Lisbon School of Economics & Management
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MASTER
MASTER'S IN FINANCE

MASTER´S FINAL WORK
PROJECT

Valuation and Risk Analysis of:

FIXED COUPON EXPRESS CERTIFICATE LINKED TO EURO STOXX 50 INDEX

CATARINA DELGADO VAZ SIMÕES MARCÃO

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SUPERVISION:
PROF. RAQUEL MEDEIROS GASPAR

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To my parents, for always believing in me.

To my little sister, who motivates me every day.

To my boyfriend, for always supporting me.

To my close family and friends.

Glossary:

CF – Cash-flow.

CL – Confidence level.

CVaR – Conditional value-at-risk.

DB – Deutsche Bank.

DIP – Down-and-in put.

EC – European Commission.

EL – Expected life.

IRR – Interest rate of return.

K – Strike.

KID – Key information document.

NPV – Net present value.

PRIIP – Packaged retail and insurance-based investment products.

SPP – Short put product.

SRI – Summary risk indicator.

VaR – Value-at-risk.

Abstract

This project has the primary objective of valuating and analysing an Express Certificate, a type of autocallable structured product. This type of product is mostly known and traded in German markets and its not a common object of academic or professional research.

The product consists of an autocallable certificate that is intended to pay a regular annual coupon along with the notional amount at maturity or early termination. The product can be called back for early termination if the underlying level is at or above the strike price on an autocallable observation date. On the last observation date, if the underlying level is below the barrier level, the payoff is directly linked to the underlying asset.

The valuation process starts with the decomposition of the Express Certificate into a callable bond along with a short position in a down-and-in put option. The pricing of the product was performed using Monte Carlo simulation with Geometric Brownian Motion. The further analysis of the certificate focuses on risk and return measures, namely stress testing, internal rate of return, value-at-risk and conditional value-at-risk and, finally, greek sensitivities.

The previously mentioned analysis was performed in 5 different models, each with 5 different volatilities, while all other assumptions remained the same. 2 of these models were the product of a factor push stress testing.

The principal conclusions drawn from the project are the consequences of an increase in volatility, which are mainly: the decrease in the product's value, the decrease of the interest rate of return, an increase in value at risk and a decrease in overall greek sensitivity.

KEYWORDS: Structured Product; Express Certificate; Monte Carlo Simulation; Greek Letters; Value-at-risk.

JEL Codes: G11; G12; G17.

Resumo

O presente projeto tem como objetivo primário a analise de um Certificado Express, um tipo de produto estruturado com possível chamada antecipada. Este tipo de produto é mais conhecido e transacionado no mercado alemão e não é um objeto comum de investigação académica ou profissional.

O produto consiste num certificado com possível chamada antecipada, estruturado de modo a pagar um cupão anual juntamente com o principal na maturidade ou data de chamada antecipada. O produto pode ser chamado antes da maturidade se o nível do ativo subjacente for igual ou superior ao preço de exercício numa data de observação. Na última data de observação, se o nível do subjacente for inferior ao nível da barreira, o pagamento final está diretamente relacionado com o nível do ativo subjacente.

O processo de avaliação do certificado inicia com a sua respetiva decomposição numa obrigação com possível chamada antecipada e uma opção de venda com barreira. Para o cálculo do valor do produto foi utilizada a simulação de Monte Carlo com movimento browniano geométrico. De seguida, é realizada uma análise de risco e retorno, onde foram calculados o valor em risco e valor em risco condicionado, as sensibilidades do produto às letras gregas e taxa de juro interna.

Os resultados foram alcançados através de 5 modelos distintos, cada um com uma volatilidade diferente. Os restantes parâmetros permaneceram inalterados. 2 dos referidos modelos estudados foram resultado de cenários de stresse onde um dos parâmetros é alterado para representar o comportamento do mercado financeiro em condições extremas.

As conclusões principais deste projeto são as consequências do aumento da volatilidade, nomeadamente: a diminuição do valor do produto, a diminuição da taxa de juro interna, um aumento no valor em risco e uma diminuição na sensibilidade do produto às letras gregas.

PALAVRAS-CHAVE: Produtos Estruturados; Certificado Express; Simulação de Monte Carlo; Letras Gregas; Valor em risco.

Códigos JEL: G11; G12; G17.

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1. INTRODUCTION

Structured products date back to 1990, when they started being traded in the UK markets. The attractiveness of these products, which combined both fixed income securities and equity derivatives, is that the allowed a less risky access to the stock markets. Europe was the pioneer market for structured investment solutions. However, the growth of these products fell after the financial crisis in 2009. As the foul and unrestrictive use of financial derivatives was one of the main drives of the crisis, the restricted regulation that was implemented resulted in a significantly reduction in the issuance of products with complex/exotic payoffs and underliers. Nowadays, although some markets have not yet recovered the volume prior to the financial crisis, the structured product markets have evolved globally, with a higher participation in Europe and Asia Pacific. Currently, the biggest structured products markets are German and Switzerland[1].

This project focuses on a specific type of structured product, auto-callable certificates. Autocallables are believed to have been introduced by BNP Paribas in the early 2000s. Autocallable structured products are non-capital guaranteed products, as the investor can lose up to all its initial investment. Autocallable certificates, such as Express Certificates, gained popularity between 2014 and 2022, due to the negative interest rate in the fixed income markets. Additionally, in a stable market environment, an autocallable's return can be more attractive than a direct investment in the underlying asset, as stated by Sie and Helmersen (2021). On the other hand, the highest risk of autocallables lies on the full downside participation on the underlying performance.

The present project is part of a specialization in financial engineering, where the main financial products studied are derivatives and structured products. Structured products are important saving instruments offered to investors by financial intermediaries, such as Deutsche Bank. The purpose of the project is to study, in detail, the Fixed Coupon Express Certificate linked to EURO STOXX 50 Index, issued by Deutsche Bank AG at 4th of February 2020. Apart from the purpose, the main group objective in the study of this autocallable is to develop a pricing model, in order to reach the theorical value for the product. Additionally, with the intent of further analysing this complex certificate, the objective is to study its risk-return profile by the means of risk and return measures.

Regarding the relevance of the proposed project, as it is not common practice for financial analyst to do a close evaluation on these financial instruments, there are no known professional nor academic work on this kind of analysis regarding a particular structured product. Throughout the research conducted to perform the analysis presented in this project report, no study was found that could be considered as comparable or benchmark. Therefore, the relevance of this project lies on taking a concrete example of a structured product and conduct

[1] More information or news of structured products can be consulted at https://www.structuredretailproducts.com.

a start-to-finish analysis, not focused only on decomposition, on probabilities, on pricing, but rather on all of them.

2. LITERATURE REVIEW

On this chapter, the focus is to present and discuss the main findings and studies of the literature in which we based our findings and methodology. The majority of the literature relies on the theoretical grounds of autocallable structured products, exotic options, pricing models and fixed income valuation.

Hull (2018) focus his work on option pricing models, the main ones being the Black-Sholes Merton model and its derivations, binomial trees, and Monte Carlo simulation. For the last pricing model, it is explained the stochastic processes, namely the Weiner and Itô's lemma behind Monte Carlo simulations. Additionally, his contributes reach all stages of an option's evaluation process, from volatility estimators to the most appropriate discounting rates for each particular analysis. Hull (2018) is the main theoretical ground for this project.

As most structured products are, in part, fixed income securities, it is relevant for this autocallable valuation to consider the work of Addams and Smith (2019) and Martellini et al. (2003). Both books cover a theoretical background on callable bonds and other fixed income securities, and how to price them. For our analysis, it is particularly important to consider both the composition of callable bonds and how to valuate them. Additionally, the contributes of the authors also gather other measures such as different yield measures and securities duration.

Célérier et al. (2021) studies the price effects of new security designs, namely, for short put products (SPP). The examples given refer to autocallable reverse convertibles and express certificates. Autocallable express certificates are identical to autocallable reverse convertibles, with the only difference being that the final settlement of a reverse convertible is with shares of the underlying instead of a cash settlement. The main conclusion of the study states that when the volume of SPP increases for a certain underlying asset, the implied volatility decreases for the strike level of the SPP, when compared to the implied volatility for other strikes.

On a study regarding a derivation on Monte Carlo simulation to price autocallables, Alm et al. (2013) followed the objective of finding a Monte Carlo algorithm that would give a stable output by finite differences. According to the study, regular Monte Carlo simulation can have instable sensitive outputs when calculating product sensitivity.

On another valuation study of autocallable structured products, Sie and Helmersen (2021) used Monte Carlo simulation to value 2 Nordic autocallable notes. The conclusion state that both products were found to be overpriced for the level of returns offered, which shows a tendency for issuers to overprice their products.

On a similar note, Ribeiro (2018) developed a case study for an autocallable structured product, contextualizing the product in the nowadays economy environment. The analysis focus on the decomposition and characteristics of the autocallable, mainly the down-and-in put and the main risks the investors face while holding the product. The case study main conclusion is that, in an economy with very low and negative interest rates, the issuers face more challenges in issuing attractive structured products.

There is also some academic criticism in which regards structured products. Graf (2019) is very critical in which regards the European legislation for packaged retail and insurance-based investment products (PRIIPs). His analysis concludes that, apart from the lack of assumptions provided by the regulation, a product evaluated following the European guidelines will result in an overestimation of risk and an underestimation of the product's performance. On the other hand, Sie and Helmersen (2021) state that structured product are targets of many critics in the Nordic markets. Most of the critics surround the overcomplexity of the products. This complexity complicates the perception of the risks incurred by the investor. Apart from that, it also makes it harder to understand the true value/benefit of the product. It is also stated that the sales documents for PRIIPS can be misleading for the potential investors.

Regarding risk analysis of structured products, Alexander (2009) focuses on measuring Value-at-Risk. The book covers multiple approaches to compute and interpret value-at-risk, such as Parametric Linear VaR, Historical VaR and Monte Carlo VaR. The methodology to perform stress testing is also discussed, along with the different characterization of stress scenarios.

3. STRUCTURED PRODUCT

On this chapter, the structured product that is the object of the whole analysis is introduced. This chapter is divided into 3 sub-chapters or sections: the product; the underlying asset; the issuer and an analysis on the key information document (KID), available in the appendix. The first point has the objective of briefly describing the product's structure along with a general description of its payoffs.

3.1. The Product

The product under analysis has the name of Fixed Coupon Express Certificate linked to EURO STOXX 50 Index. This product, issued by Deutsche Bank AG on the 4th of February 2020. The product notional amount is 100€ and the maturity date is 5 years after the issuance, on the 6th February 2025. This product has a total 0 costs, which includes no entry costs, no exit costs, and no ongoing costs. It is assumed that the product is issued at par value.

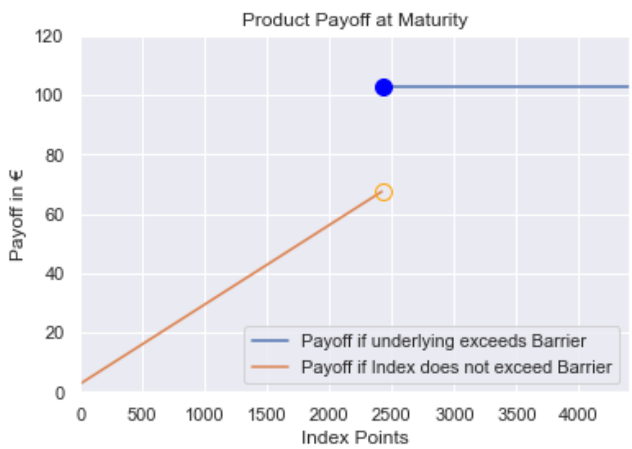
Regarding the product payoff structure, the certificate is designed to pay a fixed annual coupon (c) of 2.65€ and a cash payment upon the termination of the product. The product can terminate earlier, at each anniversary of the product (excluding the first one) if the underlying level is at or above the strike price (K), which corresponds to the underlying level at the issue date. In the case of early termination, the holder will receive the coupon accompanied by the notional amount. After the termination of the product, no further payoffs result from it.

