Lisbon School of Economics & Management
Universidade de Lisboa

MASTER

FINANCE

MASTER´S FINAL WORK

PROJECT

A STUDY ABOUT A FIXED COUPON EXPRESS CERTIFICATE LINKED TO THE EURO STOXX 50 INDEX: BACKTESTING AND PERFORMANCE ASSESSMENT

MIGUEL RAMOS DE ALMEIDA

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JUNE - 2023

Abstract

This Master's Final Work aims to study and value a Fixed Coupon Express Certificate linked to the EURO STOXX 50 Index, an autocallable structured product issued by Deutsche Bank on the 4th of February 2020. In this analysis, two paths are followed: a Monte Carlo Simulation with 10,000 trials, considering three different volatilities (σ = 7.01%; σ = 12.78%; σ = 29.25%), and a comparison between the aforementioned Monte Carlo Simulation and Historical Data on the index's performance since its inception in 1998. Aside from the valuation itself, this project also includes a probability and average lifespan analysis on the Express Certificate.

From the Monte Carlo Simulation process, we observe that a higher volatility results in a lower product's average present value and a lower probability of the investor earning back more than he or she paid for the product.

From the comparison between Historical Data and the Monte Carlo Simulation, we infer that the Monte Carlo Simulation appears to be too optimistic when dealing with the possibility of the Certificate surviving until its full maturity. Additionally, the Simulation returns a higher product's average present value and a higher probability of the investor earning back more than he or she paid for the product when compared to Historical Data on the performance of the index.

Taking into account both sets of results, the product seems slightly overpriced.

Keywords: Express Certificate; Autocallable Structured Product; Monte Carlo Simulation; Volatility; Probability

Resumo

Este Trabalho Final de Mestrado tem como objetivo estudar e avaliar um Fixed Coupon Express Certificate associado ao Índice EURO STOXX 50, um produto estruturado com possibilidade de chamada antecipada emitido pelo Deutsche Bank a 4 de fevereiro de 2020. Nesta análise, duas abordagens são seguidas: uma Simulação de Monte Carlo, com 10,000 iterações, considerando três volatilidades diferentes (σ = 7.01%; σ = 12.78%; σ = 29.25%), e uma comparação entre a supramencionada Simulação de Monte Carlo e Dados Históricos sobre a performance do índice desde a sua criação em 1998. Para além da avaliação em si, este projeto também inclui uma análise de probabilidade e da vida útil média do produto.

A partir do processo da Simulação de Monte Carlo, observamos que uma volatilidade mais elevada resulta num valor atual médio do produto mais reduzido e numa menor probabilidade de o/a investidor/investidora receber mais do que ele/ela pagou pelo produto.

A partir da comparação entre os Dados Históricos e a Simulação de Monte Carlo, inferimos que a Simulação de Monte Carlo aparenta ser demasiado otimista ao lidar com a possibilidade do produto sobreviver até à sua maturidade total. Adicionalmente, a Simulação resulta num valor atual médio do produto mais elevado e numa maior probabilidade de o/a investidor/investidora receber mais do que ele/ela pagou pelo produto quando comparada com os Dados Históricos da performance do índice.

Considerando os dois conjuntos de resultados, o produto aparenta estar ligeiramente sobreavaliado.

Palavras-chave: Express Certificate; Produto Estruturado; Simulação de Monte Carlo; Volatilidade; Probabilidade

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1. Introduction

The purpose of this Master's Final Work is to study and value a Fixed Coupon Express Certificate linked to the EURO STOXX 50 Index, issued by Deutsche Bank on the 4th of February 2020. This product was first introduced to us during the first semester of our second year in the Master in Finance, through the elective course of Case Studies in Financial Engineering, taught by Professor João Duque. We were asked at the time to conduct an analysis on this particular Express Certificate and reach a conclusion regarding its valuation.

After the first contact with the product, all the members of our group opted to proceed with an in-depth analysis of this Certificate as their Master's Final Work, keeping a common basis between everyone but with each member focusing on different research aspects. In this Master's Final Work, the individual part of the study consists of a backtesting and performance assessment, establishing a comparison between historical data gathered from the EURO STOXX 50 Index since its inception in 1998 and the results from the Monte Carlo Simulation, the pricing method employed for the valuation of the product.

A Fixed Coupon Express Certificate is a specific type of autocallable structured product. A structured product is a financial instrument comprised of two or more components, which, in this case, correspond to a bond component and a derivatives component. The bond component provides regular payments to the holders of the product, which are fixed at issuance and independent from the performance of the underlying, whereas the derivatives component's payoff is directly dependent on the performance of the underlying. In our case, the EURO STOXX 50 Index acts as the underlying to the Express Certificate. This product is classified as autocallable due to the possibility of being called back on certain dates if the underlying price is at or above a pre-specified threshold.

Even though these products are still relatively recent in the financial markets, there is already a significant amount of literature on the field, which is dissected in Section 2 of this project. However, our purpose of investigation is somewhat different and goes beyond the literature we came across when researching this subject. Instead of conducting a study on large sets of data and analysing the effects of changes in macro variables across the structured products' markets, we focus solely on our Express Certificate, building a model for its valuation fitted to its specific characteristics and examining a multitude of features that derive from that pricing method.

Aside from looking into the cash-flows generated by the product and its possible present value, the analysis includes a detailed investigation into the unconditional and conditional probabilities of the product being called back at each autocall observation date, since each termination date results in a different cash-flow structure and, therefore, a different product valuation. Furthermore, building on this probability analysis framework, we will look further into the fugit of the Certificate, which corresponds to its average lifespan.

The remainder of the text is organized as follows. Section 2 provides a brief literature review on the topic. Section 3 explains the methodology used throughout the project. Section 4 presents and discusses the main results of the analysis. Finally, Section 5 concludes and summarizes the key findings.

2. Literature Review

The main purpose of this project is to correctly value a Fixed Coupon Express Certificate, a financial instrument that can be classified as an autocallable structured product. Before entering the analysis per se, we delve into the world of structured securities, with the aim of understanding what has already been done in the field, what is currently being investigated and the different approaches proposed when it comes to the valuation of this kind of financial products.

To start with, we need to understand how structured derivatives emerged in the financial landscape and why do investors look specifically for these kinds of products. Bergstresser (2008) denotes that structured notes offer investors exposure to asset classes and investment themes that they may find difficult to obtain in other ways, allowing them to access a wide set of desired risk exposures and helping to complete otherwise incomplete markets. According to the author, these characteristics contributed significantly towards the appearance and rapid growth of structured products' markets in the late 1990s and 2000s, particularly in Europe and Asia.

Wilhelm's (2009) research on express certificates in Germany corroborates these findings, mentioning a rise of 1000% in the number of structured derivatives traded in German markets between 2001 and 2008 and putting forward another hypothesis: even though payoff structures have become increasingly more complex with time, its diversification and adaptability to different investors and different risk profiles may actually contribute to an increase in demand and trading volume for these products.

