year, 2 years, 3 years, 4 years and 5 years. The period of 3 months is the shortest and 6 months is the medial time, when considering the annualized volatility of 1 year. Additionally, years 2, 3, and 4 correspond to all possible autocall events, and finally, year 5 corresponds to the maturity of the Express Certificate.

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2}, \quad (2)$$

where the $\bar{u}$ is the mean of u_i.

Given the daily standard deviation, we calculate the annualized volatility as follows:

$$Annualized\ Volatility\ =\ Volatility\ per\ trading\ day\ \times\ \sqrt{252}\ ,$$

where 252 is assumed to be the number of trading days in a year.

Figure 2 shows the annualized volatility time series for the seven periods. Moreover, it presents the daily log fluctuation returns over the 5 years of historical data. From these fluctuations, we can see a market downturn in August 2015 and June 2016, which led to an increase in volatility.

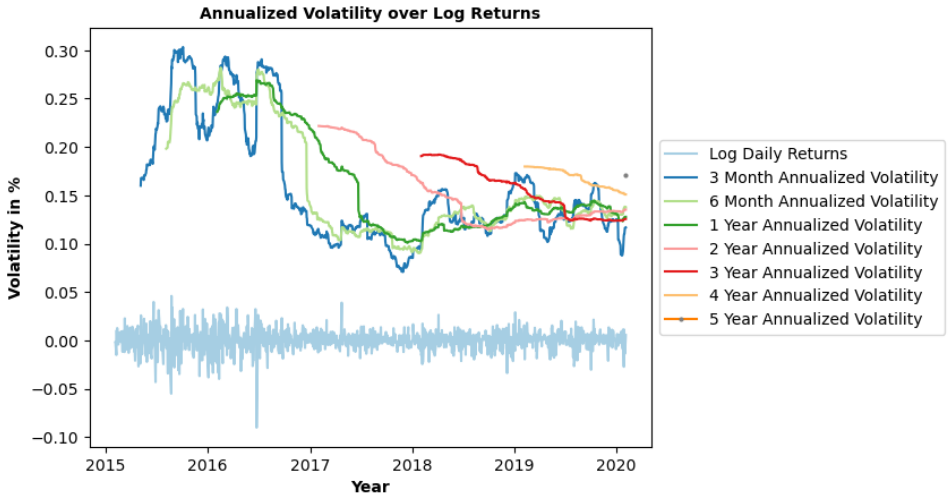

FIGURE 2: Time-series of annualised volatility over logarithmic daily returns[5].

Based on this analysis, we decided to valuate our model behavior with three levels of volatility. We considered the lowest volatility and the maximum volatility registered in our historical data, as well as the 1-year annualized volatility from one day before the product's issue date. The lowest volatility corresponds to the 3-month annualized volatility, which occurred on November 29, 2017, and the maximum volatility,

[5] We have used python software Jupyter Notebook, version 6.4.5 for the figure above.

corresponding to the 3-month annualized volatility, occurring on October 26, 2015. Therefore, the three levels of volatility are: 7,01%, 29,25% and 12,78%, respectively.

4.2. Monte Carlo Simulation

The Monte Carlo simulation is the pricing model chosen for the valuation of the Express Certificate. According to Hull (2022), the Monte Carlo simulation is a statistical model that generates n random samples of paths to reach the expected payoff discounted by a funding rate. In this valuation, in order to calculate the predicted quotes of the underlying asset, we used the formula that follows the geometric Brownian motion stochastic process, as shown in Equation 3. According to Hull (2022) and Parungrojrat & Kidsom (2019), this is a continuous time stochastic process that follows a lognormal distribution with mean, μ, and standard deviation, σ.

$$S_i = S_0 \, e^{\left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma\varepsilon\sqrt{\delta t}}, \quad (3)$$

where S_i is the predicted reference value at time T, S_0 is the initial reference value, μ is the drift-term and σ is the volatility, both with a constant value, and lastly, t is time.

Since Si follows a GBM stochastic process, we can state that ε is a random draw with N (0,1).

First, as the parameter μ is the drift-term used for the risk-free rate under a risk-neutral valuation, we use as risk-free rate a German government bond of -0,647%, with an issue date of one day before the issuance of the Express Certificate and with same maturity.

Second, the parameter δt entails the time step considered when running the simulations. Therefore, given the possibility of an early termination (through an autocall event), we chose a bimonthly time-steps over the life of the product until maturity. This time interval results in 6 time steps per year and 30 time steps up to the maturity (T $=$ 5 years). Thus, the $\delta t = 0{,}167$.

Taking into account the 30 time steps, through Excel we created 30 random variables $(\varepsilon_1, \dots, \varepsilon_{30})$ with 10,000 paths each, using the inverse cumulative normal distribution with a mean of 0 and a standard deviation of 1, as stated by Hull (2022). Therefore, we begin the calculation of the first column of predicted quotes of the underlying asset by using the initial reference level of 3.732,28. After that, when calculating each predicted quote we take into consideration the quote from the previous time-step. As a result of this reasoning, through Monte Carlo we determine the payoffs at all relevant dates that allow product decomposition and valuation.

4.3. Express Certificate Decomposition

It is mandatory for a PRIIP manufacturer, according to the European Commission Delegated Regulation (2019), to present on KID key information about the issued product. As KID is a standardized document, it is not mandatory to present the decomposition of the product. However, for valuation purposes it is necessary to have a detailed decomposition of the Express Certificate as shown in the Table I.

Component	Holder Position	Type	Maturity	Strike	Barrier
Callable Bond:	Long	Autocallable with Fixed Coupon	5 Years		
Bond	Long	Coupon Bond	5 Year		
Call Option	Short	Bermudan Option	3 Year	100%	
DIP Option	Short	European Barrier Option	5 Years	100%	65%

TABLE I: Express Certificate Decomposition.

As mentioned in the introductory chapter, according to Alm *et al.* (2013), an Express Certificate is a type of autocallable product. Adams & Smith (2019) state that usually a structured product is composed by a bond component and a derivative component. Furthermore, Ribeiro (2018) affirms that the bond component provides capital protection to the holder, while the derivative component allows the holder to benefit from the performance of the underlying asset.

An autocallable product, according to Alm *et al.* (2013) can be automatically called if the underlying reaches a certain barrier condition at the predefined observation dates. If this is the case, the buyer receives a constant payoff, and the product terminates. However, if the condition is not met, the product continues to exist until the next autocall date and so on until maturity.
A callable bond, according to Adams & Smith (2019) and Martelline *et al.* (2003) is a straight bond that contains an embedded call option. The call option gives the issuer the right, but not the obligation to redeem the product before its maturity. Due to this particularity the Express Certificate under study is not a normal callable bond, since it can be automatically called. Therefore, the product can be classified as a type of autocallable bond, as present in Table I.

Martelline *et al.* (2003) notes that when the holder buys a callable bond (long position), they also buy a bond (long position) and sell the call option to the issuer of the product (short position). This makes our product an autocallable bond that can be broken down into a bond plus a call option, as detailed in Table I.

The bond pays an annual fixed coupon of €2,65 and a principal of €100,00, if the product terminates before maturity or at maturity. For this reason, it has a maturity of 5 years, which corresponds to the period between the issue date of the product and the maturity date/term.

The call option is the component that ensures that the Express Certificate is automatically redeemed before maturity. It is only exercised if the underlying is at or above the autocall barrier on the observation dates. Since the call option has predefined autocall observation dates (2022, 2023 and 2024), it can be classified as a Bermudan-style call option with a maturity of 3 years, according to Adams & Smith (2019). Additionally, this call option is an at-the-money option as its strike level is equal to the value of the underlying asset at issuance.

Furthermore, this Express Certificate has another embedded barrier option, which according to Hull (2022) and Ribeiro (2018) is a knock-in option, as it only comes into

existence if the barrier is reached by the underlying asset. Consequently, this barrier option is a down-and-in put option, since its value is lower than the strike level, which is 65% of the value of the underlying at issuance. In addition to that, according to Bellefroid (2022), it can be classified as an at-the-money European barrier option for two reasons: first, it can only be exercised at maturity and, second, its strike value is equal (100%) to the value of the index at issuance.

At maturity, if the DIP option comes into existence, the investor is exposed to the risk of the underlying and receives a payoff linked to its performance. To avoid this, the holder sell the DIP option (short position) to the issuer. This allows the holder to protect their invested capital. However, if the underlying is at or above the barrier level at maturity or if the Express Certificate is early redeemed, the DIP option never comes into existence.