If the product survives until the maturity date, in 2025, the payoff at maturity follows the equation below,

$$CF_{year\ 5} = \begin{cases} 100 \times \dfrac{Und_M}{K} + c;\ Und_M < B \\ 100 + c;\ Und_M \geq B \end{cases}. \tag{1}$$

Figure 1 represents the graphical representation of the certificate's payoffs at maturity. If, at maturity, the underlying asset level is at or above the final barrier level, then the investor is going to receive the whole notional amount of 100€ plus the final 2.65€ coupon. On the other hand, if the underlying level is below the barrier level, the payoff is directly linked to the performance of the underlying index. In this case, as Figure 1 shows, the investor can lose almost all of its initial investment, apart from the coupons. The barrier level is set at 65% of the initial reference level, the strike price.

Figure 1. Payoff at Maturity.



3.2. The Underlying Asset

EURO STOXX 50[2] is a price index composed by the largest 50 European companies, reviewed annually. This blue-chip index gadders companies that operate in 20 different economic sectors and are from 9 European Union countries. It was founded in 1998 with a base value of 1000 points.

With respect to its composition, the three most represented sectors on EURO STOXX 50 are technology, consumer products and services, and industrial goods and services, with weightings of 15.3%, 14.8% and 13.2%, respectively. Regarding the weightings by country,

[2] The factsheet for the index (with data as of 3/2023) can be downloaded at https://www.stoxx.com/document/Indices/Factsheets/2023/March/SX5E.pdf.

France is the heaviest representation with 41.5%, followed by Germany and Netherlands, with 26.6% and 13.5%.

Figure 2. EURO STOXX 50 Historical Performance.

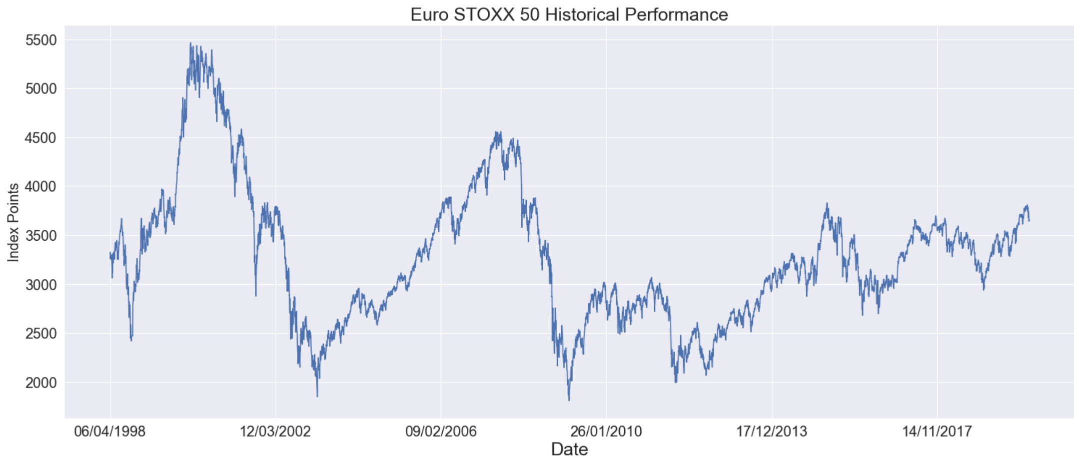


Figure 2 shows the historical performance of the index from its creation in 1998 until the issuance of the Deutsche Bank's Express Certificate in 2020. Analysing the time-series of index points, the two most relevant events, that caused a significant crash in index points after a peak, was the dot.com bubble crisis in 2000 and the financial crisis in 2009. The former originated a peak that reached 5500 index points, an all-time higher, to then crash by 3500 points until 2003. The real estate bubble in 2008 caused a peak of 4500 points and crashed by almost 2000 points in 2009.

3.3. The Issuer

The Deutsche Bank (DB) group[3] is a German bank founded in 1870. As of before the issuance of the certificate being analysed, the group was divided into 6 segments: corporate bank; investment bank; private bank; asset management; capital release unit and corporate & other. Investment bank is the segment responsible for the sale of equity linked products, such as the express certificate.

Regarding credit ratings, which is an indicator for a borrower's creditworthiness, Deutsche Bank was downgraded by Fitch in 2019. During this year, the group faced challenges regarding profitability and business model stability. This led to a decrease in counterparty assessment and deposit ratings from A- to BBB+. The counterparty rating within the credit risk is the probability of Deutsche Bank to default in a contractual obligation. This assessment is of great importance for investors who hold DB products. Deutsche Bank's structured product can be found on the X-Markets[4] platform, with all mandatory documents regarding the issuances.

[3] More information in https://www.db.com/index?language_id=1&kid=sl.redirect-en.shortcut.
[4] More information in https://www.xmarkets.db.com.

3.4. The Key Information Document

The key information document (KID)[5] is a mandatory document that must be provided by the issuer and available for the costumer in order to access information on a packaged retail and insurance-based investment products (PRIIPs). The European Commission details the information that shall be contained within this document. Apart from the description of the product and its payoff structure, the KID must also have a section on the risk, return, and costs.

The summary risk indicator (SRI) is a measure of risk which is mandatory for issuers to present in a product's KID. This indicator measures the riskiness of a PRIIP from 1 to 7, 1 being considered the lowest risk possible and 7 being the highest. This measure takes into consideration 2 components: market risk measure and credit risk. According to Deutsche Bank, the Express Certificate under analysis has a SRI of 1 out of 7, which is the lowest risk classification.

The performance scenarios consist in 4 scenarios (stress, unfavourable, moderate, and favourable), for different holding periods. Graf (2019) criticizes this analysis as the legislation methodology does not cover every possible PRIIPs characteristics, which allows for deviations in the issuer's valuation.

The methodology provided by the European Commission (EC)[6] regulation for the calculation of the SRI and performance scenarios, respectively. Graf (2019) criticizes both of these measure's regulation. In one hand, the indicator computed to reach the market risk measure, is stated to be only valid for single premium investments, resulting in overestimation of risk in regular premium investments. On the other hand, in which regards the performance scenarios, Graf (2019) found some technical errors on the formulas proposed by the EC, which results in an underestimation of the product's returns potential.

4. DATA AND METHODOLOGY

On this chapter, both the data and methodology used to reach the results of this project are discussed. This chapter is divided into 3 sub-chapters or sections: data; product valuation and risk measures. The methodology is split into the last two points, main analysis, and risk measures analysis, which corresponds to the group research and individual research, respectively.

4.1. Data

The data used throughout the valuation of the Express Certificate is mainly the underlying asset historical data. For the main analysis, we retrieved the time series of the adjusted closing values

[5] The KID for the express certificate can be found in the Appendix.
[6] Can be consulted in https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A02017R0653-20191128.

for the past 5 years of historical performance[7], counting back from the issue date of the certificate. The dataset gadders adjusted closing index points for the EURO STOXX 50 starting from 04/02/2015 until 03/02/2020, with a total of 1254 trading days.

The previously mentioned dataset is expanded for the risk measures analysis, for stress testing purposes. This extended dataset gadders all the adjusted closing values for the EURO STOXX 50 starting from the creation of the index, 06/04/1998 until 03/02/2020, summing a total of 5557 trading days.

Unfortunately, it was not possible to use data from the Deutsche Bank investment platform X-markets. This platform compiles all the PRIIP legal documents as well as their secondary markets, with closing quotes for the products being traded. These closing quotes along the life of the product would allow for a comparison between real product value and the theoretical pricing model value. However, as the Express Certificate had already terminated (due to an autocall barrier event) in February 2022, all the information about this product, including legal documents and secondary market data, was deleted. Upon contact, through email, both Deutsche Bank' X-markets and Frankfurt Stock Exchange stated that they could not provide data regarding products no longer being traded.

4.2. Methodology – Product Valuation

In this section, the focus is on explaining the rationale behind the group analysis: underlying asset volatility analysis; decomposition of the product; pricing model and product value.

4.2.1. Volatility Analysis

To perform the volatility analysis for the product's underlying asset, EURO STOXX 50 index, we followed Hull (2018) methodology for estimating volatility from historical data. Using the daily closing index points (S_i) from the past 5 years of the underlying asset historical performance before the issue date. We firstly computed the daily logarithm returns (u_i) for said period and then proceeded to calculate the moving standard deviations (s) for the following moving time intervals: 3 months; 6 months; 1 year; 2 years; 3 years; 4 years; and 5 years. The standard deviations are then used to compute the annualized volatility (σ) for each of the intervals. The calculations described followed the formulas below,

$$u_i = ln\left(\frac{S_i}{S_{i-1}}\right) \quad , for\ i = 1, 2, \ldots, n \tag{2}$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{3}$$

[7] This data can be exported from https://www.investing.com/indices/eu-stoxx50-historical-data.

$$\sigma = s \times \sqrt{252}. \quad (4)$$

The 5-year historical analysis is the recommended period to take into account while valuating a packaged retail investment and insurance product (PRIIPs), according to European regulation (2019).

Figure 3 represents the distribution of daily logarithm returns for the underlying index, EURO STOXX 50. The distribution has a center in null returns and with both tails reaching -4% and 4% returns. Additionally, there are a few negative return outliers, which drags the left tail further than the right tail.

Figure 3. Histogram of Logarithm Daily Returns.

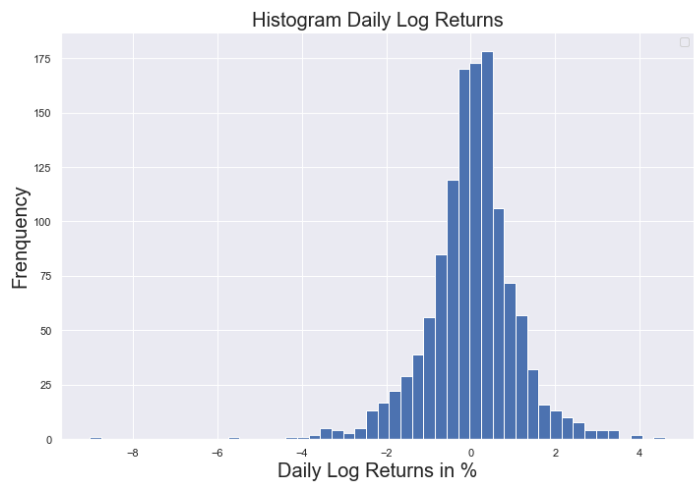
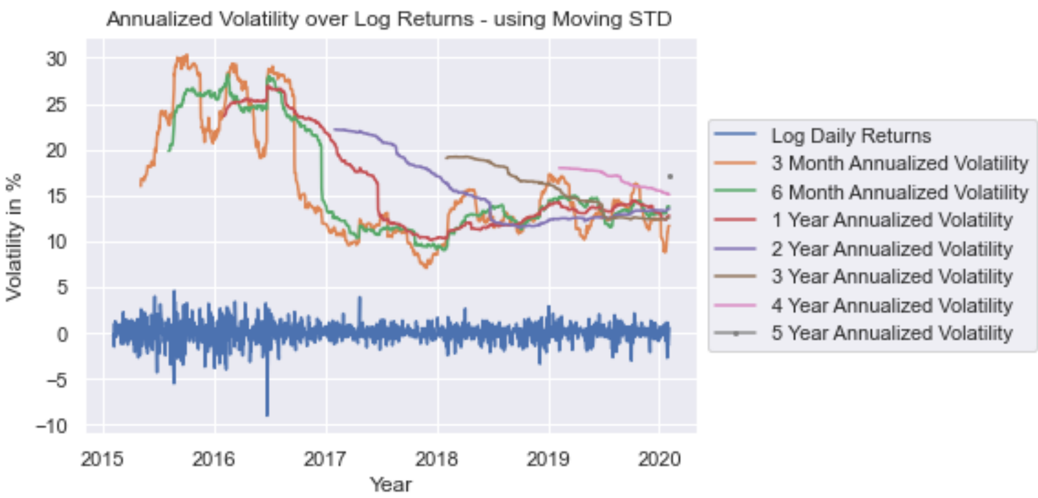


Figure 4 represents the annualized volatility time-series for all the time stamps. The choice of time intervals is based on Hull (2018), that states that generally a past performance between 3 months and 6 months is enough to capture market behavior. As an alternative, the author also advices to take into account the past performance that coincides with the maturity of the option/product being analyzed. Based on this, we choose to consider the shortest time stamp 3 month and the longest 5 years, which equals the maximum maturity of the autocallable certificate. The time stamps of 6 months and 1 year were added to account for a more intermediate past performance analysis. The 2, 3 and 4 years time stamps coincide with all the possible early maturity dates for the certificate, in case there is a barrier event at one of the yearly observation dates.