This viewpoint is supported by Ribeiro (2018), who states that structured products appear as attractive financial solutions, offering plenty of tailor-made solutions due to the numerous possible features of the option derivatives component and payoffs adapted to each risk profile.

Looking back at Wilhelm (2009), we find a contradictory remark: diversification in the payoff structure of express certificates might be appealing, but that does not imply that investors, or even the people who sell the certificates, have a full understanding of the products' characteristics. Albuquerque et al. (2015) add to this point, stating that, in spite of their growth in popularity and the opportunity autocallable securities give investors to earn high coupons in a low-yield environment, financial advisers who help sell these products associate their high yield with their complexity and are reluctant to promote them to less sophisticated investors.

Looking further into the issuance of structured products and the decision process behind the composition of a structured product, Bergstresser (2008) points out that issuances are more common for underlying securities that have recently performed well, as investors chase returns above anything else, and that are easier to hedge, which corresponds to securities with higher trading volumes and lower return standard deviation. Albuquerque et al.'s (2015) research provides a similar takeaway, in the sense that underwriters of autocallables do not appear to choose underlying assets randomly, instead opting for securities that display high volatility and perform well in the stock market, with prices at or near 52-week high values.

In his analysis, Bergstresser (2008) also puts forward that evidence on the performance of structured notes suggests that these products are sold at a significant premium when compared to their estimated valuations, contributing to a phenomenon of overpricing that we will also try to assess in our own project.

Having explored the general context of structured products and, more specifically, autocallables, Célérier et al. (2021) provide more detail on a specific type of retail financial products with characteristics that match those of our Express Certificate, called short put products. These offer a payoff that somewhat imitates that of a callable bond, since the capital is protected on the downside as long as the underlying stays beyond a barrier. As soon as the underlying goes below that barrier, the investor participates in the performance of the underlying. These products pay a fixed coupon every period until maturity and often offer an early redemption if the underlying price stays above a certain level, making them autocallable.

When it comes to pricing these derivatives, literature is mixed, but the seemingly most supported approach is the Monte Carlo Simulation. Hull (2018) states that the Monte Carlo Simulation is usually chosen to value derivatives whose payoff is dependent on the history of the underlying variable. According to the author, it is used when the payoffs depend on the path followed by the underlying variable and when they can occur at several times during the lifetime of the derivative rather than all at the end. Additionally, it is mentioned that the Monte

Carlo Simulation can be used to value European-style derivatives and can cope with a great deal of complexity as far as the payoffs are concerned.

This rationale is supported by Fries and Joshi (2008), who constructed a conditional analytic Monte Carlo pricing scheme sampling only the survival domain of the autocallable product, and Alm et al. (2013), who introduced a Monte Carlo algorithm adaptation that allows for stable differentiation through simple finite differences. Wilhelm (2009), in his analysis regarding the pricing of derivatives with truncated underlying paths, also mentions the Monte Carlo Simulation as a valuation method that can be applied when working with express certificates.

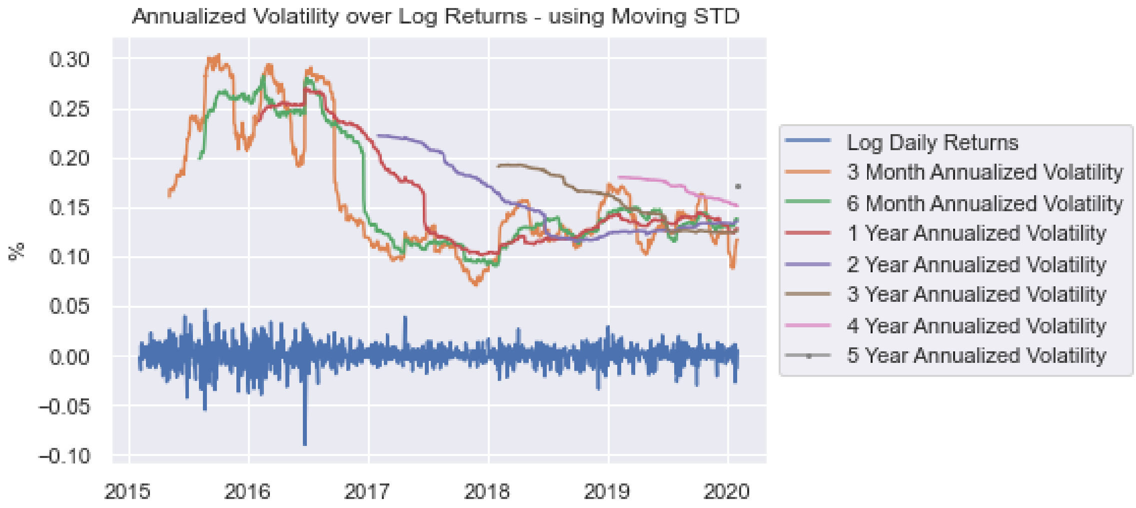
3. Methodology

3.1 Data

This project focuses on the valuation of a Fixed Coupon Express Certificate, issued by Deutsche Bank on the 4th of February 2020 and maturing on the 6th of February 2025. The EURO STOXX 50 Index acts as the underlying to the Express Certificate, since the value of the Certificate is directly dependent on the price of the index on specific observation dates throughout its lifespan. Therefore, in our analysis, we consider data regarding three key variables: underlying price, risk-free rate and cost of funding.

We gather EURO STOXX 50's historical daily adjusted closing prices from up to five years prior to the issuance of the Certificate, corresponding to an interval ranging from the 4th of February 2015 to the 4th of February 2020 and resulting in a total of 1,270 observations. After computing the underlying's daily log returns, we obtain a range of historical annualized volatilities for the five-year period (Figure 1) by multiplying the standard deviation of the daily log returns by the square root of the number of trading days in a year. For analysis' purposes, the one-year historical annualized volatility, spanning from the 4th of February 2019 to the 3rd of February 2020, is selected as the most balanced and fittest representative of the index's volatility, with a value of 12.78%.

Figure 1. Historical Annualized Volatilities



The risk-free rate represents the theoretical rate of return of an investment with zero risk associated. Therefore, we consider the yield of a Germany government bond, issued on the 3rd of February 2020, one day before the product's issuance, and with the same five-year maturity of the Express Certificate as our risk-free rate, with a value of – 0.647%.

Regarding Deutsche Bank's cost of funding, we follow the approach proposed by Aymanns et al. (2016) and define cost of funding as the ratio between total interest expense and average interest-bearing liabilities. Due to the Certificate being issued in February 2020, the ratio is calculated using values from the Bank's 2019 Annual Report for both variables, resulting in a value of 1.6021%.