4.4. Express Certificate Valuation

To value the Express Certificate, it is necessary to calculate the value of each component of the product, namely the autocallable bond and the DIP option.

In the first year (2021), the autocallable bond has no predefined autocall observation date, but it does have a coupon payment date. For this reason, we assume that in the first year the product has no risk of being redeemed. Thus, the payoff of the autocallable bond in year 1 is the present value of the coupon discounted at the cost of funding rate. According to Martellini *et al.* (2003), the present value is calculated by the following formula:

$$Present\ Value\ = \frac{CF_t}{(1+r)^n}$$

where CF_t is the future cash flows of the callable bond, r is the cost of the funding rate, and n is the number of periods.

The choice of using the funding cost rate as the discount rate is based on Hull (2022) and has a value of 1,60%. Following the formula provided by Aymanns *et al.* (2016), the value results from the division of the interest expense by the average of the interest-bearing

liabilities from Deutsche Bank's 2019 Annual Report. This report corresponds to one year before the issuance of the product, February 4, 2020.

For the following years, as described in the decomposition of the Express Certificate, the payoff depends on whether or not the trigger of the autocall is active or not. If the product is called, the autocallable bond pays the coupon plus the principal and terminates. Otherwise, if the product survives each autocall date, the autocallable bond pays the coupon and the product continues to exist. At maturity, if the final reference level is at or above the DIP barrier, it pays the last coupon plus the principal. If the final reference level is below the DIP barrier the bond still pays the coupon. With that, according to Adams & Smith (2019), the value of the autocallable bond is the average of the sum of all present values of the payoffs.

As seen in the product decomposition, the DIP option only comes into existence if the underlying at maturity is below the DIP barrier level. Consequently, according to KID, its payoff is directly linked to the performance of the underlying asset, given by the following formula:

$$Payoff\ of\ the\ DIP\ option\ = Product\ Notional\ Amount\ \times \frac{Final\ Reference\ Level}{Strike\ Level}$$

As a result of that, the value of the DIP option is also the average of the present value of all cash flows.

Finally, the value of the Express Certificate is the sum of the value of the callable bond and the value of the down-and-in put option.

4.5. The Probabilities of the Express Certificate

Since this is an autocallable product, there is a possibility of the product terminating before its maturity. For this reason, we estimate the unconditional and conditional probability of the product terminating in each year, as well as the probability of the product ending above or below the DIP barrier in year 5.

For the unconditional probabilities, we consider the possibility of the product terminating in 2022 due to an autocall barrier event. In this case, the investor receives the 2021 coupon and the second coupon from 2022, as well as the principal. For the calculation of the remaining probabilities, we apply the same logic.

According to Hájek (2003), a conditional probability is the possibility of an event occurring, knowing that another event has already occurred. For example, in this project the conditional probabilities are interpreted as the possibility of the product ending in 2023, 2024 and above or below the DIP barrier in 2025, knowing that it survived in 2022.

Based upon the unconditional probabilities, we are able to determine the expected life of the Express Certificate, also known as fugit. Fugit can be considered as the average time that the product is on the market. The expected life is calculated through the sum of the unconditional probabilities for each year times the maturity of each scenario.

4.6. Sensitivity Analysis

The main objective of this sensitivity analysis is to describe in isolation how different levels of volatility, strike and DIP barrier can affect the Express Certificate, from its change in price to the unconditional and conditional probabilities of surviving an autocall event.

As mentioned, volatility is a key parameter for both the underlying asset and the Monte Carlo simulation. In this sensitivity analysis we consider volatilities from 0%, 10%, 20% up to 100%. Using all the above parameters, we run the Monte Carlo simulation for each volatility with 30 time steps and 10.000 paths each. We obtain new EURO STOXX 50 predicted values, in order to determine, as seen previously, the future cash flows of each component of the product as well as their value. Likewise, the new unconditional and conditional probabilities are determined to verify whether the product has a lower possibility of being called and whether it is more likely to remain at or above the DIP barrier at maturity.

The Express Certificate as mentioned in the decomposition, is linked to an underlying asset, the Euro Stoxx 50 Index. In turn, the product is defined by a strike level, K, whose value is 3,732.29, being 100% equal to value of the underlying asset at issuance. It also consists of a DIP barrier, B, with a value of 2,425.982, which corresponds to 65% of the value of the underlying at issuance.

To compute the sensitivity analysis of the product in regard to the strike level, we consider strike levels starting at 70% of the value of the underlying at issuance, 80%, 90% up to 140%, taking into account the strike level of 100%, as a reference value. As for the barrier level of the DIP, we considered values starting from 30% of the value of the underlying asset at issuance, up to 100%, with a barrier of 65% serving as the reference level.

As before, we run the Monte Carlo simulation for each strike level and barrier, in order to describe the behavior of the product value as well as the unconditional and conditional probabilities.

5. RESULTS

This chapter presents all the results of the valuation of the Express Certificate, taking into account the different volatilities of 7,01%, 12,78% and 29,25%, as well as the unconditional and conditional probabilities. Additionally, this chapter presents the results of the sensitivity analysis in relation to the different levels of volatility, strikes and DIP barriers.

5.1. Pricing Results of the Express Certificate

All results presented in this subchapter take into account the different volatilities of the EURO STOXX 50 index historical data. For a better understanding, 30 time steps with 10,000 paths each, μ = -0,647% and δt= 0,167 were used to perform the Monte Carlo simulation, which is the key point to achieve these results.

Possible Scenarios	Cash-flow of the autocallable bond in 2021	Cash-flow of the autocallable bond in 2022	Cash-flow of the autocallable bond in 2023	Cash-flow of the autocallable bond in 2024	Cash-Flow in 2025		**Total Payoff**
					From the autocallable bond	From the Put	
1. Product be called in 2022	€2,65	€102,65					€105,30
2. Product be called in 2023	€2,65	€2,65	€102,65				€107,95
3. Product be called in 2024	€2,65	€2,65	€2,65	€102,65			€110,60
4. a) Product reaches maturity at or above the DIP barrier	€2,65	€2,65	€2,65	€2,65	€102,65		€113,25
4. b) Product reaches maturity below the DIP barrier							
σ = 7,01%	€2,65	€2,65	€2,65	€2,65	€2,65	€62,11	€75,36
σ = 12,78%	€2,65	€2,65	€2,65	€2,65	€2,65	€56,67	€69,62
σ = 29,25%	€2,65	€2,65	€2,65	€2,65	€2,65	€41,49	€54,74

TABLE II: Cash flow map, considering the volatilities of 7,01%, 12,78% and 29,25%.

In Table II, the cash flow from the autocallable bond is always the same in scenarios 1, 2, 3 and 4. a), independently of the volatility used. However, in scenario 4. b) the cash flow from the DIP option differs with volatility.

In the first scenario, the product is early redeemed, since the value of underlying is at or above the barrier on the predefined autocall date in 2022. In this case, the investor receives the coupons of 2021 and 2022 of €2,65 and the principal amount of €100,00, corresponding to a total payoff of €105,30, without discount.

For scenarios 2, 3 and 4. a), we follow the same reasoning. Therefore, if the product survives to the autocall observation date, the investor receives the annual coupon of €2,65 and the product continues to exist. On the following autocall date, if the product is called, the autocallable bond pays the €2,65 coupon and the principal of €100,00, proceeding to terminate. When it reaches maturity at or above the DIP barrier, the investor collects all coupons and the principal, having a maximum payoff of €113,25, without discount.

In scenario 4. b), as described in the decomposition of the product, if the Express Certificate reaches maturity below the DIP barrier, the investor receives the last coupon, but not the principal. Instead, as the DIP option comes into existence, the investor collects the payment that is directly linked to the performance of the underlying. Therefore, at a volatility of 7,01%, the investor obtains a maximum payoff of €75,36, at 12,78% they receive €69,62 and at 29,25% they obtain €54,74. The result from the cash flow of the DIP option as well as its payoff, is an approximation, since the reference level of the underlying in 2025 is still unknown.