Additionally, Figure 4 also shows the daily logarithm returns fluctuations overtime. Apart from the market crashes in August of 2015 and June of 2016, which translated into an increase in index level fluctuations and a consequentially increase in volatility, the volatility of the underlying index had a clear downward trend throughout the 5 years prior to the certificate's issuance. The period that registered the lowest volatility was 2017.

Figure 4. Time-series: Annualized Volatility over Logarithm Daily Returns (2015-2020).



4.2.2. Product Decomposition

Most structured product can be generally defined as a long position on a bond with an additional portfolio of financial derivatives, linked to an underlying asset, embedded in it. According to European legislation, issuers are not required to disclose the decomposition of their issued products. They are, however, required to present a comprehensive description of all the payoffs throughout the life of the PRIIP. For pricing purposes, it is necessary to have a decomposition, to calculate each component's value with their respective payoffs. However, the complexity of structured products many times allows for multiple approaches in terms of decomposition, as stated by Deng et al. (2014).

According to Alm et al. (2013), autocallable products have an almost identical mechanist to that of a callable bond, in which concerns the possibility of early redemption/termination and the fact that both products have a bond as a base for the product. The difference, as pointed out, is that a typical callable bond is called back by the issuer, instead of automatically, as it happens with autocallable structured products.

In terms of the derivative components of an autocallable certificate, Célérier et al. (2021) classify autocallable certificates as "short put products". This classification groups the structured products that, upon a barrier event, expose the holder to the direct performance of the underlying. Likewise Alm et al. (2013), Célérier et al. (2021) also state that short put products, imitate the payoff of a callable bond and generally pay a regular fixed coupon until the product matures.

Furthermore, both Ribeiro (2018) and Bellefroid (2022) affirm that the short put embedded in autocallable products is a short position on a Down-and-In put (DIP) option.

Table 1 summarizes the decomposition of the autocallable certificate, based on the contributes discussed above. Analyzing through a holder perspective, an investor that acquires this express certificate is buying the equivalent of a long position on a callable bond alongside a short position on a DIP option.

Table 1 - Express Certificate Decomposition, Holder Perspective.

Components	Position	Type	Maturity	Strike	Barrier
Callable Bond:	Long	(Auto)Callable with Coupon	Max 5 years		
Bond	Long	Coupon Bond	5 years		
Call Option	Short	Bermudan	Max 3 years	100%	
DIP Option	Short	European Barrier	5 years	100%	65%

According to both Martellini et al. (2003) and Adams and Smith (2019), a callable bond consists of a regular bond with an embedded call option, which the investor sells to the issuer. By buying the call option from the holder, the issuer gains the right to call back the bond, thus terminating the product before its maturity. The callable bond within our product has a maximum maturity of 5 years, the time between the issue date and the maturity date. Apart from that, the callable bond has a call protection period of 2 years, meaning it can only be called 2 years after the issue.

The regular bond has a maturity of 5 years and pays a 2.65% annual coupon as well as the notional amount at maturity or at an early termination date. The call option is the component that allows for the product to be called back for early redemption. It has yearly observation dates, at the anniversary of the product in 2022, 2023 and 2024, so it is considered a Bermudan option with maturity of 3 years. The call is automatically exercised if, at any of the observation dates the underlying asset is above its initial level, making it an at-the-money call option.

Apart from the (auto) callable bond, the product has yet another component, a down-and-in put (DIP) option. Hull (2018) defines DIP options as a regular vanilla put option that comes into existence if the underlying asset reaches the barrier level. The barrier level of a down barrier option is set below the underlying initial level. The DIP present in this express certificate is an at-the-money European barrier option, because its strike (K) is equal to the underlying initial value and it can only be exercised at maturity. If the underlying asset closes above the barrier at maturity, or the product terminates before maturity, the put option never comes into existence, having no payoff. Selling the DIP to the issuer allows the holder to have a conditional protection of the notional amount. However, if a barrier event occurs, the investor notional is exposed to the direct performance of the underlying and in risk of losing its initial investment. In this case, the option payoff is as shown in the formula below,

$$CF_{DIP} = 100 \times \frac{Und_T}{K}. \tag{5}$$

4.2.3. Pricing Model

Amongst option pricing models, the most known are closed form solutions, binomial trees and Monte Carlo simulation. According to Hull (2018), one of the limitations of closed form solutions, is that it fails to consider products with possibility for early termination. Comparing both the binomial tree and Monte Carlo methods, the latter has the advantage of being more flexible in which concerns modelling complex and path dependent payoff, such as those of a barrier option.

We priced the product through Monte Carlo simulations, which relies on a set of parameters and random variables to estimate future underlying prices through a stochastic process. The parameters used are volatility; model time-step, short-term expected return and the random variables. To calculate the simulated spot prices for the underlying asset, we use the geometric Brownian motion stochastic process. According to Hull (2018), this is the process commonly assumed for stock prices. The following formulas describe the computations performed to achieve the simulated underlying levels (S_i),

$$S_i = S_0 \, e^{\left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma \varepsilon_i \sqrt{\delta t}} \tag{6}$$

$$\delta t = \frac{T}{n}. \tag{7}$$

Regarding the time-steps (n) for the product's pricing, we chose to do bimonthly time-steps. Although the minimum required would be just 1 time-step per year (5 in total) to account for all the observation periods, we decided to do a total of 30 time-steps, which translates to 6 per year. This allows for more model accuracy and less model error because more simulations are performed within each year. Each path has, thus, 30 time-steps, and we consider a total of 10.000 paths. The random variables $\left(\varepsilon_{1;\,\ldots;}\, \varepsilon_{30}\right)$ are generated through the inverse normal cumulative distribution with a random setting, a mean value of 0 and a standard deviation of 1.

For the drift term (μ), under the pricing measure, the risk-free rate, we consider the interest rate of a 5-year Germany government bond on the 3rd of February 2020, the day before the certificate's issuance. This corresponds to a risk-free rate of -0.647%.

Finally, for our baseline model's volatility (σ), we apply a volatility of 12.78%, which corresponds to the 1-year annualized volatility 1 day before the issue date. In order to evaluate our model behavior under higher and lower volatility, we consider for the high volatility model, a volatility of 29.25%, which corresponds to the 3-month annualized volatility as of 26th of October 2015. This is the highest volatility recorded within the analyzed period. For the low volatility model, a volatility of 7.01% is applied, which corresponds to the 3-month annualized volatility as of 29th of November 2017. This was the lowest volatility recorded within the analyzed period.

The output of the Monte Carlo simulation is 10'000 paths of 30 observations of simulated underlying spot levels $(S_{1;\ \dots;}\ S_{30})$, starting with the initial reference level of 3732.28.

4.2.4. Product Value

To reach the value of the structured product, it is necessary to compute first the value of each of its components.

Firstly, the value of the callable bond, as stated by Martellini et all. (2003), is the average of the present value of the callable bond for all simulations. The present value of the bond for each simulation is the total of the discounted yearly cash-flows. To account for the possibility of early termination, at each observation date, we took into consideration the simulated underlying values.

The value of the DIP option followed the same rationale, the average of the present value of the option for all simulations. However, the DIP option only produces cash-flows if it comes into existence. This condition only comes true if the underlying level is below the initial barrier level at the maximum maturity of the product, in 2025. If this confirms, then the payoff from the option will be directly linked to the underlying index, as shown by Equation (5).

According to Hull (2018); there is not a consensus in which concerns the discount rate to be used when valuating these products. However, it is stated that it is a common approach to discount uncollateralized derivatives or products by using the issuer's average funding costs. As the holder can lose all their money when investing into this certificate, it is considered an uncollateralized product. Given that, to calculate the present value of each of the product's components, the discount rate used corresponded to the issuer's, Deutsche Bank, funding cost before the certificate was issued. According to Ayamanns et al. (2016), cost of funding is the rate which banks pay in order to obtain funds. It is also stated that this cost represents the ratio between interest expense and average interest-bearing liabilities. Using data from the 2019 Deutsche Bank's annual report[8], the cost of funding is 1.602%.

The Express Certificate value was then computed, representing the sum of both the callable bond and DIP option's value. This value also corresponds to the average of the sum of its discounted cash-flows for all 10.000 simulated underlying paths.

4.2.5. Additional Analysis

In order to further the analysis for this certificate, we computed both unconditional and conditional probabilities for each scenario the product can take. The unconditional probabilities represent the probability of the product being called for early redemption at each observation date, whilst the conditional probabilities represent the probability of the product to be called back given that it survived until a previous observation date. For example, the conditional

[8] Can be consulted in https://investor-relations.db.com/reports-and-events/annual-reports/.

probability of the certificate to terminate in 2025 given that it survived in 2022, is the ratio between the number of simulations in which the product terminated in 2025 and the number of simulations in which the product survived past the 2022's observation date.

Additionally, as the product's life accounts for different paths, each one with different possible maturities, the expected life (EL), also known as *fugit* of the product is calculated. The expected life of the product corresponds to the unconditional probability (p_i) of the product to terminate in each one of the observation dates times the maturity (t_i) corresponding to each observation,

$$EL = \sum_{i}^{n} t_i \times p_i \,. \tag{8}$$

4.3. Methodology – Risk Measures Analysis

In this section, the focus is on the rationale behind the individual analysis of the Express Certificate. This analysis mainly covers risk measures and a return analysis, with the objective of analysing the risk-return trade-off the investor must consider when investing in this structured product. This section focuses on stress testing, value-at-risk and conditional value-at-risk, internal rate of return and Greek sensitivity analysis.

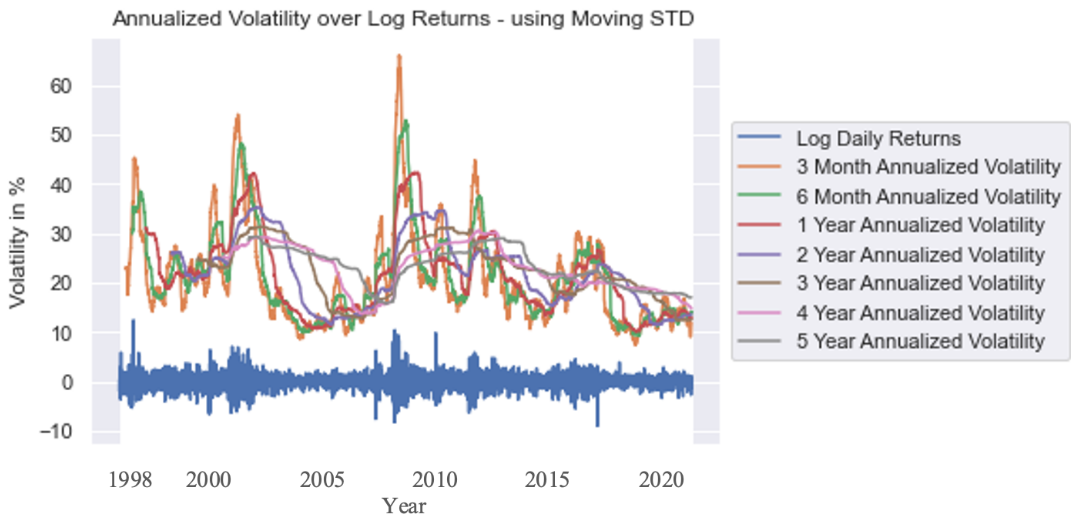
4.3.1. Stress Testing

According to Alexander (2009), the goal of stress testing is to test an investment product for market circumstances that can result in extreme losses. There are two types of stress scenarios, historical and hypothetical stress scenarios. While historical scenarios try to mimic market behaviour for a specific event that happened, hypothetical stress scenarios test for unprecedented events.