3.2 The Product

As mentioned previously, this analysis is based on a Fixed Coupon Express Certificate issued by Deutsche Bank, more specifically, the Certificate issued with the ISIN Code DE000DB9U0V9. Deutsche Bank names this product Fixed Coupon Express Certificate linked to EURO STOXX 50 Index, due to the fact that its value depends on the price of the index. The Certificate is issued at par and provides investors with a return in the form of annual fixed coupon payments plus a cash payment, dependent on the performance of the underlying, on the product's termination. Deutsche Bank classifies this product as a 1 out of 7 in the risk scale, the lowest risk class. This is calculated based on the assumption that the product is kept until maturity and does not terminate early.

The main characteristics of this product are displayed in its Key Information Document (KID), which can be consulted in Figures A.1, A.2 and A.3 in the Appendix. According to the European Insurance and Occupational Pensions Authority (EIOPA), the KID is a standardized document that provides consumer-friendly information about the key features of investment products, including what investors might gain and the risks and costs involved. Its ultimate goal is to improve transparency in the investment market.

Looking further into the Certificate, we can classify it as a structured product, featuring both a bond component and a derivatives component, in the form of a put option.

Furthermore, due to its autocallability feature, the bond can be categorized as a callable bond. There are two elements embedded in this callable bond: a regular bond, with the same five-year maturity as the product, that pays an annual coupon of 2.65€ plus a principal of 100€ (equal to the product's notional amount) at maturity or when the product is called back and nothing afterwards; a Bermudan call option, with a maturity of three years starting in February 2021 and with the possibility of being exercised on any observation date from 2022 to 2024. The holder of the Certificate has a long position on the callable bond.

The put option can be described as a European down-and-in put option, with a maturity of five years and a payoff dependent on the underlying's price on the final observation date of February the 3rd 2025, the only date when the option can be exercised. The strike is set at 100% and the barrier level at 65% of the initial underlying price (S_0) of 3,732.28 points. The holder of the Certificate has a short position on the put option.

The Express Certificate is issued on the 4th of February 2020 and its maximum maturity date is on the 6th of February 2025. However, it can be called back and terminate earlier if a certain criteria is met.

Deutsche Bank defines four autocall observation dates, distancing approximately one year from one another: 2nd of February 2022; 1st of February 2023; 1st of February 2024; 3rd of February 2025. Additionally, for each autocall observation date, there is a corresponding autocall barrier level and an autocall payment date (Table 1).

Table 1. Autocall Observation Dates, Barrier Levels and Payment Dates

Autocall Observation Dates	Autocall Barrier Levels	Autocall Payment Dates
2 February 2022	3,732.28 (100% of S_0)	7 February 2022
1 February 2023	3,732.28 (100% of S_0)	6 February 2023
1 February 2024	3,732.28 (100% of S_0)	6 February 2024
3 February 2025	2,425.982 (65% of S_0)	6 February 2025

If, on any autocall observation date, the underlying price is at or above the relevant autocall barrier level, the product terminates. In the event of early termination, the holder of the product receives a final coupon payment of 2.65€ plus a cash payment of 100€ corresponding to the principal of the bond on the immediately following autocall payment date. Afterwards, no more coupon payments are made.

The coupon payments are not linked to the performance of the index and are fixed at 2.65€. The coupon payment dates correspond to the autocall payment dates, with the exception of 2021, when the product pays the coupon but there is no possibility of it being called back. Therefore, the coupon payment dates are: 8th of February 2021; 7th of February 2022; 6th of February 2023; 6th of February 2024; 6th of February 2025.

If the product survives until maturity, two possibilities arise. If the underlying price on the last autocall observation date (S_T) is at or above the barrier level, set at 2,425.982 points (65% of S_0), the holder receives a cash payment equal to 100€. However, if S_T is below the barrier level, the holder receives a cash payment equal to the product notional amount (100€) multiplied by S_T divided by S_0. Additionally, in both scenarios, the holder also receives a final coupon of 2.65€.

The condition described previously determines the existence of five possible scenarios for termination: (i) the product terminates in 2022; (ii) the product terminates in 2023; (iii) the product terminates in 2024; (iv) the product terminates in 2025 and the underlying price on the final autocall observation date is at or above the barrier level; (v) the product terminates in 2025 and the underlying price on the final autocall observation date is below the barrier level. Each of these scenarios results in a different payoff for the holder of the Certificate:

i. If the product terminates in 2022, the holder receives a coupon payment of 2.65€ in 2021 plus a final coupon payment of 2.65€ and a cash payment of 100€ in 2022.

ii. If the product terminates in 2023, the holder receives a coupon payment of 2.65€ in 2021 and 2022, plus a final coupon payment of 2.65€ and a cash payment of 100€ in 2023.

iii. If the product terminates in 2024, the holder receives a coupon payment of 2.65€ in 2021, 2022 and 2023, plus a final coupon payment of 2.65€ and a cash payment of 100€ in 2024.

iv. If the product terminates in 2025 and S_T is equal or higher than 65% of S_0, the holder receives a coupon payment of 2.65€ in 2021, 2022, 2023 and 2024, plus a final coupon payment of 2.65€ and a cash payment of 100€ in 2025.

v. If the product terminates in 2025 and S_T is lower than 65% of S_0, the holder receives a coupon payment of 2.65€ in 2021, 2022, 2023 and 2024, plus a final coupon payment of 2.65€ and a cash payment of $100€ \times \frac{S_T}{S_0}$ in 2025.

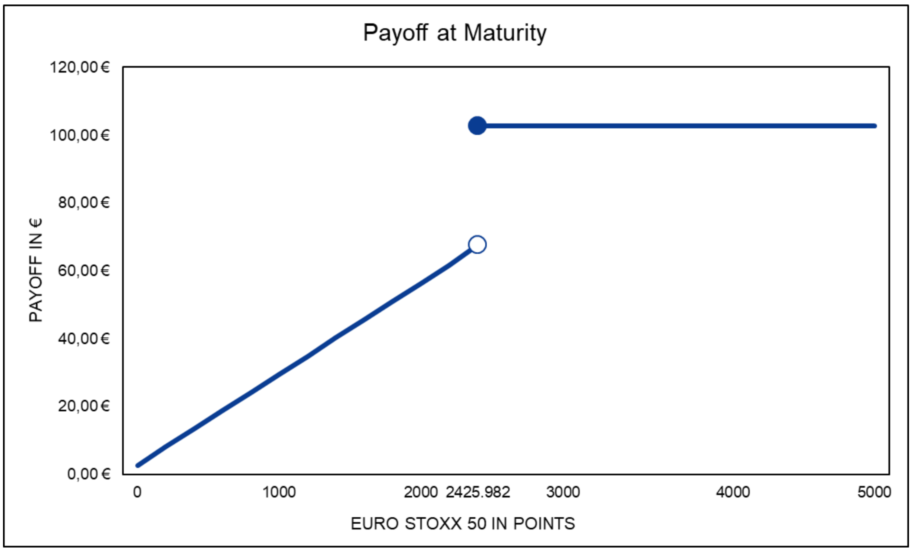
We conclude that, up until the final autocall observation date, all possible cash-flows originate from the long position on the callable bond. The short position on the put option only results in a cash-flow for the holder if the product terminates in 2025 and the final underlying price is below the barrier level. These scenarios and the respective associated cash-flows are summarized on the following Cash-Flow Map (Table 2).