Condition	Probability of being called in 2022	Probability of being called in 2023	Probability of being called in 2024	Probability of being called in 2025		Sum of all probabilities
				Above the barrier	Below the barrier	
Unconditional						
σ = 7,01%	41,87%	9,29%	5,19%	43,00%	0,65%	100,00%
σ = 12,78%	42,63%	9,47%	5,27%	32,53%	10,10%	100,00%
σ = 29,25%	39,86%	8,69%	4,91%	14,65%	31,89%	100,00%
Conditional						
Product survives 2022						
σ = 7,01%		15,98%	8,93%	73,97%	1,12%	100,00%
σ = 12,78%		16,51%	9,19%	56,70%	17,61%	100,00%
σ = 29,25%		14,45%	8,16%	24,36%	53,03%	100,00%
Product survives 2023						
σ = 7,01%			10,63%	88,04%	1,33%	100,00%
σ = 12,78%			11,00%	67,91%	21,09%	100,00%
σ = 29,25%			9,54%	28,47%	61,98%	100,00%
Product survives 2024						
σ = 7,01%				98,51%	1,49%	100,00%
σ = 12,78%				76,31%	23,69%	100,00%
σ = 29,25%				31,48%	68,52%	100,00%

TABLE III: Unconditional and Conditional Probabilities.

The unconditional probability present in Table III shows that the Express Certificate has a probability of 41,87% and 42,63% to be called in 2022 with a volatility of 7,01% and 12,78%, respectively. This scenario corresponds to what actually happened to the product when it was on the market. Even so, if the product had reached maturity, it has a

probability of 43,00% with a volatility of 7,01% and 32,53% with a volatility of 12,78% of being at or above the DIP barrier. This allows the investor to receive the maximum return from the Express Certificate. Otherwise, at a volatility of 29,25%, and assuming that the product survives in 2022, it has a likelihood of 31,89% of being below the DIP barrier at maturity.

The conditional probabilities confirm the scenario, knowing that the product survives in 2022, at a volatility of 7,01% and 12,78%, it has a probability of 73,97% and 56,70% to reach the maturity at or above the DIP barrier. Meanwhile, at a higher volatility, the scenario changes and the product has 53,03% of probability of meeting maturity below the DIP barrier. In the meantime, the other scenarios demonstrate that, at the two lower volatilities, the product has a higher probability of being at or above the barrier. In the case of the volatility of 29,25%, the scenario reverses.

As mentioned in the subchapter of the probabilities of the Express Certificate, we can also calculate the expected life of the product, also known as the fugit, from the conditional probabilities, given by Table IV:

Volatility	7,01%	12,78%	29,25%
Expected Life of the Express Certificate (in years)	3,51	3,48	3,58

TABLE IV: Expected Life of the Express Certificate, considering the volatilities of 7,01%, 12,78% and 29,25%.

Through Table IV, we can conclude that considering the higher volatility of 29,25%, the product is on the market for a longer period of time, as the possibility of the product terminating in 2022 decreases.

Figure 3 represents the payoff distribution at maturity, considering volatilities of 7,01%, 12,78% and 29,25%. The graphics reflect the cash-flows of the last year of the product, knowing that it survives in year 4. Therefore, Figure 3 represents the final coupon and principal that comes from the autocallable bond, and also the cash-flow from the option.

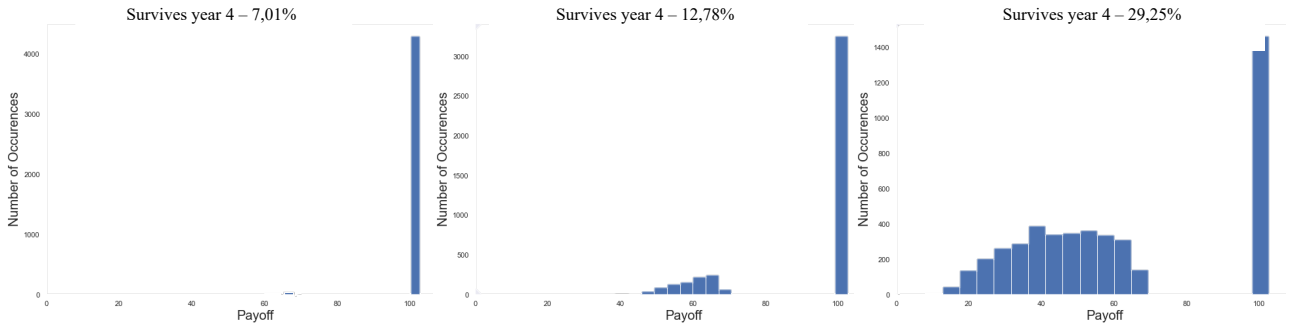

FIGURE 3: Product Payoffs Distribution in 2025, corresponding to volatilities of 7,01%, 12,78%, and 29,25%[6].

With the lowest volatility and with a volatility of 12,78% it can be seen that the majority of the number of occurrences (x-axis) are concentrated on the payoff of €105,30. Therefore, this proves what was described in the conditional probabilities, i.e., at the two lowest volatilities it is more likely that the product will meet its maturity at or above the DIP barrier, allowing the investor to collect the maximum return.

However, as expected from the conditional probabilities at a volatility of 29,25%, the payoff distribution starts to increase with the number of occurrences, since it is likely to have more cases reaching maturity below the DIP barrier.

The Figure 4 below represents the zoom in of Figure 3 of the cash-flows resulting from the DIP option and the final coupon, i.e., only the cases where the Express Certificate ends below the barrier in 2025 are represented. From this we can conclude, that the higher is the volatility, the lower ate the payoffs of the DIP option. In the last chart, we can notice a higher number of occurrences with the lowest payoff.

[6] We have used python software Jupyter Notebook, version 6.4.5 for the figure above.

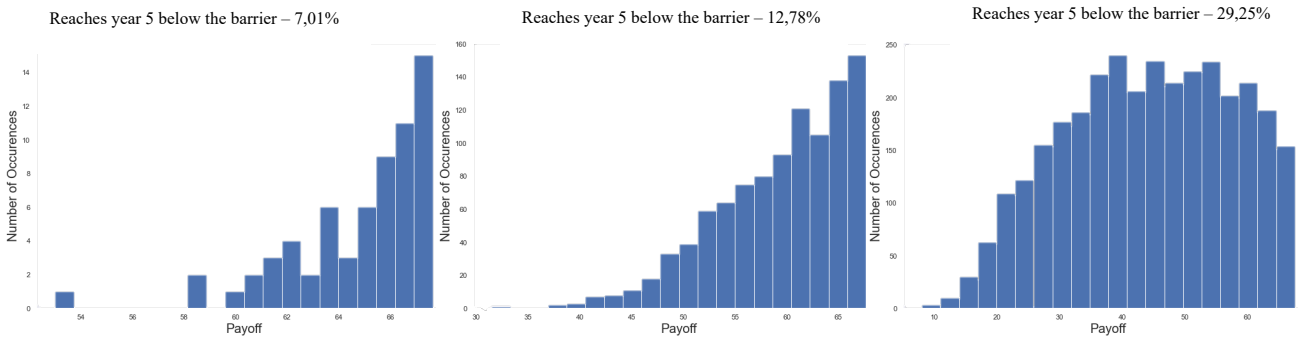

FIGURE 4: Product Payoffs Distribution of the DIP option in 2025, corresponding to volatilities of 7,01%, 12,78% and 29,25%[7].

Table V shows that the value of the autocallable bond is €102,93 when the volatility is 7,01%, €94,17 when the volatility is 12,78%, and €74,15 at a volatility of 29,25%. Moreover, at the same volatilities, the DIP has a value of €0,37, €5,29, and €12,22, respectively. It can be concluded that the value of the DIP option increases, while its payoffs decrease as volatility increases. Hull (2022) explains that this happens due to the fact that the probability of the DIP barrier being reached decreases with the increase in volatility, as observed in the conditional probabilities. For this reason, the holder sell the DIP to the issuer at a higher price, protecting their invested capital. As can be seen from the valuation of the Express Certificate, the value of the product under study is the sum of the autocallable bond and the down-and-in put option, with values of €103,30, €99,46 and €86,37, respectively, given by Table V. Overall, the value of the Express Certificate decreases as volatility increases.

Components Value	σ = 7,01%	σ = 12,78%	σ = 29,25%
Autocallable Bond	€102,93	€94,17	€74,15
DIP Option	€0,37	€5,29	€12,22
Express Certificate	€103,30	€99,46	€86,37

TABLE V: Pricing results of the Express Certificate.

[7] We have used python software Jupyter Notebook, version 6.4.5 for the figure above.

5.2. Pricing Results of the Sensitivity Analysis

This subchapter presents the results of the sensitivity analysis, describing in isolation how different levels of volatility, σ, strike, K, and the DIP barrier, B, affect the unconditional and conditional probabilities and the value of the Express Certificate.