In order to define the historical stress scenario, and following the same methodology as the volatility analysis, we consider the entire history of EURO STOXX 50 and calculated the annualized volatility. The volatilities were calculated using moving standard deviations (STD), like in Equation (2), (3), and (4). The objective being to find the highest 1-year annualized volatility, given that this was the criteria for the choice of the volatility for our baseline model. Figure 5 shows that the highest 1-year annualized volatility (red line) occurred during September of 2009. The volatility used for the historical stress scenario is 42.29%.

To define the hypothetical stress scenario, Alexander (2009) suggests a common hypothetical event, called a six-sigma event. The six-sigma event volatility is calculated by multiplying the baseline model volatility's standard deviation by 6. Based on this calculation, the volatility used for the hypothetical stress scenario is 76.68%.

Figure 5. Time-series: Annualized Volatility over Logarithm Daily Returns (1998-2020).



Both the historical and hypothetical stress scenarios are examples of single factor push methodology for stress testing, as described by Alexander (2009), where a single factor, in this case volatility, is pushed in a way to generate losses that exceed normal market behaviour. These two scenarios are then used to compute probabilities, expected life, value-at-risk, internal rate of return and Greek letters sensitivity.

4.3.2. Value-at-Risk and Conditional Value-at-Risk

Hull (2018) defines value-at-risk as the attempt to summarize risk into a single number. It represents for a given confidence level (CL), that losses higher than the value-at-risk do not occur in the next n days. To Alexander (2009), value-at-risk has the advantage of being a universal risk measure that can be compared within different markets and exposures. Value-at-risk can be computed by subtracting the initial investment by the $(1 - CL)$ percentile of total payoffs for a given time. This follows the expression below,

$$VaR_{5\ years} = 100 - P_{(1-CL)}. \tag{9}$$

Taking into account the payoff structure of this certificate, the holder only has its initial investment's value-at-risk if the product is to survive to its maximum maturity, in 2025. This is because of the product's conditional principal protection, that can only be broken if the product survives until 2025 and if the final underlying reference level in below the 65% barrier. For this reason, the value-at-risk is calculated for a time of 5 years, and only having into consideration the payoffs from the simulations in which the product survives until 2025.

Despite of the many advantages of this measure, both Hull (2008) and Alexander (2009) agree that the disadvantage of value-at-risk is that it fails to acknowledge the amount of loss incurred in case that value-at-risk is exceeded. The solution to account these losses is to resort to the measure conditional value-at-risk (CVaR) or expected shortfall, which is the initial investment minus the average loss if the VaR is exceeded. This follows the equation below,

$$CVaR_{5\,years} = 100 - \frac{\sum\left[payoffs < P_{(1-CL)}\right]}{n}. \tag{10}$$

This analysis is performed from the investor's perspective. Additionally, both value-at-risk and conditional value-at-risk are computed considering undiscounted product payoffs, due to the negative risk-free rate. It is assumed that a rational investor would not invest in a risk-free investment with negative returns, so a discount/capitalization rate of 0% was considered.

4.3.3. Internal Rate of Return

The internal rate of return (IRR) is the rate in which the net present value (NPV) of an investment is matched to zero. This measure is computed for each scenario the product can take, according to the following formula,

$$0 = \sum_{n=0}^{N} \frac{CF_n}{(1+IRR)^n}. \tag{11}$$

The results are quite predictable for the majority of the product's scenarios, given that the coupon is constant, and the notional payment is certain. However, if the product reaches the maximum maturity of 5 years, and the final reference level is below the DIP barrier, the internal rate of return is going to differ from the constant. For this case, the internal rate of return is going to be computed for each simulation, and the final IRR to be considered is going to be the average of all the simulations. Additionally, the internal rate of return for the product is going to be the weighted probability of the average IRR for below the barrier and the constant IRR for the other product paths.

4.3.4. Greek Sensitivity

The "greek letters", as stated by Hull (2018), are risk measures that represent an option or product sensitivity to different variables that can influence an investment product's value. For this sensitivity analysis, we determined the sensitivity of the certificate with respect to 4 variables, which also corresponds to 4 greek letters. Delta represents the sensitivity to a change in the underlying asset. Gamma represents the sensitivity to a change in the delta when a change in the underlying asset is applied. Vega measures the sensitivity with respect to changes in volatility is applied. Lastly, Rho measures the sensitivity of an option's value to a change in the interest rate, in this case, the risk-free rate.

According to Hull's (2018) methodology, in order to compute Greek letters (G) using Monte Carlo simulation, a small change need to be applied to the desired sensible variable (x) and then compare the value with the changed parameter ($\hat{f}^*$) to the original option's value ($\hat{f}$),

$$G = \frac{\hat{f}^{*} - \hat{f}}{\Delta x}. \tag{12}$$

This sensitivity analysis is performed for the product, as well as for its components, the DIP option and the callable bond. For Delta, an increase of 1% unit is applied to the underlying asset. For Gamma, it was considered the difference in Delta for a 1% unit change in the underlying asset. For Vega, an increase of 1% is applied to the volatility. For Rho, an increase of 1% was applyed to the risk-free rate.

5. RESULTS

In this chapter, the focus is presenting the results reached through the analysis described in the previous chapter. The results will be divided into the following groups: Cash-Flow analysis; probability analysis; payoff distribution; product expected life; pricing results; value-at-risk and conditional value-at-risk; interest rate of return and greek sensitivity analysis.

5.1. Cash-Flow Analysis

Table 2 shows the Cash-Flow map for all the outcomes the express certificate can have, which includes all the possible early termination dates and the path dependency of the DIP option if the product reaches the maximum maturity of 5 years.

Due to the call protection period, the bond cannot be called in 2021, meaning its only possible payoff is the 2.65€ coupon payment. For 2022, 2023 and 2024, the bond can be called for early termination in the yearly observation dates if the underlying level is at or above the initial level. If called, the holder will receive the coupon paying along with the notional amount. After the product is called, no further payments are made. If the bond is not called on any of the autocallable dates, the investor will receive a yearly coupon.

If the certificate survives until the maturity date, in 2025, and the underlying level is below the 65% barrier, the bond does not pay the full 100 € notional, only 2.65 €, the value corresponding to the last coupon. In this case, the payoff that the holder receives from the bond are the 5 annual coupons, which totals 13.25€.

Given that the DIP option only comes into existence if the product survives until 2025 and the underlying closes below the barrier level, it only yields a payoff if these conditions are met. The cash-flow from the DIP is directly linked to the underlying performance and shown in Equation (5). The payoff for the DIP shown in Table 2 represents the average for all the simulation paths, as well as for the different volatilities we considered for the evaluation of the certificate. As the underlying volatility increases, the more extreme the simulated underlying values, which results in lower cash-flows from the option.

Table 2. Express Certificate Cash-Flow Map (undiscounted).

Product Scenarios	CF 2021	CF 2022	CF 2023	CF 2024	CF 2025		Total Payoff
					Bond	DIP	
1. Terminates in 2022	€ 2,65	€ 102,65					€ 105,30
2. Terminates in 2023	€ 2,65	€ 2,65	€ 102,65				€ 107,95
3. Terminates in 2024	€ 2,65	€ 2,65	€ 2,65	€ 102,65			€ 110,60
4. Terminates in 2025:							
4.1. Above barrier	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 102,65		€ 113,25
4.2. Below barrier:							
a) σ = 7,01%	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 62,11*	€ 75,36
b) σ = 12,78%	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 56,67*	€ 69,92
c) σ = 29,25%	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 41,49*	€ 54,74
d) σ = 42,29%	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 26,82*	€ 40,07
e) σ = 76,68%	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 2,65	€ 13,64*	€ 28,89

** Average Values.*

For the investor, the highest possible total payoff is 113.25€, if the product survives until 2025 above the barrier level. However, the lowest possible total payoff would be only 13.25€, which corresponds only to the sum of all the coupons, if the underlying closes at 0 points, given that the product offers no capital protection for the holder. This is merely a theoretical low, as we do not have any of the 10.000 simulations to reach this null value.

5.2. Probability Analysis

Table 3 shows the unconditional and conditional probabilities of the product being terminated in each possible autocallable observation date.

Regarding unconditional probabilities, across all of the volatilities, the product is much more likely to be called back on the very first observation date or survive until the maturity date. Additionally, when the volatility increases, the unconditional probability of the product to reach maximum maturity rises (going from 43% to 61%), while the probability of the product being called back in 2022 drops from 41% to 28%. The probability of the certificate being called back in 2023 and 2024 are relatively low, although the former is more likely than the latter. This fact might be because the higher simulations are “cut out” in the first observation date, leaving not only a smaller sample going forward but also a sample with lower values. This increases the chance of the product to survive until 2025.

In which concerns conditional probabilities, the analysis and results are similar to the unconditional probabilities. Mainly, if the product survives 2022, its probability of terminating in 2025 increases greatly.

Table 3. Conditional Probabilities for Each Autocall Date.

Condition:	Terminates 2022	Terminates 2023	Terminates 2024	Terminates 2025			Total
					Above Barrier	Below Barrier	
Low Volatility Model (σ = 7,01%)							
Unconditional	41,87%	9,29%	5,19%	43,65%	43,00%	0,65%	100%
Survives 2022		15,98%	8,93%	75,09%	73,97%	1,12%	100%
Survives 2023			10,63%	89,37%	88,04%	1,33%	100%
Survives 2024				100%	98,51%	1,49%	100%
Baseline Volatility Model (σ = 12,78%)							
Unconditional	42,63%	9,47%	5,27%	42,63%	32,53%	10,10%	100%
Survives 2022		16,51%	9,19%	74,31%	56,70%	17,61%	100%
Survives 2023			11,00%	89,00%	67,91%	21,09%	100%
Survives 2024				100%	76,31%	23,69%	100%
High Volatility Model (σ = 29,25%)							
Unconditional	39,86%	8,69%	4,91%	46,59%	14,65%	31,89%	100%
Survives 2022		14,45%	8,16%	77,39%	24,36%	53,03%	100%
Survives 2023			9,54%	90,46%	28,47%	61,98%	100%
Survives 2024				100%	31,48%	68,52%	100%
Historical Stress Model (σ = 42,29%)							
Unconditional	36,89%	8,14%	4,29%	50,58%	10,02%	40,66%	100%
Survives 2022		12,90%	6,80%	80,30%	15,88%	64,43%	100%
Survives 2023			7,80%	92,20%	18,23%	73,97%	100%
Survives 2024				100%	19,77%	80,23%	100%
Hypothetical Stress Model (σ = 76,68%)							
Unconditional	28,90%	6,10%	3,11%	61,89%	4,48%	57,41%	100%
Survives 2022		8,58%	4,37%	87,05%	6,30%	80,75%	100%
Survives 2023			4,78%	95,22%	6,89%	88,32%	100%
Survives 2024				100%	7,20%	92,76%	100%

If the product survives until the year of 2025, there are two path the product can follow, closing below or above the barrier. For lower volatilities, most of the simulations fall above the barrier, because the lower volatility does not allow for many low simulated spot levels. However, as the volatility rises, the probability of the product to survive until 2025 below the barrier increase to the vast majority of the cases. In the hypothetical stress scenario's volatility, the simulated values become extremely low, hence why there is a low probability of the product terminating above the barrier.

Analysing our main model, with a volatility of 12,78%, the probability of the investor losing money, which only happens if the product survives until 2025 and stays below the barrier, is 10%. Conditional to surviving until 2025, this probability increases to 23%.