Table 2. Cash-Flow Map

Possible Scenarios	Cash-Flow in 2021	Cash-Flow in 2022	Cash-Flow in 2023	Cash-Flow in 2024	Cash-Flow in 2025		Total Payoff
	From the Bond	From the Bond	From the Bond	From the Bond	From the Bond	From the Put	
1. Product Terminates in 2022	2.65€	102.65€					105.30€
2. Product Terminates in 2023	2.65€	2.65€	102.65€				107.95€
3. Product Terminates in 2024	2.65€	2.65€	2.65€	102.65€			110.60€
4. a) Product Terminates in 2025 Above the Barrier	2.65€	2.65€	2.65€	2.65€	102.65€		113.25€
4. b) Product Terminates in 2025 Below the Barrier	2.65€	2.65€	2.65€	2.65€	2.65€	$100€ \times \frac{S_T}{S_0}$	$5 \times 2.65€ + 100€ \times \frac{S_T}{S_0}$

The figure below (Figure 2) depicts the product's payoff structure at maturity, considering it survives until 2025. The line with the constant slope, on the right, represents the scenarios where S_T is equal to or higher than 65% of S_0 (2,425.982 points) and the holder receives 102.65€ in 2025, whilst the line with the positive slope, on the left, represents the scenarios where S_T is lower than 65% of S_0 and the holder receives 2.65€ plus a cash payment equal to $100€ \times \frac{S_T}{S_0}$ in 2025.

Figure 2. Payoff at Maturity



3.3 Pricing Model

Regarding the pricing of the product, due to a malleability that allows to fit the specific characteristics of the product, we follow the rationale put forward by Hull (2018), which suggests that the Monte Carlo Simulation is the ideal valuation method when dealing with derivatives whose payoffs are dependent on the history of the underlying variable.

In order to perform the Monte Carlo Simulation, we use the Geometric Brownian Motion to generate the possible prospective prices for the underlying, with the following formula:

$$S_{kn} = S_0 \times e^{((\hat{\mu}-\frac{\sigma^2}{2}) \times \Delta_t + \sigma \times \varepsilon_{kn} \times \sqrt{\Delta_t})},$$

where:

- S_{kn} is the generated underlying price for the kth timestep in the nth trial;
- S_0 is the initial underlying price, equal to 3,732.28 points;
- $\hat{\mu}$ is the risk-free rate;
- σ is the volatility;
- Δ_t is the interval between each timestep, measured in years;
- ε_{kn} is the generated random variable for the kth timestep in the nth trial.

The risk-free rate functions as the drift term of the Geometric Brownian Motion, corresponding to the yield of a five-year Germany government bond issued one day before the issuance of the product, with a value of – 0.647%. The volatility takes the historical one-year annualized volatility's value of 12.78%, obtained through the aforementioned computation of the index's daily log returns. Although not a parameter of the model itself, the Bank's cost of funding is calculated using the approach described in Section 3.1 and amounts to a value of 1.6021%.

We decide to introduce a timestep every two months, resulting in a total of 30 timesteps across the five years of the product's life. This means that, for every path, the Geometric Brownian Motion generates 30 prospective underlying prices, with an interval of two months from one another. However, due to the fact that Δ_t is measured in years, its value equals $\frac{1}{6}$ of a year. Even though annual timesteps are more commonly used, we opted for bi-monthly timesteps, which allow us to obtain a more insightful representation of the possible evolution of the index's price throughout time.

For each individual trial (n), 30 random variables denoted by ε_{kn}, one for each timestep (k), are generated through the inverse function of the normal cumulative distribution, with a random probability, a mean of zero and a standard deviation of one. This introduces the randomness effect into our model.

With all the inputs determined, the Monte Carlo Simulation is then repeated 10,000 times. This process starts on the Certificate's issue date and continues until its maturity date, eventually coinciding with all the product's autocall observation dates. This is the basis for our analysis: for every autocall observation date, comparing the 10,000 generated underlying prices with the corresponding autocall barrier level. From that comparison, a multitude of paths are followed.

From a valuation standpoint, the Monte Carlo process enables us to not only determine the cash-flows that the Certificate produces to its holders but also to correctly value each individual component of the structured product. The value of the callable bond is equal to the present value of the cash-flows generated from the long position on the bond, taking into account the callable component of the product, averaged throughout the 10,000 trials. The value of the put option is equal to the present value of the cash-flows generated from the short position on the put option, averaged throughout the 10,000 trials. The sum of the two components' value is naturally equal to the structured product's present value. All the cash-flows are discounted to their present value using Deutsche Bank's cost of funding of 1.6021%.

Additionally, we elaborate a probability analysis in order to test the likelihood of each product termination scenario, since, as shown in Table 2, each scenario leads to a different cash-flow structure and, therefore, a different product valuation. This analysis is carried out through the computation of both the unconditional probabilities of each scenario coming to fruition and the conditional probabilities dependent on certain previous events throughout the life of the product, for example, the probabilities of the Certificate terminating in 2023, 2024 and 2025 given that it did not terminate in 2022. Taking this analysis into account also allows us to compute the average lifespan of the Certificate, also known as fugit, using the unconditional probabilities of the product terminating in each year.

Simultaneously, we run the exact same Monte Carlo Simulation and the exact same analysis considering an underlying volatility of 7.01% and 29.25%. These values correspond, respectively, to the lowest and highest values of the three-month historical annualized volatility displayed in Figure 1.

3.4 Backtesting and Performance Assessment

In this section, the previously dissected Monte Carlo Simulation and Geometric Brownian Motion are put aside and the focus shifts to a backtesting analysis on the performance of the EURO STOXX 50 Index. Furthermore, the main objective is to understand, through the analysis of historical data instead of generated prospective prices, how an Express Certificate with similar characteristics to the structured product we have dealt with throughout this project would have behaved if it had been issued on any other date and how do both analyses compare.

In order to proceed with this analysis, historical adjusted daily closing prices of the index are collected since its launch in 1998, corresponding to an interval ranging from the 6th of April 1998 to the 30th of December 2020 and resulting in a total of 5,788 observations. Each date in that interval functions as a potential issue date for the Certificate to be considered in this backtesting process. Dates from 2021 onwards were not considered as potential issue dates for the product due to the fact that at least two more years would be needed to reach the first autocall observation date, which corresponds to the first time in the product's lifetime where the possibility of termination exists.