As seen in the methodology, volatility is a key parameter for both the underlying and the Monte Carlo simulation. For the sensitivity analysis of the volatility, we consider volatilities from 0%, 10%, 20% up to 100%, as shown on the x-axis of Figure 5, present below. In addition, for each volatility, Figure 5 demonstrates the unconditional probability of the product being called in each year and whether it is more likely to reach maturity at, above, or below the DIP barrier (through each bar).

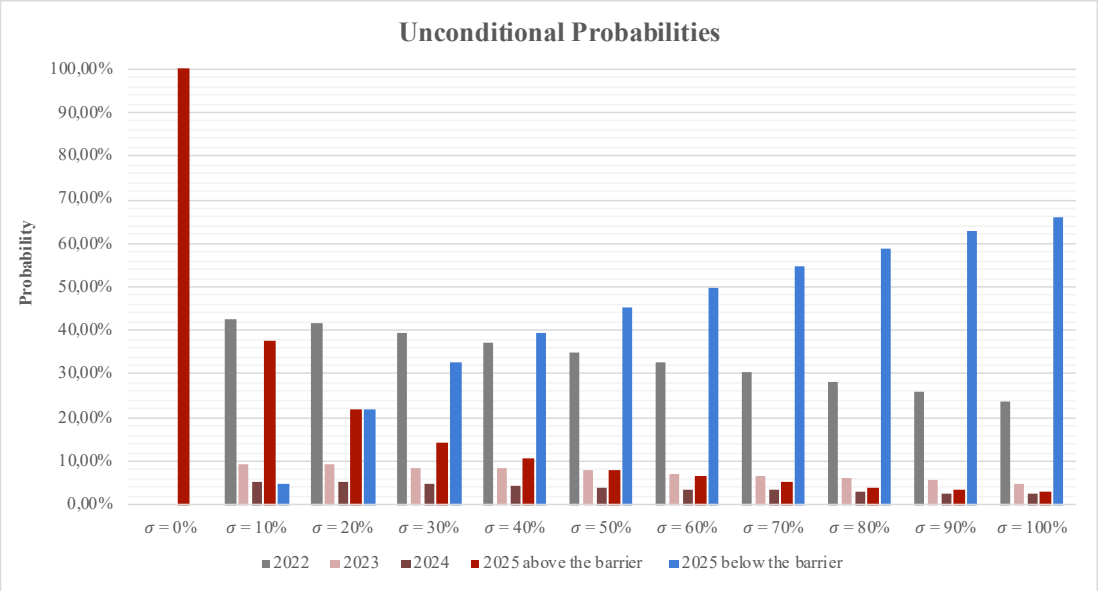

FIGURE 5: Unconditional probabilities considering volatilities from 0% up to 100%[8].

With a volatility of 0%, the product is expected to be called in 2022 with a likelihood of 100%, as result of the drift-term present in the Equation (3). However, as the drift-term used in this project is negative (-0,647), it can be observed in the Monte Carlo simulation that the predicted values decrease slightly over the 30 time steps considered. This results

[8] I used Microsoft Excel software, version 16.75 to create the figures in this subchapter.

in the Express Certificate having a 100% possibility of reaching maturity at or above the DIP barrier.

Figure 5 further shows that, when considering σ = 10% and σ = 20%, the possibility of the product being called in 2022 is over 40%. However, at σ = 10%, the product also has a higher chance of reaching maturity at or above the barrier. In contrast, with a σ = 20%, the product has the same probability of being above or below the DIP barrier in 2025.

Using the remaining volatilities, the probability of the product being called in 2022 starts to decrease. Conversely, from a σ = 40% to a σ = 100%, the possibility that the Express Certificate achieves its maturity below the DIP barrier increases.

Based on the unconditional probabilities, we can see that independently of the volatility used, the likelihood of the product being called in 2022 is significantly higher when compared to 2023 and 2024. Furthermore, we can also see that as volatility increases, the probability of the product reaching maturity below the DIP barrier grows, according to Table A.I, in the appendix.

To better understand the details, Figure 6 demonstrates the conditional probabilities taking into account some volatilities. As observed with the unconditional probabilities, when σ = 0% and σ = 10% are used, the product has a higher probability of reaching maturity at or above the DIP barrier, knowing that it has already survived the autocall date in 2022. Figure 6 also shows that the possibility of the product achieving maturity below the barrier is very high when volatilities up to 50% are used.

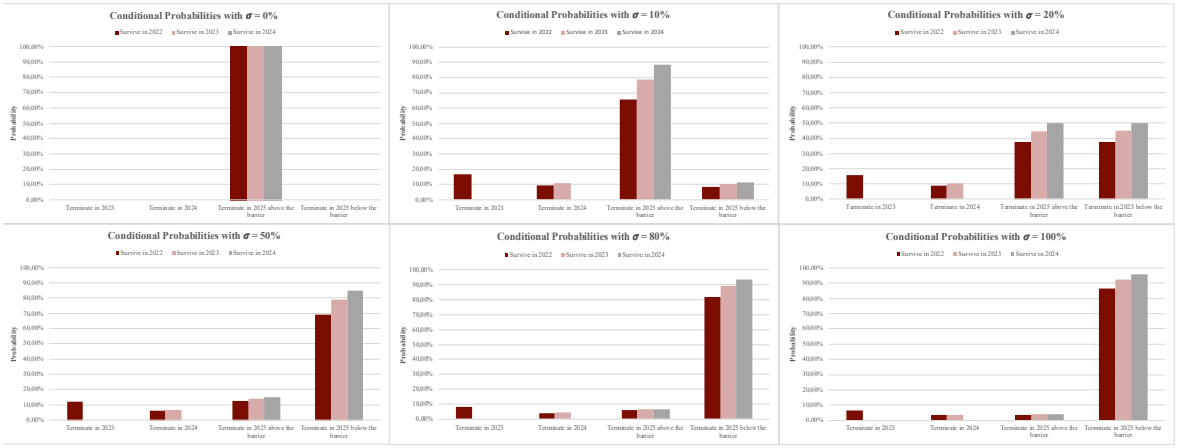

FIGURE 6: Conditional probabilities, considering the volatilities of 0%, 10%, 20%, 50%, 80%, and 100%.

As for the pricing results, Figure 7 below shows that, as expected, the value of the autocallable bond declines as volatility increases, as does the value of the Express Certificate itself. The situation is different for the value of the DIP option, whose value increases from σ = 0% to σ = 30% and then slowly declines with the remaining volatilities. In the previous pricing results of the Express Certificate, we found that the value of the DIP option increases at volatilities of 7,01%, 12,78% and 29,25%. Meanwhile, in this sensitivity analysis, it is possible to conclude that the value of the option starts to decrease with the increase of the volatility, due to the fact that the chance of the product meeting the DIP barrier is very low, as shown in more detailed in Table A.II in the appendix.

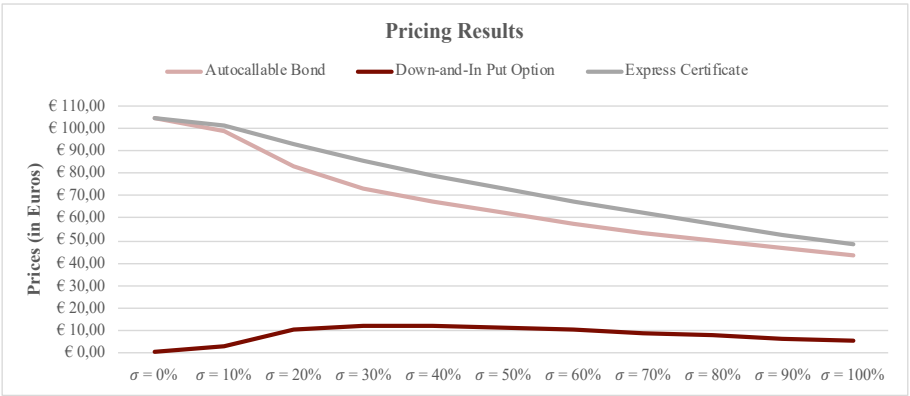

FIGURE 7: Pricing results of the sensitivity analysis of the different volatility levels.

For the sensitivity analysis of the strike level, we take into account strike levels from 70% of the value of the underlying asset at issuance up to 140%, having a K=100% as reference value.