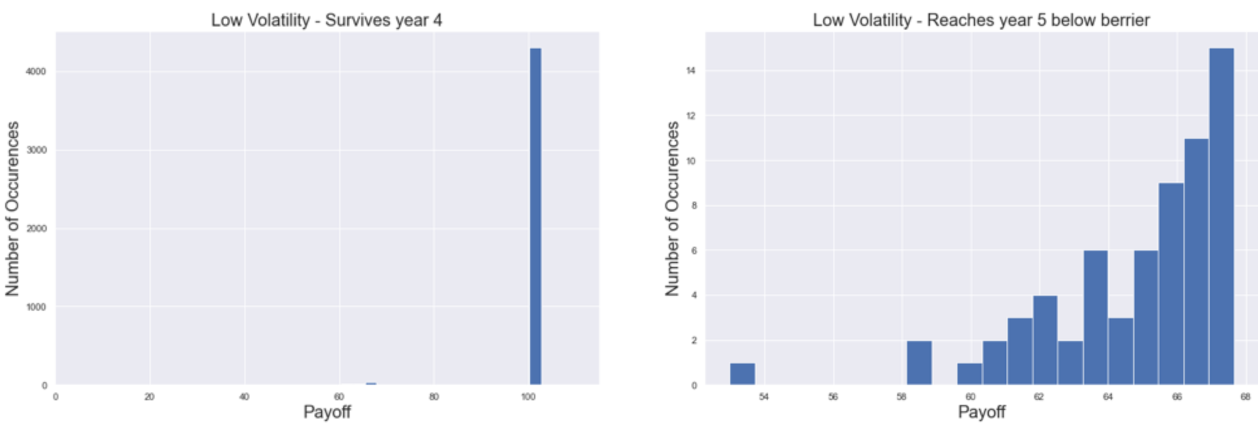
5.3. Payoff Distribution

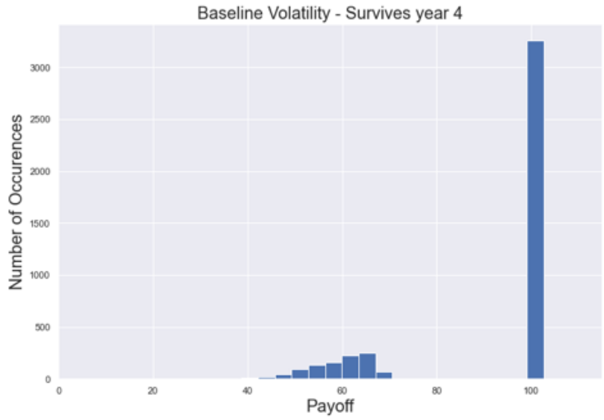
Figure 6 shows histograms for the distribution of the nominal payoff for each simulation, filtered for the ones in which the product survived until the maturity date. The decision behind filtering for this specific product scenario is because the payoffs are not discrete, as it is the case when the product is called back in 2022, 2023 and 2024.

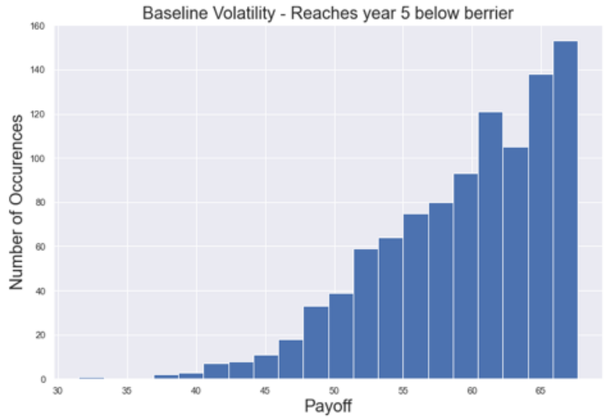
On the right-hand side of Figure 6, the graphs represent the undiscounted payoffs under the conditional that the certificate survives until the last year. On this side, it is possible to visualize the difference in occurrences of the product to close above or below the barrier, conditional to surviving 2024.

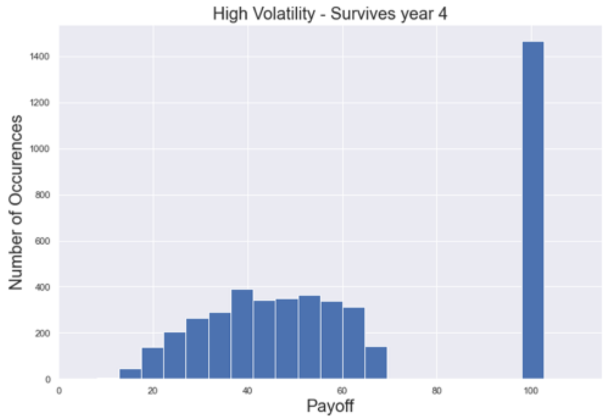
On the left side of Figure 6, the graphs show a zoom in for only the payoffs if the product closes below the barrier. The payoff distribution for the low volatility model is of an ascendent nature, with few cases with low payoffs and more occurrences with higher payoffs. This distribution gradually changes with the increasing volatility. In the hypothetical stress scenario, the payoff distribution is the complete inverse as the distribution under low volatility. In this last left-side graph, there majority of the cases yield lower payoffs, where the investor loses almost all the initial investment, and the minority higher payoffs.

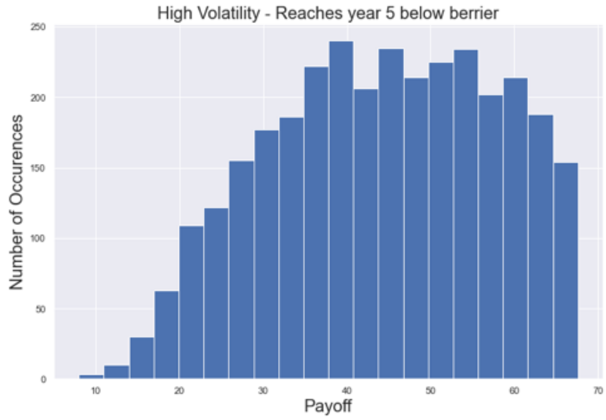
Figure 6. Payoff distribution at Maximum Maturity.

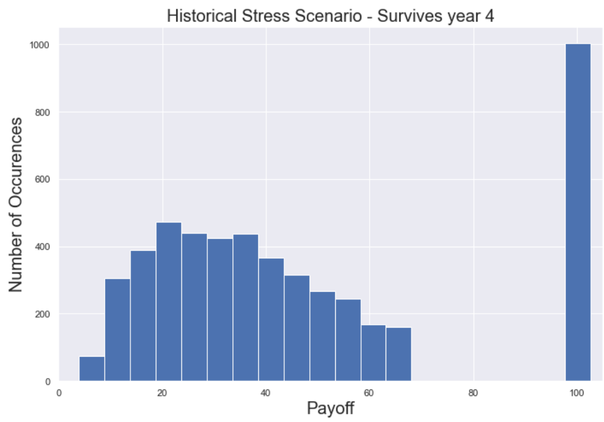


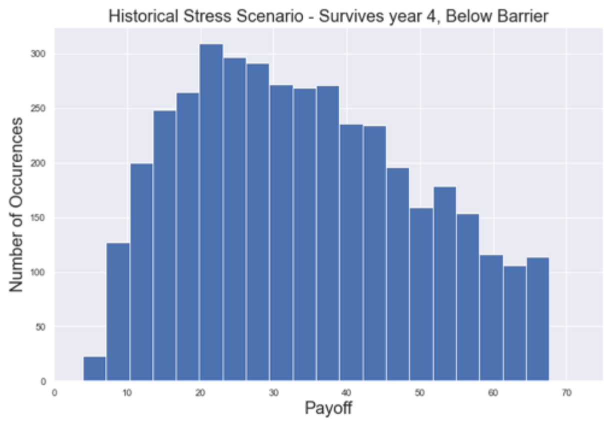

Baseline Volatility - Survives year 4
Number of Occurences
Payoff

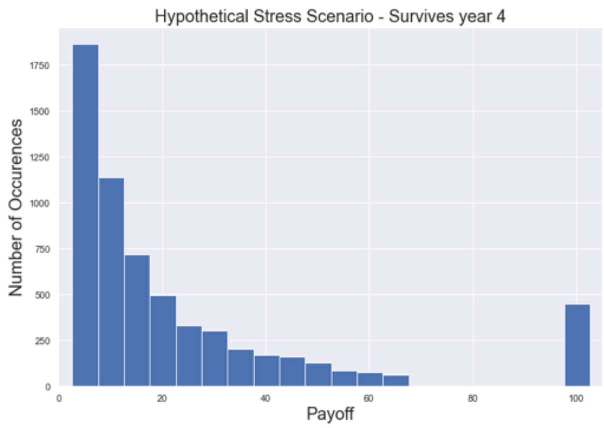

Baseline Volatility - Reaches year 5 below berrier
Number of Occurences
Payoff

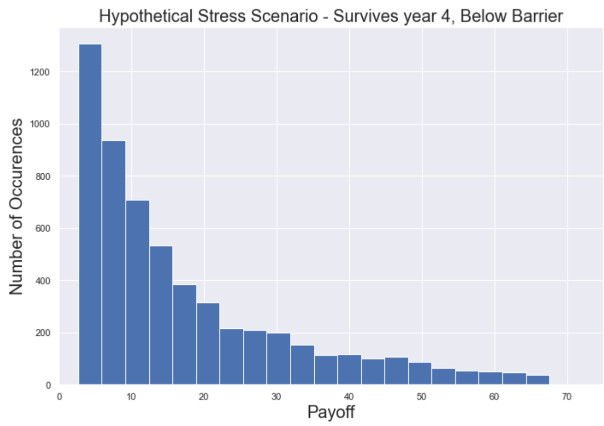

High Volatility - Survives year 4
Number of Occurences
Payoff


High Volatility - Reaches year 5 below berrier
Number of Occurences
Payoff


Historical Stress Scenario - Survives year 4
Number of Occurences
Payoff


Historical Stress Scenario - Survives year 4, Below Barrier
Number of Occurences
Payoff


Hypothetical Stress Scenario - Survives year 4
Number of Occurences
Payoff


Hypothetical Stress Scenario - Survives year 4, Below Barrier
Number of Occurences
Payoff

5.4. Expected Life

Table 4 represents the expected life of the product considering the different models. As the volatility increases, the investor can expect his holding period to increase as well. The expected life (*fugit*) of the certificate increases from 3.5 to almost 4 years because the higher the volatility, the lower it is the probability of the product being called in the first observation date.

Table 4. Express Certificate Expected Life (fugit).

Model	Volatility	Expected Life (*fugit*)
Low Volatility	σ = 7,01%	3,506 Years
Baseline Model	σ = 12,78%	3,479 Years
High Volatility	σ = 29,25%	3,581 Years
Historical Stress Scenario	σ = 42,29%	3,688 Years
Hypothetical Stress Scenario	σ = 76,68%	3,980 Years

This is an important measure for the investor because it allows to manage the reinvestment risk of this certificate. Reinvestment risk is the risk the holder faces of not being able to re-apply the capital at the same or higher return rate in case the product is called back before its maturity. In this case, the higher the volatility the lower the reinvestment risk, because the probability of the product to reach maximum maturity increases. This decreases the need for the investor to reinvest its capital.

5.5.Pricing Results

Table 5 represents the value of both components of the express certificate, the callable bond and the DIP option, and the final product value. At issuance, the express certificate has a value of 99.46 €, following the assumptions of our baseline model (σ = 12.78%).

Table 5. Express Certificate Pricing Results.

Components	Value				
	σ = 7,01%	σ = 12,78%	σ = 29,25%	σ = 42,29%	σ = 76,68%
Callable Bond	€ 102, 93	€ 94,18	€ 74,15	€ 66,15	€ 50,97
DIP Put Option	€ 0,37	€ 5,29	€ 12,22	€ 11,79	€ 7,83
Product Value	€ 103,30	€ 99,46	€ 86,37	€ 77,94	€ 58,80

The value of the callable bond is inversely related to the volatility. This is due to the fact that if the product survives until maximum maturity, the bond does not pay the principal if the final reference value is below the barrier level.

Regarding the DIP value, it shows an increasing in value until a volatility of 29,25%, and then a decreasing value for the stress scenarios. The value of the DIP is the average payoff from all the simulations, which is based on two aspects: the number of simulations in which the option comes into existence, and the underlying final reference level. For lower volatilities (from σ = 7.01% to σ = 29.25%), the number of cases in which the product terminates below the barrier increases and the final reference levels do not fall to extreme values, which results in an increase in the put values. However, from there on to the stress volatilities, although there are a lot of cases in which the barrier comes into existence, the final values of the underlying reaches very low values in many of the simulations, which causes the option value to decrease. This is because the payoff from the DIP is directly related to the underlying level.

The final product value is the sum of both components, which also corresponds to the average sum of discounted cash-flows from the whole product. The product value has an inverse relationship in which concerns volatility.