From 1998 to 2020, the average number of trading days in a year were 255, so the autocall observation dates are set in a way that they are separated by 255 trading days each. For simplification purposes, even though there is no possibility of the product being called back on that date, the first observation date is set 255 days after the issue date. For a product issued on the first trading day of the index, the 6th of April 1998, the autocall observation dates are the following: 13th of April 1999; 6th of April 2000; 6th of April 2001; 11th of April 2002; 11th of April 2003. The same rationale is applied to potential issue dates until the 30th of December 2020 (Table 3).

Table 3. Potential Historical Issue Dates, Observation Dates and Prices

Issue Date	Price	1st Obs. Date	Price	2nd Obs. Date	Price	3rd Obs. Date	Price	4th Obs. Date	Price	5th Obs. Date	Price
06/04/98	3314.68	13/04/99	3725.4	06/04/00	5179.96						
07/04/98	3325.81	14/04/99	3699.33	07/04/00	5259.52						
…	…	…	…	…	…	…	…	…	…	…	…
18/12/09	2871.22	22/12/10	2869.63	19/12/11	2202.95	13/12/12	2627.66	12/12/13	2928.12		
21/12/09	2926.05	23/12/10	2864.52	20/12/11	2262.39	14/12/12	2630.54	13/12/13	2921.92	12/12/14	3067.32
…	…	…	…	…	…	…	…	…	…	…	…
29/12/20	3581.37	29/12/21	4284.83	27/12/22	3832.89						
30/12/20	3571.59	30/12/21	4306.07	28/12/22	3808.82						

Apart from the first autocall observation date, the conditions are the same as the ones described in Section 3.2. On the second, third and fourth year, the barrier is equal to the price of the underlying at issuance. For a product issued on the 6th of April 1998, that would imply a barrier level of 3,314.68 points for those years. On the fifth year, however, the barrier is set at the same 65% of the initial underlying price. For a Certificate issued on the 6th of April 1998, that would imply a barrier level of 2,154.452 points for that final year.

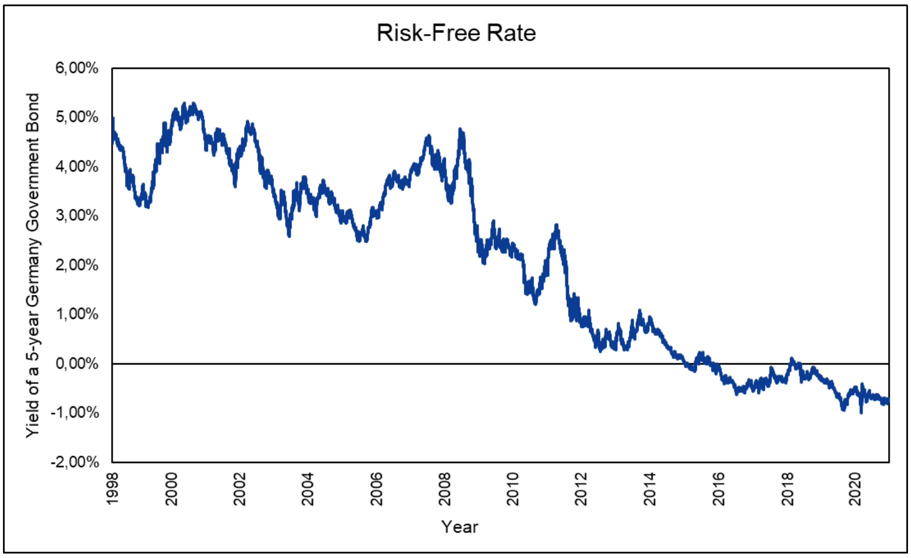
The cash-flow generation process is also equivalent to the one described previously. On the first observation date, the product pays the coupon of 2.65€ regardless of the performance of the underlying. If the underlying price is at or above the barrier on the second, third or fourth observation date, the product pays the coupon of 2.65€ plus the principal of 100€ and terminates immediately. Otherwise, it pays only the coupon of 2.65€ and survives for at least one more year. If the underlying price is at or above the barrier on the fifth observation date, the product pays the coupon of 2.65€ plus the principal of 100€ and terminates. Otherwise, it pays the coupon of 2.65€ plus a cash payment equal to the product notional amount multiplied by the final underlying price divided by the initial underlying price.

Regarding the product issued on the 6th of April 1998, it pays a coupon of 2.65€ after the first observation date and, because the index's price on the second observation date is above the barrier level, it pays another coupon of 2.65€ plus a principal of 100€ and terminates straight after the second observation date.

Repeating this process for the 5,788 possible issue dates considered leads to an analysis similar to the one produced with the Monte Carlo Simulation, in the sense that one can also reach a valuation of the Express Certificate based on the underlying's historical prices and their comparison with the barrier levels. However, a discount rate needs to be introduced in order to properly compare cash-flows originated in different years. In previous sections, we defined the yield of a Germany government bond issued on the day prior to the issuance of the product and with the same five-year maturity as the risk-free rate, and used it as the drift term for the Geometric Brownian Motion. Cash-flows were then discounted to their present value using Deutsche Bank's cost of funding.

However, for this historical analysis, there is no Geometric Brownian Motion or Monte Carlo Simulation, and so the risk-free rate is used directly to discount the product's cash-flows to their present values instead of the Bank's cost of funding. For a product issued on the 6th of April 1998, the risk-free rate is equal to the yield of a five-year Germany government bond issued on the 5th of April 1998, taking a value of 4.47%. The graph below depicts the historical evolution of the yield of a five-year Germany government bond, from April 1998 to December 2020 (Figure 3).

Figure 3. Risk-Free Rate Historical Evolution



The main purpose of this analysis is to create a historical data framework to compare with the Monte Carlo Simulation. Therefore, it makes sense to focus our attention on the same aspects mentioned in the end of Section 3.3: product valuation; unconditional and conditional probability analysis associated with the termination scenarios; average lifespan of the Express Certificate using the previously calculated probabilities.