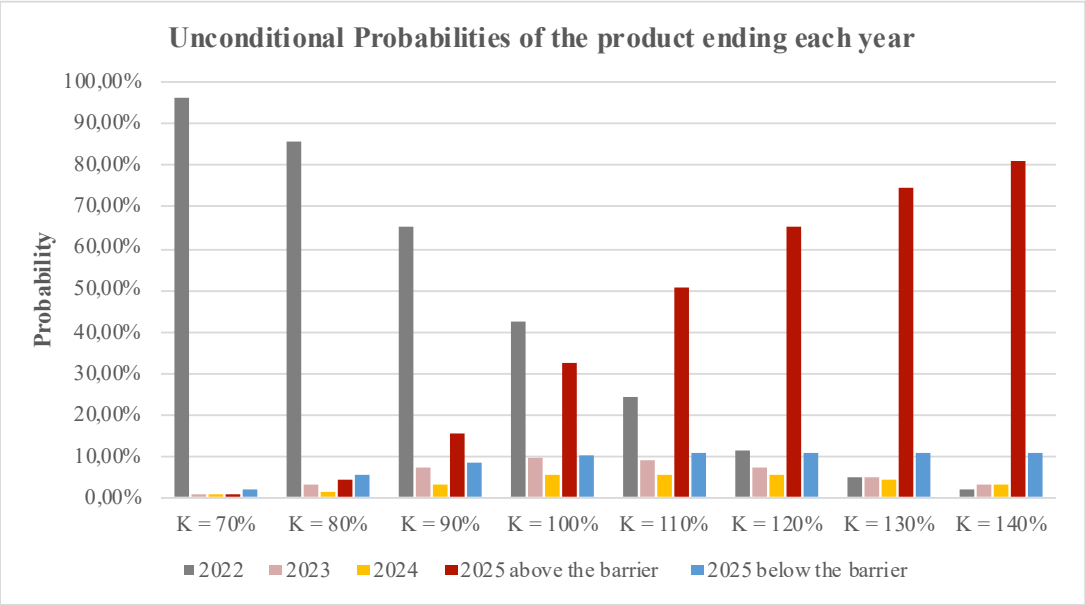

FIGURE 8: Unconditional probabilities considering strike levels from 70% up to 140% of the value the underlying asset at issuance.

In Figure 8, it is possible to observe that using strikes levels below K=100% results in the Express Certificate presenting a higher possibility of being called in 2022. However, as seen previously, a strike level of 100% increases the probability of the product reaching maturity at or above the DIP barrier.

Meanwhile, considering a strike level from 110% changes the scenario and the likelihood of the product being called in 2022 starts to gradually decline. Furthermore, at these levels, the product has a higher chance of achieving maturity at or above the DIP barrier.

Due to the unconditional probabilities, it is possible to conclude that independently of the strike level used, the possibility of the product reaching maturity below the DIP barrier is very low. Unconditional probabilities are presented in Table A.III in more detail at appendix.

Below, Figure 9 represents the conditional probabilities concerning some of the strike levels. Knowing that the product survives the autocall date in 2022, using a K=70% the product has a higher probability of reaching maturity below the DIP barrier. When using strike levels from K=90%, there is a high possibility of the product reaching maturity at or above the barrier.

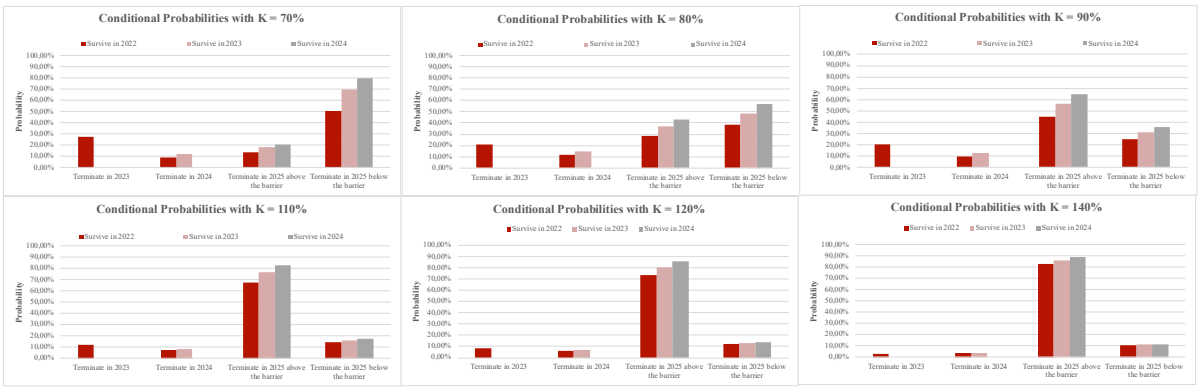

FIGURE 9: Conditional probabilities, considering the strike levels from 70% up to 90% and from 110% up to 140%.

A sensitivity analysis of the strike levels shows that the value of the Express Certificate hardly changes independently of the strike used. Instead, it remains constant over the various levels of K, as can be seen in Figure 10.

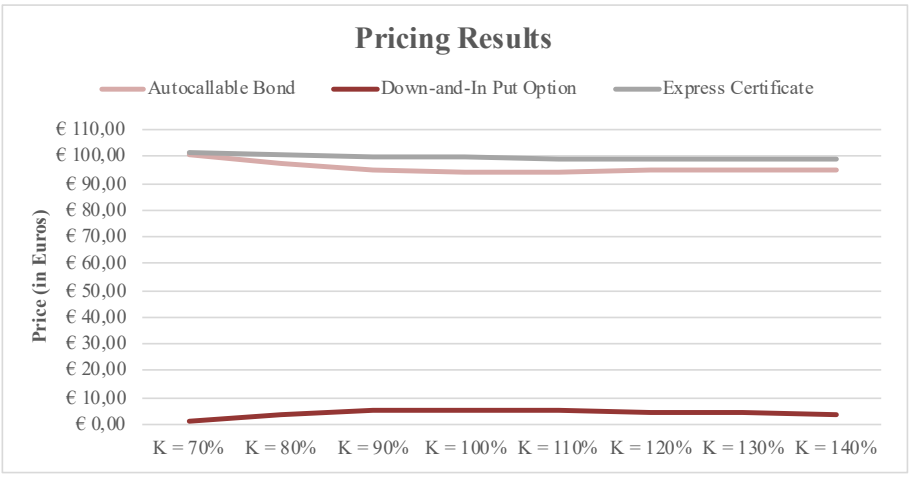

FIGURE 10: Pricing results of the sensitivity analysis of the strike level.

Furthermore, the value of the autocallable bond decreases slightly from K=80% to K=110% and then tends to remain constant. In the case of the DIP option, its value starts to increase with the increase of the strike levels and subsequently slowly declines from K=100%. It is possible to see the pricing results in more detail in Table A.IV in appendix.

The Express Certificate is also defined by a barrier level of the DIP option. For this reason, we perform a sensitivity analysis of different barrier levels starting at 30% of the value of the underlying asset at issuance up to 100%, with a B=65% as a reference level.

The left graph in Figure 11 shows the unconditional probabilities of the Express Certificate in each year. As the DIP barrier can only come into existence in 2025 the probability of the product being called in 2022, 2023, and 2024 remains the same considering the different barrier levels. For this reason, the right chart in Figure 11 demonstrates the unconditional probabilities of the Express Certificate in 2025 only.

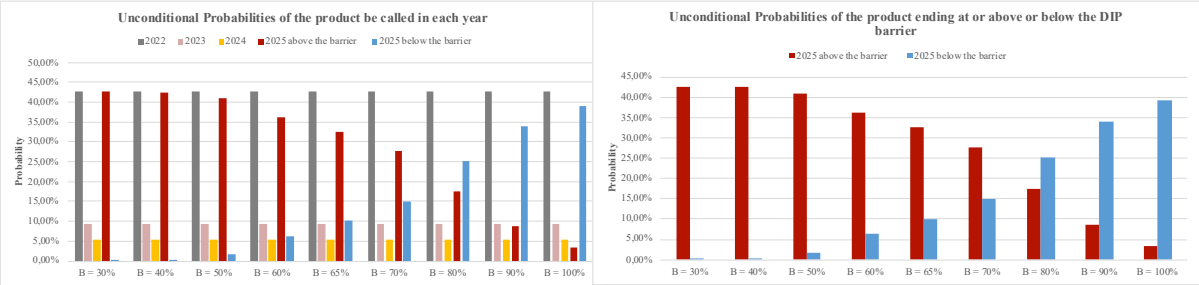

FIGURE 11: Unconditional probabilities considering barrier levels from 30% to 100% of the value of the underlying asset at issuance.