5.6. Value-at-Risk and Conditional Value-at-Risk

For Figure 7, Figure 8 and Table 6, all values can be interpreted either by amount or percentage of initial investment, since the unit notional amount is 100 €. The inputs are solely the total payoff received conditional to the product surviving the autocallable date in 2024.

Figure 7 shows the 5-year value-at-risk (VaR) for the confidence levels of 90%, 95%, 97.5% and 99%. Before the fifth year, the investor does not have any value-at-risk when investing in this certificate, because the principal is always granted. The graphs show the money loss for each percentile of returns.

For low volatility, the probability of losing money is so low that there is only value-at-risk for the 1% worst returns. As volatility increases, the VaR increases gradually as well. In the most extreme scenario, the hypothetical stress scenario, the investor loses 85% of its initial investment if his returns are placed within the 1st percentile. Additionally, as the volatility increases, the closer are the VaR for the confidence levels analysed. The figure also allows to see the cut-off percentile for which the investor ceases to have any Value-at-Risk. While the cut-off point for the low volatility model is close to the 2nd percentile, for the baseline model is close to the 25 percentile. In the most extreme case, the cut-off only occurs near the 90 percentile, with VaR greater than 50% up until a confidence interval of 20%.

Figure 8 represents the results already presented in Figure 7 but in a different way, in which is simpler to understand the conditional value-at-risk (CVaR) outputs. The CVaR in these histograms can be interpreted as the average of the payoffs to the right of the respective VaR dashed line. This corresponds to the average loss the investor faces if the loss is greater than the VaR. This view also allows a better analysis at the way the values converge closes as volatility rises. On the hypothetical stress scenario, the left tail-heavy distribution makes the VaRs almost indistinguishable.

Figure 7. 5-Year Value-at-Risk.

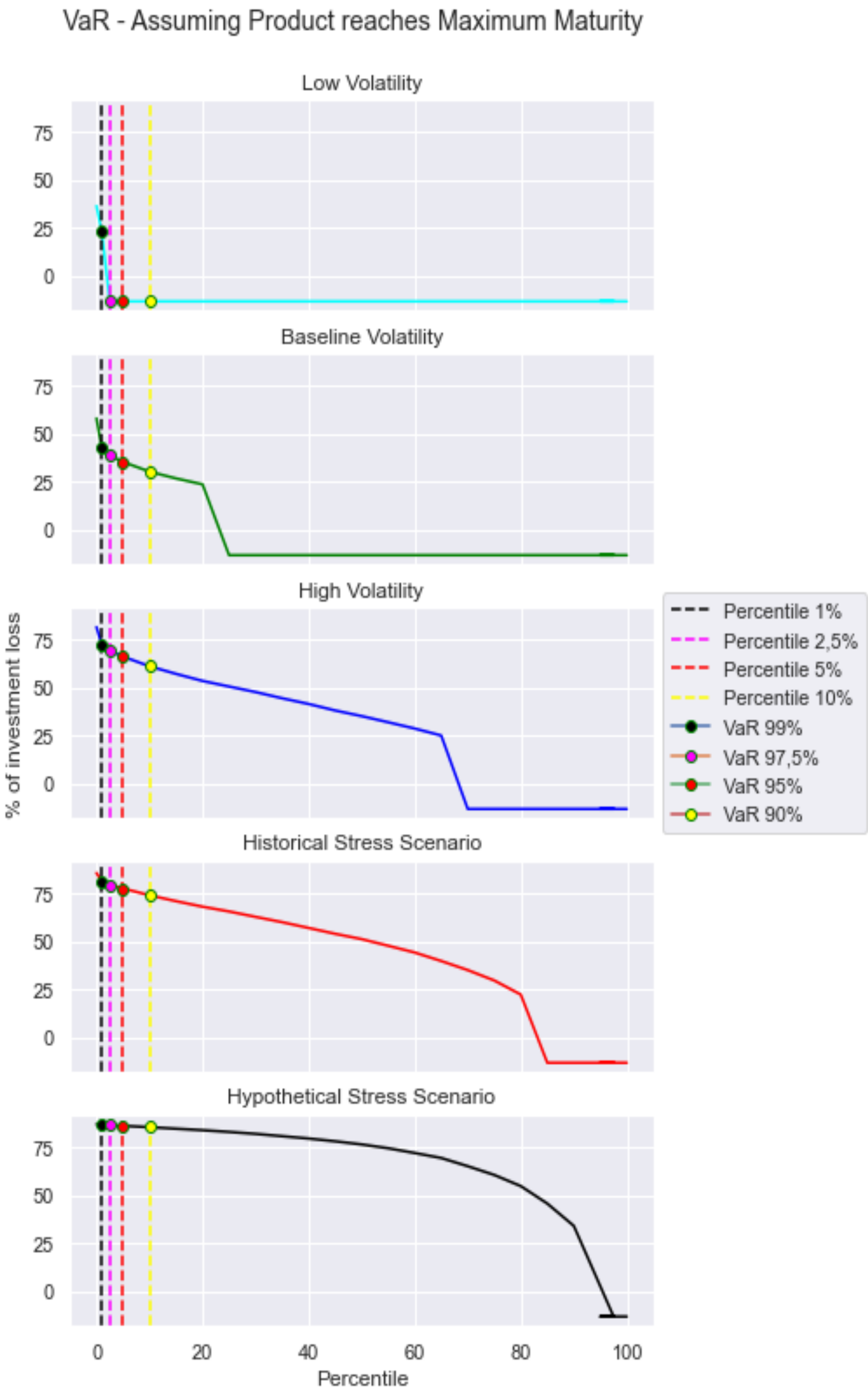


Figure 8. Conditional Value-at-Risk.

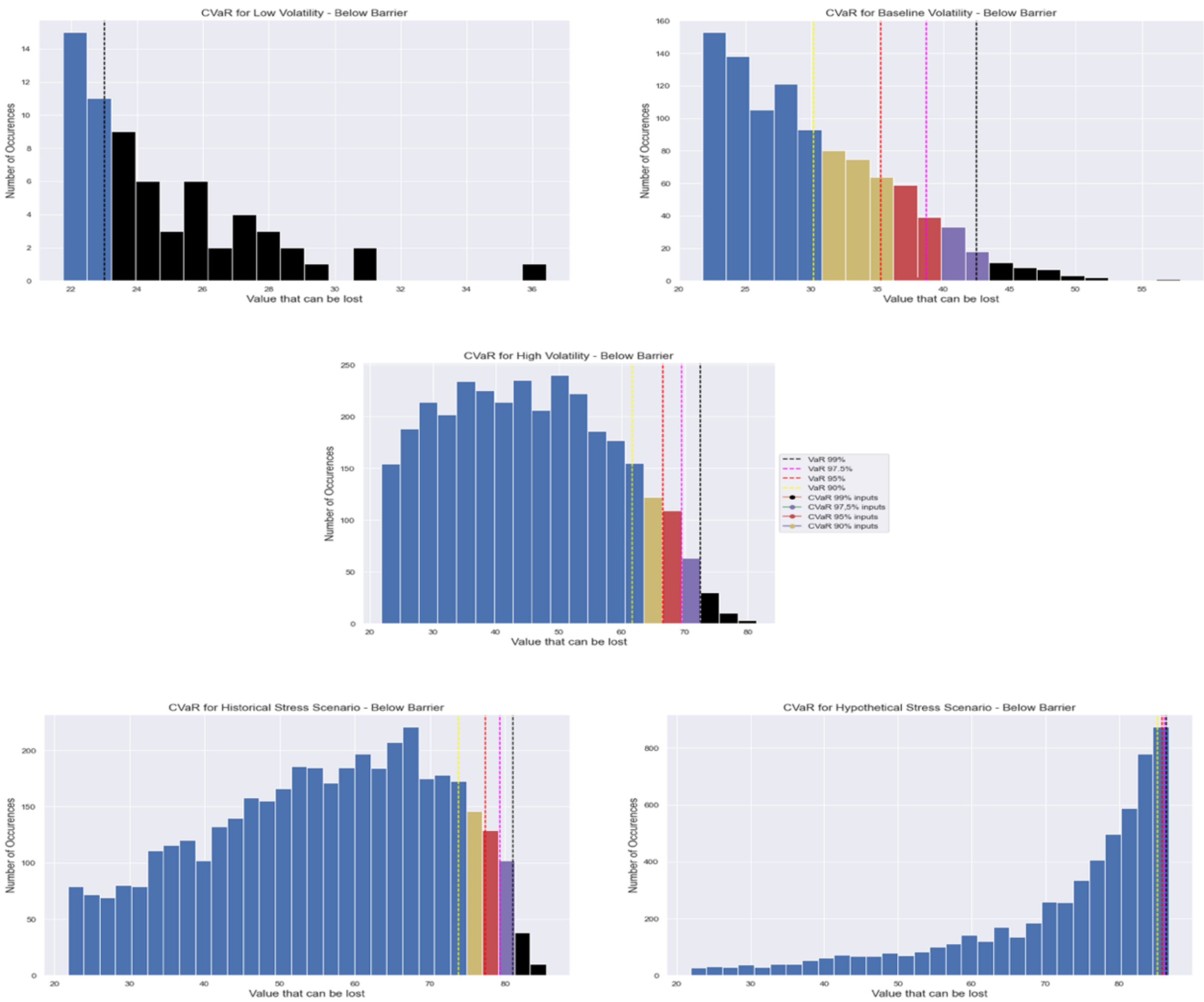


Table 6 compiles all the exact values for VaR and CVaR. Looking at the evolution of both measures in the first 3 models, the rate of increase is more accentuated than the evolution in the last 2 stress models. This indicates that after a certain volatility, such as 42.29%, where the VaR is already almost all the initial investment, a rise in volatility triggers a way less sensitive response for both VaR and CVaR. Additionally, CVaR converges with VaR as the confidence level rises.

Table 6. 5-year VaR and CVaR.

Confidence Level	σ = 7,01%		σ = 12,78%		σ = 29,25%		σ = 42,29%		σ = 76,68%	
	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR	VaR	CVaR
99%	23,08	25,84	42,52	45,94	72,46	74,71	81,08	82,39	86,43	86,56
97,5%			38,72	42,58	69,50	72,30	79,36	81,07	86,24	86,41
95%			35,26	39,73	66,52	70,16	77,41	78,56	85,92	86,24
90%			30,21	36,19	61,71	67,12	73,83	77,61	85,21	85,90