4. Results

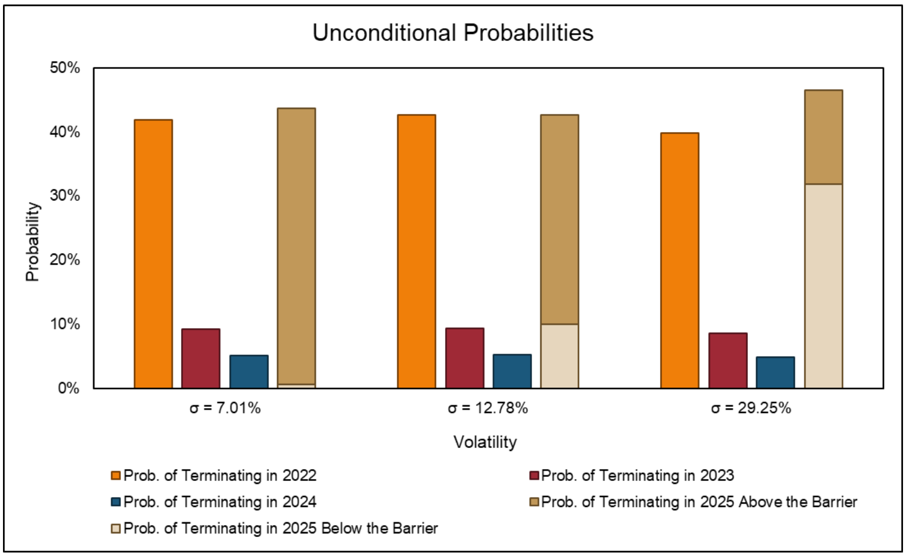
In this chapter, two different sets of results are analysed, with the first one emerging exclusively from the Monte Carlo Simulation, considering the aforementioned volatilities (σ) of 7.01%, 12.78% and 29.25%, and the second one arising from the comparison between the central Monte Carlo Simulation (σ = 12.78%) and Historical Data on the performance of the EURO STOXX 50 Index since its inception.

4.1 Monte Carlo Simulation (σ = 7.01%; σ = 12.78%; σ = 29.25%)

The first aspect to be covered is the unconditional probability of each termination scenario happening. As previously mentioned, there are five possible scenarios for termination: the product terminates in 2022; the product terminates in 2023; the product terminates in 2024; the product terminates in 2025 and the underlying price on the final autocall observation date is at or above the barrier level; the product terminates in 2025 and the underlying price on the final autocall observation date is below the barrier level.

The unconditional probability of each scenario coming to fruition is obtained by dividing the number of trials in the Monte Carlo Simulation where the product terminates in each situation by the total number of trials of 10,000. Additionally, for 2025, the probabilities of the product terminating above or below the barrier level are computed in the same manner, with the sum of the two equalling the probability of the product terminating in 2025. Figure 4 summarizes the results of this analysis, considering the three volatilities already mentioned.

Figure 4. Unconditional Probabilities – Monte Carlo Simulation



The results are quite similar across all volatilities. The two most likely scenarios are for the product to terminate in 2022 or 2025, combining for more than 85% of all terminations. This implies that it is unlikely for the product to terminate in either 2023 or 2024. However, disparities arise when we look further into the cases where the product terminates in 2025. Although the overall probability of terminating in the fifth year is similar, the split between trials where the Certificate terminates above and below the barrier level varies significantly with volatility.

Considering only the cases where the product survives until 2025:

- For σ = 7.01%, the probability of the product terminating above the barrier is 98.51%, computed through the division of the unconditional probability of the product terminating above the barrier in 2025 (43.00%) by the unconditional probability of the product terminating in 2025 (43.65%), and the probability of the product terminating below the barrier is 1.49%.
- For σ = 12.78%, the probability of the product terminating above the barrier is 76.31% and the probability of the product terminating below the barrier is 23.69%.
- For σ = 29.25%, the probability of the product terminating above the barrier is 31.48% and the probability of the product terminating below the barrier is 68.52%.

Therefore, we conclude that the higher the volatility considered, the higher the probability of the Express Certificate terminating below the barrier level in 2025. All relevant values regarding this analysis can be consulted in Table A.1 in the Appendix.

Moving to the conditional probability analysis (Figure 5), the conclusions remain practically unchanged and still very similar across volatilities: given that the product does not terminate in 2022, the most likely scenario is for it to terminate in 2025, and the same applies to 2023. The significant discrepancies emerge once again when we look further into the cases where the product terminates in the fifth year. The finding stated in the previous paragraph is corroborated: the higher the volatility considered, the higher the probability of the Express Certificate terminating below the barrier level in 2025. All relevant values regarding this analysis can be consulted in Table A.2 in the Appendix.

Figure 5. Conditional Probabilities – Monte Carlo Simulation

Having established the probability analysis framework, we can proceed with the study of the product's average lifespan, the fugit. Fugit is calculated by multiplying the probability of each termination scenario coming to fruition by the number of years that span since the Certificate's issuance until each scenario's termination date and then adding them up. This means we multiply the unconditional probability of the product terminating in 2022 by 2, the number of years elapsed between issuance and termination in that scenario, the unconditional probability of the product terminating in 2023 by 3 and so on, adding all values up in the end to obtain the product's average lifespan. Table 4 displays the results of these computations.