From B=30% up to B=70%, the product has a higher likelihood of being at or above the DIP barrier in 2025. This way, the investor can collect the maximum payoff of the Express Certificate, while from a B=80%, the opposite happens.

Moreover, the left chart in Figure 11 shows that the product has a higher possibility of being called in 2022. Considering a B=30% and B=40%, the Express Certificate presents the same probability of being called in 2022 and met the maturity at or above the barrier. Table A.V presents the unconditional probabilities in a more detailed form in appendix.

Figure 12 below presents the conditional probabilities in year 5 of the barrier levels of 40%, 70%, 80% and 100%. As expected from the unconditional probabilities, under the condition that the product survives in 2022, it has a higher chance of being at or above the DIP barrier considering lower barrier levels. This probability reverses considering barriers from B=80%.

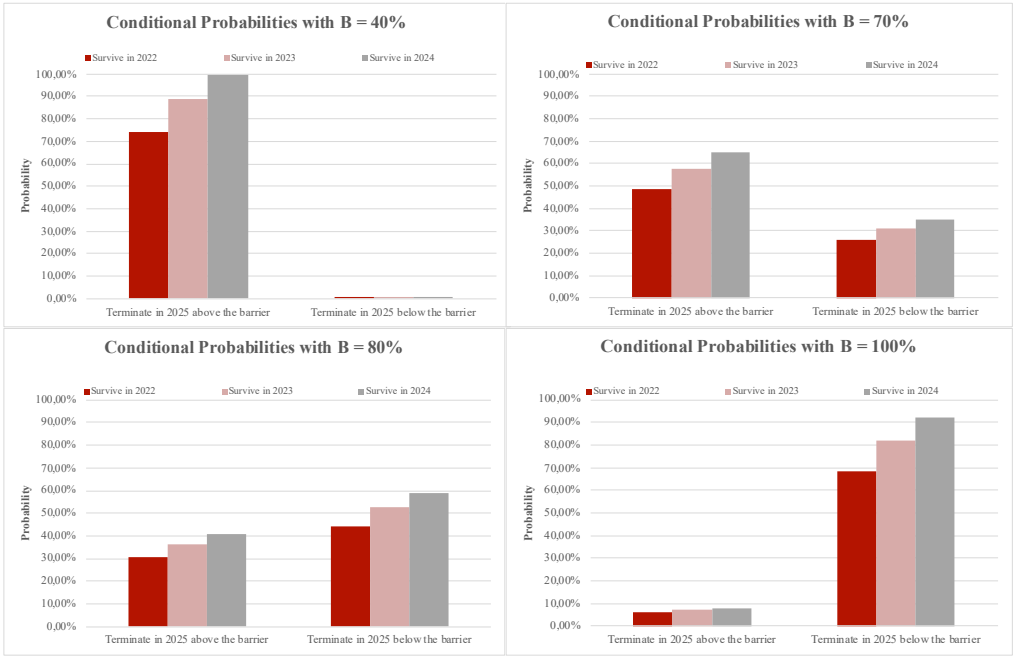

FIGURE 12: Conditional probabilities considering the barrier levels of 40%, 70%, 80% and 100%.

The pricing results of this sensitivity analysis present in Figure 13 reflect what was confirmed in the unconditional and conditional probabilities.

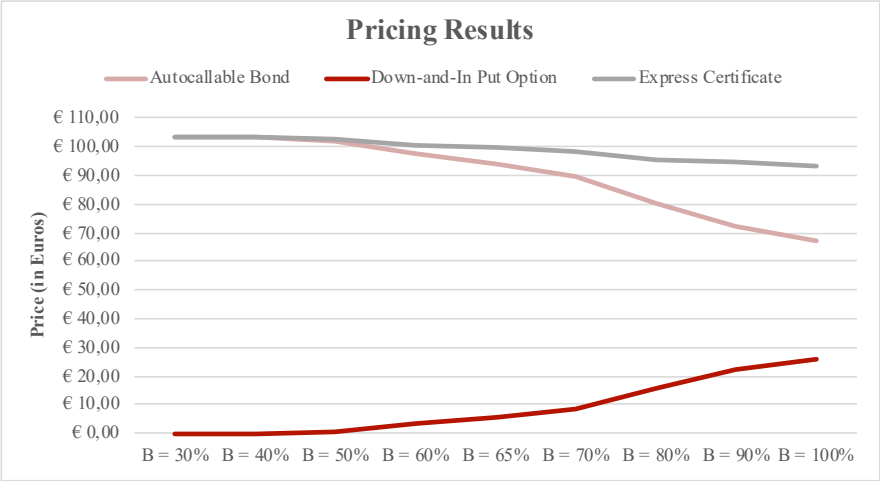

FIGURE 13: Pricing results of the sensitivity analysis of the barrier levels.

It can be seen that the value of Express Certificate itself, the value of the autocallable bond, and the DIP option are constant from B=30% up to B=50%. After that, from B=60%, the value of the Express Certificate starts to slowly decrease and the value of the autocallable bond declines with the increase of the barrier levels. Consequently, the value of the DIP option increases significantly. As we have seen previously, the behavior of the value of the DIP increases as expected, since from B=70% it is almost impossible for the product to meet the DIP barrier. For a deeper analysis, Table A.VI in appendix presents all the prices over the different barrier levels.

6. CONCLUSION

In financial markets, there is a desire among investors for alternative investments, and structured products are a welcome choice. This project described an Express Certificate, which is one of the modern structured products. The return of this product is derived from the performance of an underlying asset and bridges the gap between a pure investment in bonds, which offers a return with low risk, and a pure investment in derivative instruments, which brings higher returns but also a higher risk. Due to the complex structure of these types of products, a lot of knowledge is required to understand their payoff and decomposition.

The goal of this project was to perform a valuation of the Express Certificate using the Monte Carlo simulation as a pricing model, as well as a sensitivity analysis of different levels of volatility, strike and barrier. Once the product under study is linked to an underlying asset, the historical volatilities of 7,01%, 12,78%, and 29,25% are applied to the pricing model. This allows us to perform the predicted values for the calculation of the futures cash flows from the autocallable bond and from the DIP option.

When calculating the unconditional probabilities, we can see that the product has a higher probability of being called in 2022. Afterwards, in the conditional probabilities, knowing that the product already survived in 2022, the whole scenario changes and at lower volatilities, the product has a higher probability of reaching maturity at or above the DIP barrier.

Through the pricing results of the Express Certificate, we can conclude that the value of the product decreases with the increase of the volatility from €103,30, €99,46 to €86,77. Additionally, the value of the autocallable bond decreases and consequently the value of the DIP option increases.

In the unconditional probabilities of the sensitivity analysis of the volatility, considering σ=0%, we can verify that a simple change in a signal in the drift-term can affect what was expected in 2022, which was that of the product terminating with 100% of the guarantee.

Moreover, we can observe that considering a volatility starting from 20%, the probability of the product reaching maturity below the DIP barrier is high, reflected in its value, which starts to decline.

Regarding the sensitivity analysis of the strike level, we can note that from K=100%, the Express Certificate has a higher chance of reaching maturity at or above the DIP barrier. However, it can be observed that the value of the product, the autocallable bond and the DIP option remain constant with respect to the strike levels. Concerning the barrier level, from B=30% to B=70% there is a probability of the product reaching maturity at or above the DIP barrier. After B=70% the scenario changes together with the value of the Express Certificate.

Finally, through all the analysis performed in this project, given the unconditional probabilities it appears that the Express Certificate has a higher possibility of being called in 2022. In the real market, this situation actually occurred, and the product was called and terminated in 2022. This leads to the following question: does the product have a risk design of being called on the first autocall date? With this in mind, for a future research other autocallable products could be analysed in order to verify this risk.

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APPENDIX

FIGURE A.1: Key Information Document (KID) of the Express Certificate.

Key Information Document

Purpose

This document provides you with key information about this investment product. It is not marketing material. The information is required by law to help you understand the nature, risks, costs, potential gains and losses of this product and to help you compare it with other products.

Product

Product name	Fixed Coupon Express Certificate linked to EURO STOXX 50® Index (Price Index)
Product identifiers	ISIN: DE000DB9U0V9 | WKN: DB9U0V
PRIIP manufacturer	Deutsche Bank AG. The product issuer is Deutsche Bank AG, Frankfurt.
Website	www.db.com/contact
Telephone number	Call +49-69-910-00 for more information.
Competent authority of the PRIIP manufacturer	German Federal Financial Supervisory Authority (BaFin)
Date of production	7 December 2021

You are about to purchase a product that is not simple and may be difficult to understand.