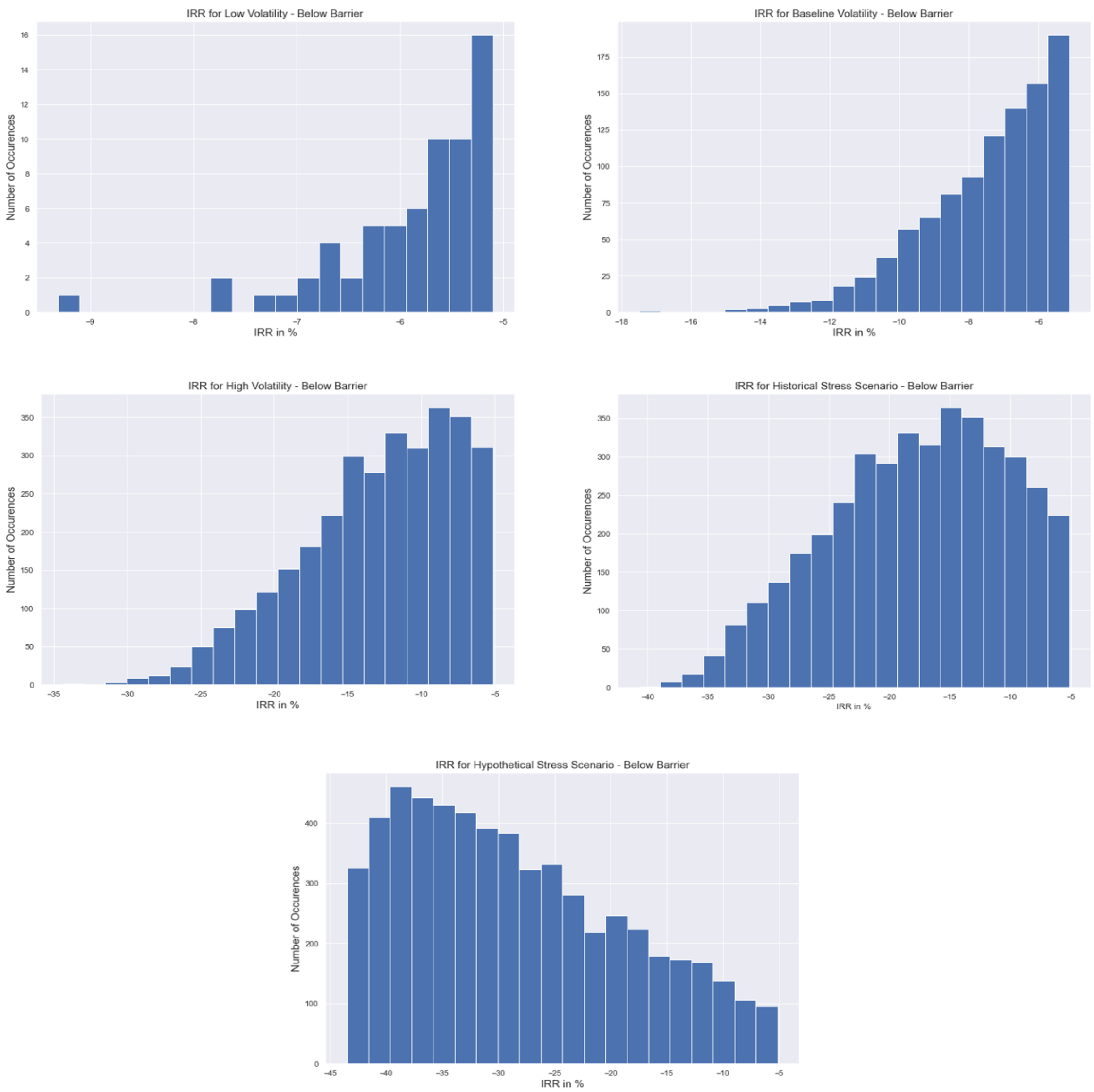
For our baseline model, the one we agree represents better the market behaviour at the time of the issuance of the certificate, the investor has 99% certainty (at issuance) that no more than 72% of the initial investment will be loss in the next 5 years.

5.7.Interest Rate of Return

Figure 9 shows the distribution of the interest rate of return (IRR) of the express certificate, for the investor. The figure only represents the IRR for the simulations in which the product survived until the last year and closed below the barrier. For the other possible terminations of the product, the IRR has a constant value of 2.65%, which is equal to the annual coupon rate.

For the cases represented in the graphs, the IRR is always going to be negative, because if the underlying level is below the barrier level in the maturity date, the investor is bound to receive less than the notional amount.

Figure 9. IRR Distribution.



These distributions behave in a similar way to those of the payoffs in 2025, as the lower the payoffs, the lower the IRR is going to be. In the graphs, there is a gap between an IRR of 0 and 5, due to the gap in the possible payoffs at the maturity date, analysed in Figure 1. The evolution of IRR follows an inverse relationship with volatility, as the IRR get progressively lower as volatility increases. The lowest limit for IRR starts at -10% for the lower volatility and reaches almost -45% for the hypothetical stress scenario. The extreme scenario is not as peaked as the payoffs distribution because it accounts for the 5 coupons received, which contributes to smooth the most extreme simulations.

Table 7 shows the average values for the IRR in each one of the different volatilities, first only for the cases in which the product closes below the barrier, and then for the express certificate, weighting in the probabilities of the product to terminate early or at maturity above the barrier. This will contribute to a rise in the IRR given that the IRR is positive for the previously mentioned outcomes.

For the baseline model, the certificate would be profitable with a return of 1.6%. For the ongoing models, the product would not yield positive returns. In the case of the Express certificate, the lower the volatility, the higher the return, with a cap at 2.65%.

Table 7. Express Certificate IRR.

Average IRR	σ = 7,01%	σ = 12,78%	σ = 29,25%	σ = 42,29%	σ = 76,68%
Year 5- Below Barrier	-5,88%	-7,47%	-12,94%	-17,66%	-28,52%
Product	2,59%	1,63%	-2,32%	-5,61%	-15,24%

5.8. Greek Sensitivity Analysis

Table 8 compiles the sensitivity values for the 4 greek letters, for all the volatilities analysed throughout this project. These sensitivities are divided into the two components of the certificate, and them presented as the total sensitivity for the product.

Figure 10 represents the evolution of the greek letters in function of volatility. In this figure, the volatility is presented into multiples of 10, and sensitivities were computed based on those volatilities. The graphs do not feature the same volatilities/sensitivities of the models presented in Table 8. The objective of Figure 10 is to study the evolution of the sensibilities of the product within a regular scale.

For delta, an increase in 1% of the underlying level is applied. For the DIP option, an increase in the underlying level results in a decrease of the option's value. This happens because as the underlying level increases, the probability of the product to be called back in the first observation date increases as well. Consequently, the probability of the of the product to reach the maximum maturity, when the put option can come into existence, decreases. The delta sensitivity of the DIP, however, decreases as volatility goes up. Regarding the delta for the callable bond, its evolution is inverse to the one of the option's. The callable bond has long delta, because it's value benefits from the product being called prior to maturity, or at maturity above the barrier level.

Table 8. Greek Letters of Express Certificate Components.

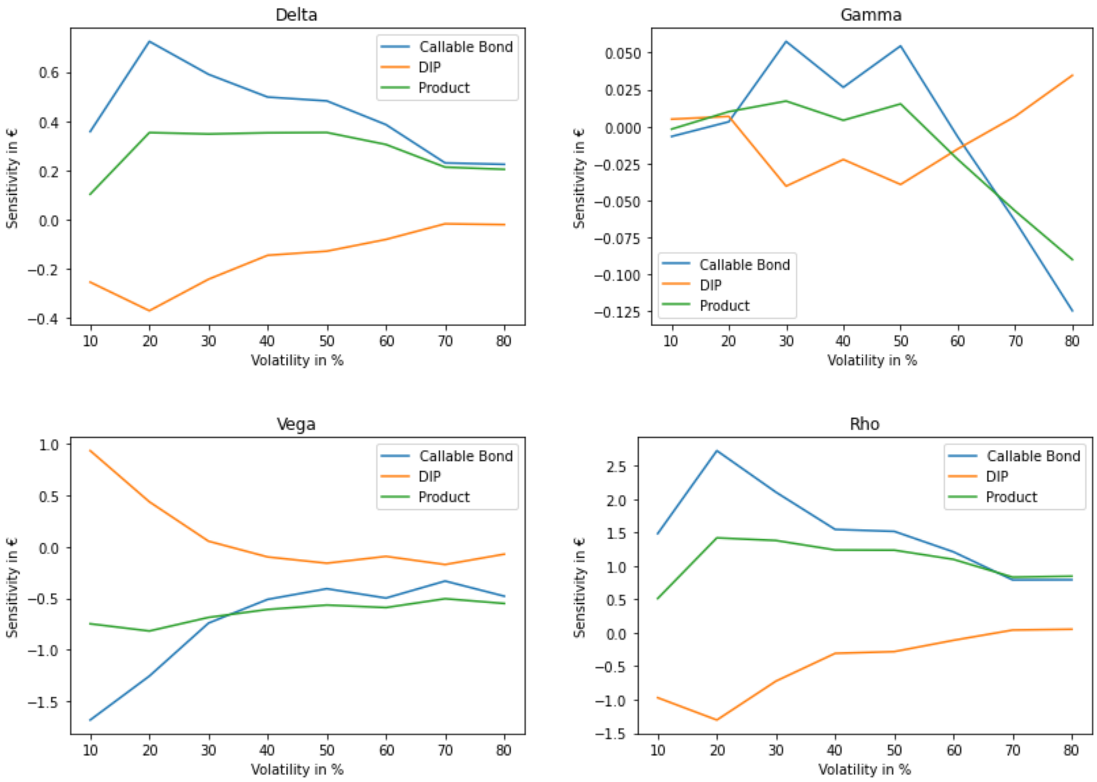
Greek Letters	σ = 7,01%	σ = 12,78%	σ = 29,25%	σ = 42,29%	σ = 76,68%
Down-and-In Put Greeks					
Delta	-0,087 €	-0,435 €	-0,178 €	-0,081 €	-0,025 €
Gamma	0,005 €	0,005 €	0,079 €	0,064 €	0,044 €
Vega	0,649 €	0,990 €	0,083 €	-0,059 €	-0,074 €
Rho	-0,240 €	-1,387 €	-0,677 €	-0,350 €	0,030 €
Callable Bond Greeks					
Delta	0,024 €	0,700 €	0,474 €	0,390 €	0,259 €
Gamma	-0,017 €	-0,007 €	-0,155 €	-0,117 €	-0,155 €
Vega	-1,076 €	-1,909 €	-0,802 €	-0,600 €	-0,507 €
Rho	0,130 €	2,359 €	2,009 €	1,639 €	0,836 €
Express Certificate Greeks					
Delta	-0,062 €	0,264 €	0,297 €	0,308 €	0,233 €
Gamma	-0,012 €	-0,002 €	-0,076 €	-0,053 €	-0,110 €
Vega	-0,427 €	-0,919 €	-0,718 €	-0,659 €	-0,580 €
Rho	-0,111 €	0,972 €	1,332 €	1,290 €	0,866 €

For the gamma sensitivity, it was considered the change in delta for a 1% increase in the underlying value. This is the weakest sensitivity of the product. This low gamma values mean that the rate of change in delta is very low. In practical terms, a low gamma, low change is delta, allows for a less volatile hedging. Low gamma portfolios/positions, need to be adjusted less frequently.

For vega, an increase in 1% on the volatility is applied. For the DIP option, an increase in volatility reflects in an increase of the option's value, up until a volatility of 30%, because it increases the probability of the put option to come into existence. However, the sensibility flattens from 40% volatility onwards. For the callable bond, the sensibility to volatility is the inverse of the option's, because as volatility increases so does the probability of the product being called earlier or at maturity above the barrier.

For rho, an increase of 1% in the risk-free rate is applied. Given that the original risk-free was slightly negative, this small 1% change is enough to result in a positive risk-free rate. The risk-free rate was only applied as a drift term for geometrical Brownian motion. This signal change from negative to positive is going to result in a slight positive change in the simulated underlying levels, instead of a negative one. This is the reason why the evolution of rho sensitivities is so similar (almost identical) to the evolution in delta sensitivities. The difference is that the sensitivities are bigger for rho than for delta, despite the similarities in the evolution.

Figure 10. Evolution of the Express Certificate Greek Letters, over Volatility.



The sensitivity for the whole express certificate is merely the sum of its component's sensibilities. Given that the callable bond accounts for the majority of the certificate's value, the product's greeks will follow an evolution more similar to the callable bond, rather than the DIP option.

6. CONCLUSION

On this project, we analyse the composition of an express certificate, a type of autocallable structured product. Based on the assumptions and parameters defined for the Monte Carlo simulation model, we reached a product value of 99.46 €, which is almost at par with the assumed issue price of 100 €. We also concluded that, although the product has 3 early autocallable dates, the probability of the product terminating in the first one is disproportionally bigger than the rest.

In which regards the risk-return characteristics of this certificate, the holder of the product will not get a higher return than 2.65% annually. Although the returns are capped, the losses can be of almost all of the initial investment of 100€, being the minimum payoff 13.25€, the sum of the 5 coupons.

Apart from the main evaluation model, the baseline model, the analysis of 4 alternative models with different volatilities, allowed us to study the effects of an increase in volatility. Regarding the main conclusions, the increase in the underlying volatility results in the following:

(i) Decreases the probability of the product to be called for early termination; Increases the probability of the certificate to terminate below the barrier at maturity.
(ii) Increases the Expected life of the Express Certificate.
(iii) Decreases the overall value of the product; Decreases the value of the callable bond component; Increases the value of the DIP option until a cut-off point (near 30% volatility) and then the value decreases with further increases in volatility.
(iv) Increases the 5-year Value-at-Risk and, consequentially, the Conditional Value-at-Risk of the investment in the certificate.
(v) Decreases the internal rate of return of the product.
(vi) Decreases the product sensitivity to underlying price (delta), to interest rate (rho) and to volatility (vega).