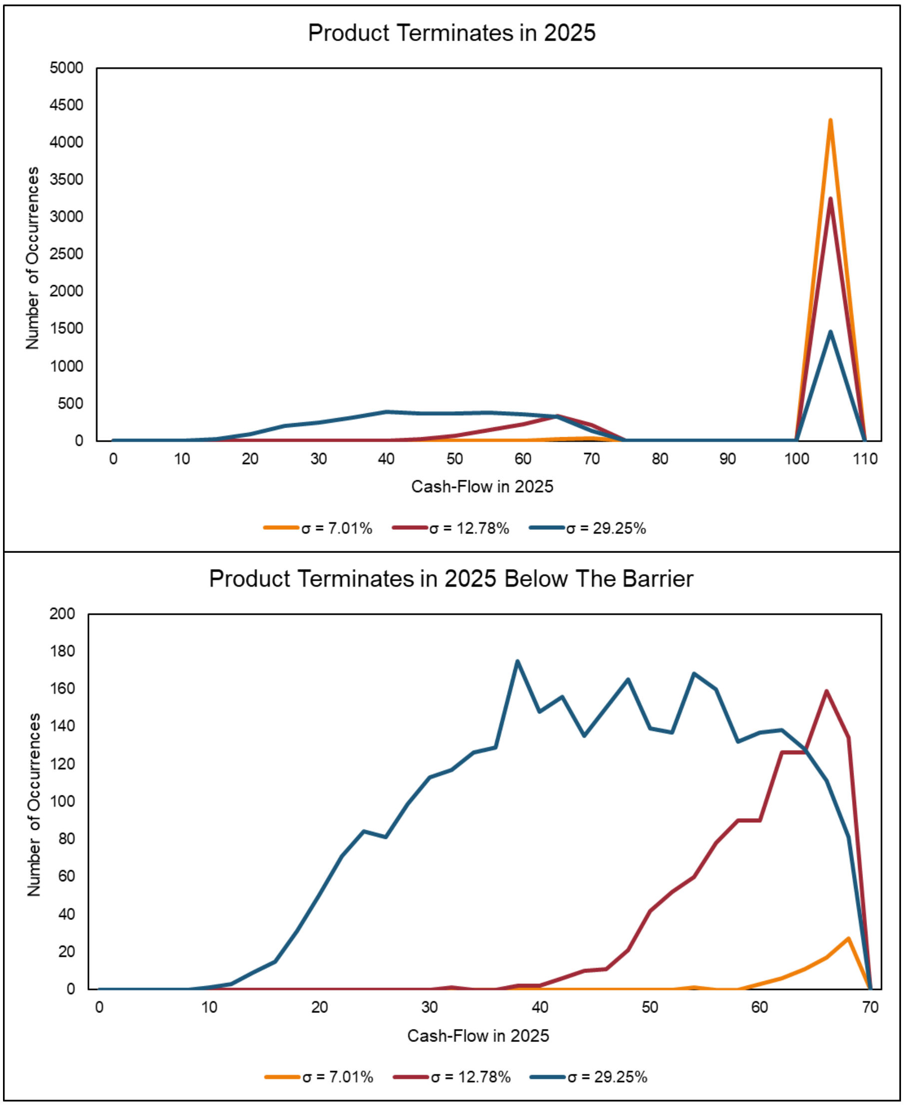
Table 4. Fugit - Monte Carlo Simulation

Volatility	**Fugit (years)**
7.01%	3.5062
12.78%	3.479
29.25%	3.5813

Similarly to the probability analysis, results are fairly consistent throughout the volatilities: considering σ = 7.01%, the Certificate has an average lifespan of 3.5062 years; considering σ = 12.78%, the Certificate has an average lifespan of 3.479 years; considering σ = 29.25%, the Certificate has an average lifespan of 3.5813 years. There is no clear relation between volatility and expected lifespan of the product.

The next topic to cover is the cash-flow generated by the Certificate when the product reaches 2025. We consider only the cash-flow from that final year in this analysis due to the fact that 2025 is the only year where there is uncertainty about the amount the holder of the product receives. If the product survives until 2025 and the underlying price on the final autocall observation date (S_T) is equal to or higher than 2,425.928 points (65% of S_0, the initial underlying price), then the holder receives 102.65€ in 2025. If the product survives until 2025 and S_T is lower than 2,425.928 points, the holder receives 2.65€ plus $100€ \times \frac{S_T}{S_0}$ in 2025. Figure 6 depicts the fifth-year cash-flows generated by the product in the trials where it survives until 2025, for all three volatilities considered.

Figure 6. Cash-Flow in 2025 – Monte Carlo Simulation

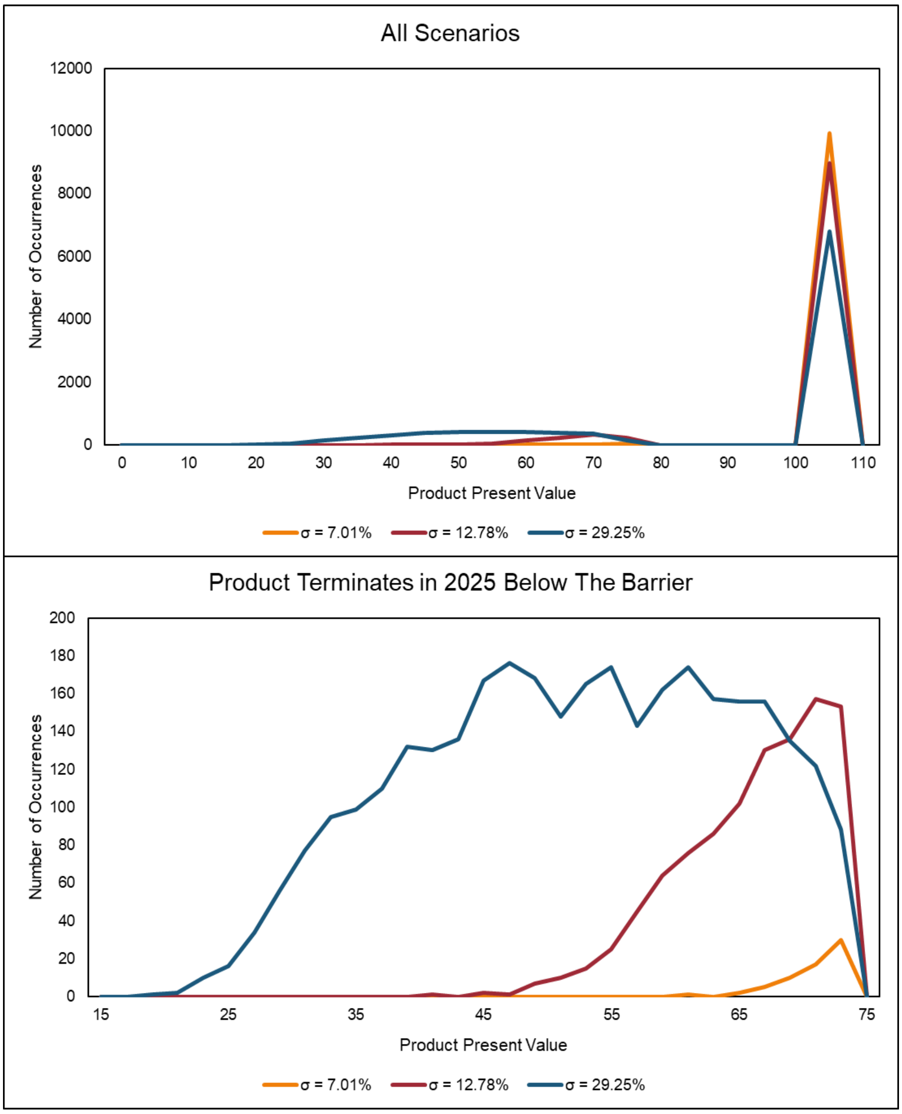


The first graph in Figure 6 represents all the instances where the Certificate terminates in 2025, whereas the second graph represents only the instances where the Certificate terminates in 2025 and S_T is below the barrier level. We have already inferred from the probability analysis that the higher the volatility, the more instances there are where the product terminates below the barrier in 2025. Delving now into the cash-flows, we can see that a higher volatility leads to a lower average cash-flow in 2025 and a wider range of cash-flows if the product terminates below the barrier in the final year.

With σ = 7.01%, the average cash-flow in 2025 is 102.09€, and if the product terminates below the barrier, cash-flows vary from 52.99€ to 67.65€. With σ = 12.78%, the average cash-flow in 2025 is 92.38€, and if the product terminates below the barrier, cash-flows vary from 31.50€ to 67.65€. With σ = 29.25%, the average cash-flow in 2025 is 62.56€, and if the product terminates below the barrier, cash-flows vary from 8.02€ to 67.59€. All relevant values regarding this analysis can be consulted in Table A.3 in the Appendix.