1. What is this product?

Type

German law governed certificates

Objectives

(Terms that appear in **bold** in this section are described in more detail in the table(s) below.)

The product is designed to provide a return in the form of (1) regular fixed coupon payments and (2) a cash payment on termination of the product. The timing and amount of this payment will depend on the performance of the **underlying**.

Early termination following an autocall: The product will terminate prior to the **maturity date** if, on any **autocall observation date**, the **reference level** is at or above the relevant **autocall barrier level**. On any such early termination, you will on the immediately following **autocall payment date** receive, in addition to a final coupon payment, a cash payment equal to the autocall payment of EUR 100. No coupon payments will be made on any date after such **autocall payment date**. The relevant dates and **autocall barrier levels** are shown in the table(s) below.

Autocall observation dates	*Autocall barrier levels*	*Autocall payment dates*
2 February 2022	3,732.28	7 February 2022
1 February 2023	3,732.28	6 February 2023
1 February 2024	3,732.28	6 February 2024
3 February 2025	2,425.982	Maturity date

Coupon: If the product has not terminated early, on each **coupon payment date** you will receive a coupon payment of EUR 2.65. The coupon payments are not linked to the performance of the **underlying**. The relevant dates are shown in the table(s) below.

Coupon payment dates
8 February 2021
7 February 2022
6 February 2023
6 February 2024
Maturity date

Termination on the maturity date: If the product has not terminated early, on the **maturity date** you will receive:

1. if the **final reference level** is at or above the **barrier level**, a cash payment equal to EUR 100; or
2. if the **final reference level** is below the **barrier level**, a cash payment directly linked to the performance of the **underlying**. The cash payment will equal (i) the **product notional amount** multiplied by (ii) (A) the **final reference level** divided by (B) the **strike level**.

Under the product terms, certain dates specified above and below will be adjusted if the respective date is either not a business day or not a trading day (as applicable). Any adjustments may affect the return, if any, you receive.

When purchasing this product during its lifetime, the purchase price may include accrued coupon on a pro rata basis.

You do not have any entitlement to a dividend from the **underlying** and you have no right to any further entitlement resulting from the **underlying** (e.g., voting rights).

Underlying	EURO STOXX 50 (Price return index) (ISIN: EU0009658145)	**Strike level**	3,732.28
Underlying market	Equity	**Barrier level**	2,425.982
Product notional amount	EUR 100	**Reference level**	The closing level of the **underlying** as per the **reference source**
Product currency	Euro (EUR)	**Reference source**	STOXX Limited, Zurich
Underlying currency	Euro (EUR)	**Final reference level**	The **reference level** on the **valuation date**
Issue date	4 February 2020	**Valuation date**	3 February 2025
Initial reference level	3,732.28	**Maturity date / term**	6 February 2025

The issuer may terminate the product with immediate effect in the event of obvious written or mathematical errors in the terms and conditions or if certain extraordinary events provided in the terms and conditions occur. Examples of extraordinary events include (1) material changes, particularly in connection with the **underlying**, including where an index ceases to be calculated, and (2) events, in

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particular due to changes in certain external conditions that hinder the issuer in meeting its obligations in connection with the product or – depending on the terms and conditions of the security – otherwise affect the product and/or the issuer. In case of immediate termination, the return (if any) may be significantly lower than the purchase price, but will reflect the product's market value and, if higher, any minimum redemption (alternatively, in some cases the corresponding compounded amount may be paid out at the product's scheduled maturity). Instead of immediate termination, the issuer may also amend the terms and conditions.

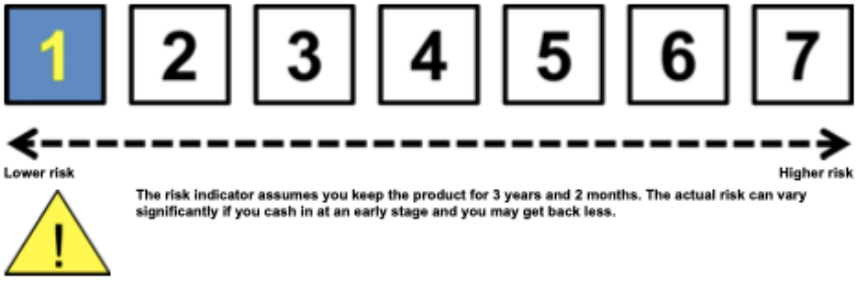
Provided that in the event of any inconsistency and/or conflict between the foregoing paragraph and any applicable law, order, rule or other legal requirement of any governmental or regulatory authority in a territory in which this product is offered, such national requirements shall prevail.

Intended retail investor

The product is intended for private clients who pursue the objective of general capital formation/asset optimization and have a medium-term investment horizon. This product is a product for clients who have sufficient knowledge and / or experience to make an informed investment decision. The investor can bear losses up to the total loss of the capital invested and attaches no importance to capital protection.

2. What are the risks and what could I get in return?

Risk indicator



The risk indicator assumes you keep the product for 3 years and 2 months. The actual risk can vary significantly if you cash in at an early stage and you may get back less.

The summary risk indicator is a guide to the level of risk of this product compared to other products. It shows how likely it is that the product will lose money because of movements in the markets or because we are not able to pay you.

We have classified this product as 1 out of 7, which is the lowest risk class. This rates the potential losses from future performance at a very low level, and poor market conditions are very unlikely to impact our capacity to pay you.

This product does not include any protection from future market performance so you could lose some or all of your investment.

If we are not able to pay you what is owed, you could lose your entire investment.

Performance scenarios

Market developments in the future cannot be accurately predicted. The scenarios shown are only an indication of some of the possible outcomes based on recent returns. Actual returns could be lower.

Investment: EUR 10,000

Scenarios		*1 year*	*2 years*	*3 years and 2 months (Recommended holding period)*
Stress scenario	**What you might get back after costs**	EUR 8,339.17	EUR 9,058.79	EUR 10,161.35
	Average return each year	-16.61%	-4.82%	0.51%
Unfavourable scenario	**What you might get back after costs**	EUR 9,977.56	EUR 9,962.67	EUR 10,161.35
	Average return each year	-0.22%	-0.19%	0.51%
Moderate scenario	**What you might get back after costs**	EUR 10,171.53	EUR 10,182.25	EUR 10,161.35
	Average return each year	1.72%	0.91%	0.51%
Favourable scenario	**What you might get back after costs**	EUR 10,174.80	EUR 10,208.30	EUR 10,161.35
	Average return each year	1.75%	1.04%	0.51%

This table shows the money you could get back over the next 3 years and 2 months under different scenarios, assuming that you invest EUR 10,000.

The scenarios shown illustrate how your investment could perform. You can compare them with the scenarios of other products. The scenarios presented are an estimate of future performance based on evidence from the past on how the value of this investment varies, and are not an exact indicator. What you get will vary depending on how the market performs and how long you keep the product. The stress scenario shows what you might get back in extreme market circumstances, and it does not take into account the situation where we are not able to pay you.

The figures shown include all the costs of the product itself, but may not include all the costs that you pay to your advisor or distributor. The figures do not take into account your personal tax situation, which may also affect how much you get back.

3. What happens if Deutsche Bank AG, Frankfurt is unable to pay out?

You are exposed to the risk that the issuer might be unable to fulfil its obligations in respect of the product – e.g. in the event of insolvency (inability to pay / over-indebtedness) or an administrative order of resolution measures. In case of a crisis of the issuer such an order can also be issued by a resolution authority in the run-up of an insolvency proceeding. In doing so, the resolution authority has extensive intervention powers. Among other things, it can reduce rights of the investors to zero, terminate the product or convert it into shares of the issuer and suspend rights of the investors. With regard to the basic ranking of the issuer's obligations in the event of action by the resolution authority, please see www.bafin.de and search for the keyword "Haftungskaskade". A total loss of your capital invested is possible. The product is a debt instrument and as such is not covered by any deposit protection scheme.

4. What are the costs?

The Reduction in Yield (RIY) shows what impact the total costs you pay will have on the investment return you might get. The total costs take into account one-off, ongoing and incidental costs.

The amounts shown here are the cumulative costs of the product itself, for three different holding periods. They include potential early exit penalties. The figures assume you invest EUR 10,000. The figures are estimates and may change in the future.