There were many challenges throughout the realization of this process. The lack of literature on specialized empirical analysis on structured products challenged us in a sense that we did not have any concrete benchmark for the project. This led to many assumptions, especially on the decomposition of the product, where the literature is not consensual. On the second part of the analysis, the lack of literature on exotic greek letters limited the analysis in a sense that there is no comparison or benchmark to make the analysis more robust. Additionally, the lack of cooperation from Deutsche Bank in providing its secondary market quotes for the express certificate, limited us from performing the relevant analysis of comparing our theoretical prices with the price quotes.

For future studies on express certificates, it would be relevant to test another pricing model. According to European regulation, the recommended model relies on simulations, but with historical returns (with replacement). To further the risk analysis, it would also be relevant to account for spot shocks, and risk-free rate shocks in the stress testing scenarios, instead of only volatility shocks. Additionally, for future work, it would be relevant to analyse a potential hedging strategy for the Express Certificate and study the effects of a barrier shift for this product.

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8. APPENDIX

Figure A.1 - Fixed Coupon Express Certificate KID.

Key Information Document

Purpose

This document provides you with key information about this investment product. It is not marketing material. The information is required by law to help you understand the nature, risks, costs, potential gains and losses of this product and to help you compare it with other products.

Product

Product name	Fixed Coupon Express Certificate linked to EURO STOXX 50® Index (Price Index)
Product identifiers	ISIN: DE000DB9U0V9 | WKN: DB9U0V
PRIIP manufacturer	Deutsche Bank AG. The product issuer is Deutsche Bank AG, Frankfurt.
Website	www.db.com/contact
Telephone number	Call +49-69-910-00 for more information.
Competent authority of the PRIIP manufacturer	German Federal Financial Supervisory Authority (BaFin)
Date of production	7 December 2021

You are about to purchase a product that is not simple and may be difficult to understand.

1. What is this product?

Type

German law governed certificates

Objectives

(Terms that appear in **bold** in this section are described in more detail in the table(s) below.)

The product is designed to provide a return in the form of (1) regular fixed coupon payments and (2) a cash payment on termination of the product. The timing and amount of this payment will depend on the performance of the **underlying**.

Early termination following an autocall: The product will terminate prior to the **maturity date** if, on any **autocall observation date**, the **reference level** is at or above the relevant **autocall barrier level**. On any such early termination, you will on the immediately following **autocall payment date** receive, in addition to a final coupon payment, a cash payment equal to the autocall payment of EUR 100. No coupon payments will be made on any date after such **autocall payment date**. The relevant dates and **autocall barrier levels** are shown in the table(s) below.

Autocall observation dates	*Autocall barrier levels*	*Autocall payment dates*
2 February 2022	3,732.28	7 February 2022
1 February 2023	3,732.28	6 February 2023
1 February 2024	3,732.28	6 February 2024
3 February 2025	2,425.982	Maturity date

Coupon: If the product has not terminated early, on each **coupon payment date** you will receive a coupon payment of EUR 2.65. The coupon payments are not linked to the performance of the **underlying**. The relevant dates are shown in the table(s) below.

Coupon payment dates
8 February 2021
7 February 2022
6 February 2023
6 February 2024
Maturity date

Termination on the maturity date: If the product has not terminated early, on the **maturity date** you will receive:

1. if the **final reference level** is at or above the **barrier level**, a cash payment equal to EUR 100; or
2. if the **final reference level** is below the **barrier level**, a cash payment directly linked to the performance of the **underlying**. The cash payment will equal (i) the **product notional amount** multiplied by (ii) (A) the **final reference level** divided by (B) the **strike level**.

Under the product terms, certain dates specified above and below will be adjusted if the respective date is either not a business day or not a trading day (as applicable). Any adjustments may affect the return, if any, you receive.

When purchasing this product during its lifetime, the purchase price may include accrued coupon on a pro rata basis.

You do not have any entitlement to a dividend from the **underlying** and you have no right to any further entitlement resulting from the **underlying** (e.g., voting rights).

Underlying	EURO STOXX 50 (Price return index) (ISIN: EU0009658145)	**Strike level**	3,732.28
Underlying market	Equity	**Barrier level**	2,425.982
Product notional amount	EUR 100	**Reference level**	The closing level of the **underlying** as per the **reference source**
Product currency	Euro (EUR)	**Reference source**	STOXX Limited, Zurich
Underlying currency	Euro (EUR)	**Final reference level**	The **reference level** on the **valuation date**
Issue date	4 February 2020	**Valuation date**	3 February 2025
Initial reference level	3,732.28	**Maturity date / term**	6 February 2025

The issuer may terminate the product with immediate effect in the event of obvious written or mathematical errors in the terms and conditions or if certain extraordinary events provided in the terms and conditions occur. Examples of extraordinary events include (1) material changes, particularly in connection with the **underlying**, including where an index ceases to be calculated, and (2) events, in

Page 1 DocID: 5b69c223-e8c3-44eb-8db2-dbc9701d0685 particular due to changes in certain external conditions that hinder the issuer in meeting its obligations in connection with the product or – depending on the terms and conditions of the security – otherwise affect the product and/or the issuer. In case of immediate termination, the return (if any) may be significantly lower than the purchase price, but will reflect the product's market value and, if higher, any minimum redemption (alternatively, in some cases the corresponding compounded amount may be paid out at the product's scheduled maturity). Instead of immediate termination, the issuer may also amend the terms and conditions.

Provided that in the event of any inconsistency and/or conflict between the foregoing paragraph and any applicable law, order, rule or other legal requirement of any governmental or regulatory authority in a territory in which this product is offered, such national requirements shall prevail.

Intended retail investor

The product is intended for private clients who pursue the objective of general capital formation/asset optimization and have a medium-term investment horizon. This product is a product for clients who have sufficient knowledge and / or experience to make an informed investment decision. The investor can bear losses up to the total loss of the capital invested and attaches no importance to capital protection.

2. What are the risks and what could I get in return?

Risk indicator

1	2	3	4	5	6	7

Lower risk **Higher risk**

The risk indicator assumes you keep the product for 3 years and 2 months. The actual risk can vary significantly if you cash in at an early stage and you may get back less.

The summary risk indicator is a guide to the level of risk of this product compared to other products. It shows how likely it is that the product will lose money because of movements in the markets or because we are not able to pay you.

We have classified this product as 1 out of 7, which is the lowest risk class. This rates the potential losses from future performance at a very low level, and poor market conditions are very unlikely to impact our capacity to pay you.

This product does not include any protection from future market performance so you could lose some or all of your investment.

If we are not able to pay you what is owed, you could lose your entire investment.

Performance scenarios

Market developments in the future cannot be accurately predicted. The scenarios shown are only an indication of some of the possible outcomes based on recent returns. Actual returns could be lower.

Investment: EUR 10,000				
Scenarios		***1 year***	***2 years***	***3 years and 2 months (Recommended holding period)***
Stress scenario	**What you might get back after costs**	EUR 8,339.17	EUR 9,058.79	EUR 10,161.35
	Average return each year	-16.61%	-4.82%	0.51%
Unfavourable scenario	**What you might get back after costs**	EUR 9,977.56	EUR 9,962.67	EUR 10,161.35
	Average return each year	-0.22%	-0.19%	0.51%
Moderate scenario	**What you might get back after costs**	EUR 10,171.53	EUR 10,182.25	EUR 10,161.35
	Average return each year	1.72%	0.91%	0.51%
Favourable scenario	**What you might get back after costs**	EUR 10,174.80	EUR 10,208.30	EUR 10,161.35
	Average return each year	1.75%	1.04%	0.51%

This table shows the money you could get back over the next 3 years and 2 months under different scenarios, assuming that you invest EUR 10,000.

The scenarios shown illustrate how your investment could perform. You can compare them with the scenarios of other products. The scenarios presented are an estimate of future performance based on evidence from the past on how the value of this investment varies, and are not an exact indicator. What you get will vary depending on how the market performs and how long you keep the product. The stress scenario shows what you might get back in extreme market circumstances, and it does not take into account the situation where we are not able to pay you.

The figures shown include all the costs of the product itself, but may not include all the costs that you pay to your advisor or distributor. The figures do not take into account your personal tax situation, which may also affect how much you get back.

3. What happens if Deutsche Bank AG, Frankfurt is unable to pay out?

You are exposed to the risk that the issuer might be unable to fulfil its obligations in respect of the product – e.g. in the event of insolvency (inability to pay / over-indebtedness) or an administrative order of resolution measures. In case of a crisis of the issuer such an order can also be issued by a resolution authority in the run-up of an insolvency proceeding. In doing so, the resolution authority has extensive intervention powers. Among other things, it can reduce rights of the investors to zero, terminate the product or convert it into shares of the issuer and suspend rights of the investors. With regard to the basic ranking of the issuer's obligations in the event of action by the resolution authority, please see www.bafin.de and search for the keyword "Haftungskaskade". A total loss of your capital invested is possible. The product is a debt instrument and as such is not covered by any deposit protection scheme.

4. What are the costs?

The Reduction in Yield (RIY) shows what impact the total costs you pay will have on the investment return you might get. The total costs take into account one-off, ongoing and incidental costs.

The amounts shown here are the cumulative costs of the product itself, for three different holding periods. They include potential early exit penalties. The figures assume you invest EUR 10,000. The figures are estimates and may change in the future.

Costs over time

Investment: EUR 10,000			
Scenarios	*If you cash in after 1 year*	*If you cash in after 2 years*	*If you cash in at the end of the recommended holding period*
Total costs	EUR 0.00	EUR 0.00	EUR 0.00
Impact on return (RIY) per year	0.00%	0.00%	0.00%

The costs shown in the table above represent how much the expected costs of the product would affect your return, assuming the product performs in line with the moderate performance scenario.

The person selling you or advising you about this product may charge you other costs. If so, this person will provide you with information about these costs, and show you the impact that all costs will have on your investment over time.

Composition of costs

The table below shows:

- The impact each year of the different types of costs on the investment return you might get at the end of the recommended holding period.
- The meaning of the different cost categories.

The table shows the impact on return per year.			
One-off costs	*Entry costs*	0.00%	The impact of the costs already included in the price.
	Exit costs	0.00%	The impact of the costs of exiting your investment when it matures.
Ongoing costs	*Portfolio transaction costs per year*	0.00%	The impact of the costs of us buying and selling underlying investments for the product.
	Other ongoing costs	0.00%	The impact of the costs that we take each year for managing your investments.

5. How long should I hold it and can I take money out early?

Recommended holding period: 3 years and 2 months

The product aims to provide you with the return described under "1. What is this product?" above. However, this only applies if the product is held to maturity. It is therefore recommended that the product is held until 6 February 2025 (maturity).

The product does not guarantee the possibility to disinvest other than by selling the product either (1) through the exchange (where the product is listed) or (2) off-exchange. No fees or penalties will be charged by the issuer for any such transaction. However if you sell the product in the secondary market you will incur a bid/offer spread. By selling the product before its maturity, you may receive back less than you would have received if you had kept the product until maturity.

Exchange listing	Börse Stuttgart and Deutsche Börse AG	**Last exchange trading day**	31 January 2025 (Börse Stuttgart) and 31 January 2025 (Deutsche Börse AG)
Smallest tradable unit	1 unit	**Price quotation**	Units

In volatile or unusual market conditions, or in the event of technical faults/disruptions, the purchase and/or sale of the product can be temporarily hindered and/or suspended and may not be possible at all.

6. How can I complain?

Any complaint regarding the conduct of the person advising on, or selling, the product can be submitted directly to that person.

Any complaint regarding the product or the conduct of the manufacturer of this product can be submitted in writing at:

Jurisdiction	Postal address	Email address	Website
Germany	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Austria	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Luxembourg	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com

7. Other relevant information

Any additional documentation in relation to the product and in particular the prospectus, any supplements thereto and the final terms are published on the manufacturer's website (www.xmarkets.db.com/DocumentSearch ; after entering of the respective ISIN or WKN), all in accordance with legal requirements. In order to obtain more detailed information - and in particular details of the structure and risks associated with an investment in the product - you should read these documents. These documents are also available free of charge from Deutsche Bank AG, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany, in accordance with legal requirements.