Costs over time

Investment: EUR 10,000			
Scenarios	***If you cash in after 1 year***	***If you cash in after 2 years***	***If you cash in at the end of the recommended holding period***
Total costs	EUR 0.00	EUR 0.00	EUR 0.00
Impact on return (RIY) per year	0.00%	0.00%	0.00%

The costs shown in the table above represent how much the expected costs of the product would affect your return, assuming the product performs in line with the moderate performance scenario.

The person selling you or advising you about this product may charge you other costs. If so, this person will provide you with information about these costs, and show you the impact that all costs will have on your investment over time.

Composition of costs

The table below shows:

- The impact each year of the different types of costs on the investment return you might get at the end of the recommended holding period.
- The meaning of the different cost categories.

The table shows the impact on return per year.			
One-off costs	*Entry costs*	0.00%	The impact of the costs already included in the price.
	Exit costs	0.00%	The impact of the costs of exiting your investment when it matures.
Ongoing costs	*Portfolio transaction costs per year*	0.00%	The impact of the costs of us buying and selling underlying investments for the product.
	Other ongoing costs	0.00%	The impact of the costs that we take each year for managing your investments.

5. How long should I hold it and can I take money out early?

Recommended holding period: 3 years and 2 months

The product aims to provide you with the return described under "1. What is this product?" above. However, this only applies if the product is held to maturity. It is therefore recommended that the product is held until 6 February 2025 (maturity).

The product does not guarantee the possibility to disinvest other than by selling the product either (1) through the exchange (where the product is listed) or (2) off-exchange. No fees or penalties will be charged by the issuer for any such transaction. However if you sell the product in the secondary market you will incur a bid/offer spread. By selling the product before its maturity, you may receive back less than you would have received if you had kept the product until maturity.

Exchange listing	Börse Stuttgart and Deutsche Börse AG	**Last exchange trading day**	31 January 2025 (Börse Stuttgart) and 31 January 2025 (Deutsche Börse AG)
Smallest tradable unit	1 unit	**Price quotation**	Units

In volatile or unusual market conditions, or in the event of technical faults/disruptions, the purchase and/or sale of the product can be temporarily hindered and/or suspended and may not be possible at all.

6. How can I complain?

Any complaint regarding the conduct of the person advising on, or selling, the product can be submitted directly to that person.

Any complaint regarding the product or the conduct of the manufacturer of this product can be submitted in writing at:

Jurisdiction	Postal address	Email address	Website
Germany	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Austria	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Luxembourg	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com

7. Other relevant information

Any additional documentation in relation to the product and in particular the prospectus, any supplements thereto and the final terms are published on the manufacturer's website (www.xmarkets.db.com/DocumentSearch ; after entering of the respective ISIN or WKN), all in accordance with legal requirements. In order to obtain more detailed information - and in particular details of the structure and risks associated with an investment in the product - you should read these documents. These documents are also available free of charge from Deutsche Bank AG, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany, in accordance with legal requirements.

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TABLE A.I: Detailed unconditional probabilities considering considering volatilities from 0% up to 100%.

Unconditional Probabilities	Probability of being called in 2022	Probability of being called in 2023	Probability of being called in 2024	Probability of being called in 2025		Sum of all probabilities
				Above the barrier	Below the barrier	
$\sigma = 0\%$	0,00%	0,00%	0,00%	100,00%	0,00%	100,00%
$\sigma = 10\%$	42,63%	9,44%	5,28%	37,76%	4,89%	100,00%
$\sigma = 20\%$	41,60%	9,23%	5,13%	21,97%	22,07%	100,00%
$\sigma = 30\%$	39,59%	8,64%	4,87%	14,33%	32,57%	100,00%
$\sigma = 40\%$	37,41%	8,23%	4,35%	10,71%	39,30%	100,00%
$\sigma = 50\%$	34,93%	7,90%	3,93%	8,12%	45,12%	100,00%
$\sigma = 60\%$	32,70%	7,20%	3,60%	6,53%	49,97%	100,00%
$\sigma = 70\%$	30,48%	6,47%	3,26%	5,14%	54,65%	100,00%
$\sigma = 80\%$	28,08%	6,00%	2,93%	4,11%	58,88%	100,00%
$\sigma = 90\%$	25,77%	5,51%	2,80%	3,33%	62,59%	100,00%
$\sigma = 100\%$	23,56%	4,95%	2,62%	2,82%	66,05%	100,00%

TABLE A.II: Detailed pricing results of the sensitivity analysis of the volatility.

Components Value	$\sigma = 0\%$	$\sigma = 10\%$	$\sigma = 20\%$	$\sigma = 30\%$	$\sigma = 40\%$	$\sigma = 50\%$	$\sigma = 60\%$	$\sigma = 70\%$	$\sigma = 80\%$	$\sigma = 90\%$	$\sigma = 100\%$
Autocallable Bond	€105,00	€98,99	€83,16	€73,53	€67,39	€62,10	€57,70	€53,46	€49,64	€46,29	€43,18
DIP Option	€0,00	€2,68	€10,14	€12,30	€11,97	€11,14	€9,87	€8,70	€7,45	€6,21	€5,12
Express Certificate	€ 105,00	€ 101,67	€ 93,30	€ 85,83	€ 79,36	€ 73,24	€ 67,57	€ 62,16	€ 57,09	€ 52,51	€ 48,30

TABLE A.III: Detailed unconditional probabilities considering strike levels from 70% up to 140%.

Unconditional Probabilities	Probability of being called in 2022	Probability of being called in 2023	Probability of being called in 2024	Probability of being called in 2025		Sum of all probabilities
				Above the barrier	Below the barrier	
K = 70%	96,29%	1,02%	0,33%	0,49%	1,87%	100,00%
K = 80%	85,61%	3,05%	1,66%	4,15%	5,53%	100,00%
K = 90%	65,51%	6,97%	3,41%	15,55%	8,56%	100,00%
K = 100%	42,63%	9,47%	5,27%	32,53%	10,10%	100,00%
K = 110%	24,54%	8,87%	5,33%	50,73%	10,53%	100,00%
K = 120%	11,59%	7,19%	5,38%	65,25%	10,59%	100,00%
K = 130%	5,12%	4,97%	4,41%	74,88%	10,62%	100,00%
K = 140%	1,96%	2,87%	3,36%	81,18%	10,63%	100,00%

TABLE A.IV: Detailed pricing results of the sensitivity analysis of the different strike levels.

Components Value	K = 70%	K = 80%	K = 90%	K = 100%	K = 110%	K = 120%	K = 130%	K = 140%
Autocallable Bond	€100,41	€97,29	€94,99	€94,17	€94,32	€94,68	€94,90	€95,03
DIP Option	€1,30	€3,51	€4,95	€5,29	€5,02	€4,63	€4,29	€3,99
Express Certificate	€ 101,71	€ 100,80	€ 99,94	€ 99,46	€ 99,34	€ 99,31	€ 99,19	€ 99,02

TABLE A.V: Detailed unconditional probabilities considering barrier levels from 30% to 100%.

				Probability of being called in 2025		
Unconditional Probabilities	Probability of being called in 2022	Probability of being called in 2023	Probability of being called in 2024	Above the barrier	Below the barrier	Sum of all probabilities
B = 30%	42,63%	9,47%	5,27%	42,62%	0,01%	100,00%
B = 40%	42,63%	9,47%	5,27%	42,50%	0,13%	100,00%
B = 50%	42,63%	9,47%	5,27%	40,99%	1,64%	100,00%
B = 60%	42,63%	9,47%	5,27%	36,27%	6,36%	100,00%
B = 65%	42,63%	9,47%	5,27%	32,53%	10,10%	100,00%
B = 70%	42,63%	9,47%	5,27%	27,74%	14,89%	100,00%
B = 80%	42,63%	9,47%	5,27%	17,43%	25,20%	100,00%
B = 90%	42,63%	9,47%	5,27%	8,67%	33,96%	100,00%
B = 100%	42,63%	9,47%	5,27%	3,45%	39,18%	100,00%

TABLE A.VI: Detailed pricing results of the sensitivity analysis of the different barrier levels.

Components Value	B = 30%	B = 40%	B = 50%	B = 60%	B = 65%	B = 70%	B = 80%	B = 90%	B = 100%
Autocallable Bond	€103,49	€103,38	€101,99	€97,63	€94,17	€89,75	€80,23	€72,14	€67,32
DIP Option	€0,00	€0,04	€0,70	€3,12	€5,29	€8,27	€15,42	€22,27	€25,83
Express Certificate	€ 103,50	€ 103,43	€ 102,68	€ 100,75	€ 99,46	€ 98,02	€ 95,65	€ 94,41	€ 93,14