The cash-flow analysis just described, albeit significant, is merely nominal, as it does not incorporate the discount rate needed to compute the present value of the cash-flows generated by the Certificate and, therefore, to correctly value it. Keeping this in mind, we discount the product's cash-flows using Deutsche Bank's cost of funding of 1.6021%. With this step, we are now able to study the product's present value (Figure 7).

Figure 7. Product's Present Value – Monte Carlo Simulation



The first graph in Figure 7 represents the present value of the Express Certificate taking into consideration every termination scenario and, therefore, all 10,000 Monte Carlo trials. The second graph represents the present value of the Express Certificate considering only the cases where the product terminates in 2025 below the barrier level. This scenario is highlighted here because it is the only one where the present value is not known from the get-go. For all other termination scenarios, due to the payoffs being linear and not determined by a formula dependent on the underlying's performance, there is no uncertainty surrounding the product's present value, regardless of the volatility considered:

- If the product terminates in 2022, its present value is always 102.05€, equal to the present value of the coupon payment of 2.65€ in 2021 plus the coupon and principal payment of 102.65€ in 2022.
- If the product terminates in 2023, its present value is always 103.05€, equal to the present value of the coupon payment of 2.65€ in 2021 and 2022 plus the coupon and principal payment of 102.65€ in 2023.
- If the product terminates in 2024, its present value is always 104.03€, equal to the present value of the coupon payment of 2.65€ in 2021, 2022 and 2023 plus the coupon and principal payment of 102.65€ in 2024.
- If the product terminates in 2025 above the barrier, its present value is always 105.00€, equal to the present value of the coupon payment of 2.65€ in 2021, 2022, 2023 and 2024 plus the coupon and principal payment of 102.65€ in 2025.

For these four scenarios, volatility does not directly influence the Certificate's present value in each individual case, but it does play a role in the probability of each of them happening, which in turn has an impact on the product's average present value. However, when we take into account the possibility of the product terminating in 2025 below the barrier, volatility has an undeniable effect on the product's present value, due to the fact that the payoff in that situation is dependent on the index's performance.

Considering all scenarios, with $\sigma = 7.01\%$, the product's average present value is 103.30€ and the probability of the present value being higher than the product's notional amount of 100€ is 99.35%. This means that, in 99.35% of the cases, the holder of the Certificate earns back more than what he or she paid for the product. With $\sigma = 12.78\%$, the product's average present value is 99.46€ and the probability of the present value being higher than the notional is 89.90%. With $\sigma = 29.25\%$, the product's average present value is 86.37€ and the probability of the present value being higher than the notional is 68.11%.

If we examine only the trials where the product terminates in 2025 below the barrier, with $\sigma = 7.01\%$, the product's average present value is 70.00€. With $\sigma = 12.78\%$ it is 64.98€ and with $\sigma = 29.25\%$ it is 50.96€. Naturally, there is no possibility of the holder of the Certificate earning back more than what he or she paid for in this scenario. All relevant values regarding this analysis can be consulted in Table A.4 in the Appendix.

The main takeaway from this assessment is that the higher the volatility, the lower the product's average present value and the lower the probability of the product's holder earning back more than what he or she paid for the product.

Having looked into the Certificate's valuation as a whole, when we explore the valuation of its individual components (Table 5), another conclusion arises: the higher the volatility considered, the lower the value of the callable bond and the higher the value of the put option. This happens due to the increase in the probability of the product terminating in the fifth year below the barrier associated with a higher volatility. This implies that there are more instances where the put option originates a cash-flow and, therefore, it increases in value, whereas the callable bond decreases for the same reason.

Table 5. Present Value Decomposition – Monte Carlo Simulation

Component	Average Present Value		
	σ = 7.01%	**σ = 12.78%**	**σ = 29.25%**
Callable Bond	102.93€	94.17€	74.15€
Put Option	0.37€	5.29€	12.22€
Structured Product	103.30€	99.46€	86.37€

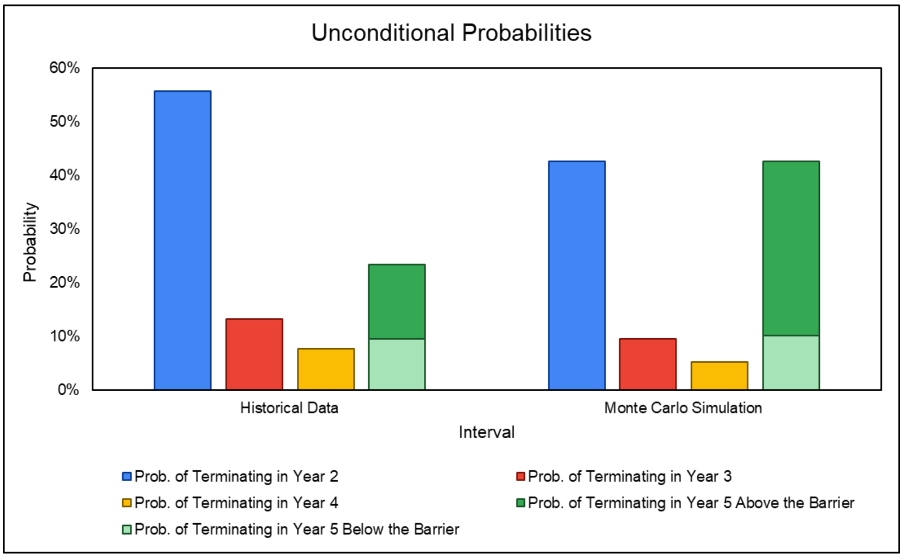
4.2 Historical Data vs. Monte Carlo Simulation (σ = 12.78%)

In this section, the analysis is identical in every aspect to the one described above, except for one major difference. Instead of comparing Monte Carlo Simulation's results for different volatilities, the parallel is now drawn between the central Monte Carlo Simulation, with σ = 12.78%, and Historical Data on the performance of the EURO STOXX 50 Index since its inception in 1998.

One aspect worth mentioning is that Historical Data on the index only allows us to obtain a total of 5,788 observations, whereas the Monte Carlo Simulation generates 10,000 different paths. In order to take this difference into account and establish a coherent comparison between both sets of data, we change the y axis in graphs regarding the product's cash-flows and present value (Figures 10 and 13, respectively) from number of occurrences to percentage of occurrences.

Following the same line of thought as before, the first aspect to cover is the unconditional probability of each termination scenario coming to fruition. Due to the inclusion of Historical Data in the analysis, we no longer classify the scenarios as terminating in 2022, 2023, 2024 or 2025. The five possible scenarios for termination are now: the product terminates in Year 2; the product terminates in Year 3; the product terminates in Year 4; the product terminates in Year 5 and the underlying price on the final autocall observation date is at or above the barrier level; the product terminates in Year 5 and the underlying price on the final autocall observation date is below the barrier level. Figure 8 displays the results for this analysis.

Figure 8. Unconditional Probabilities – Historical Data vs. Monte Carlo Simulation



The results are somewhat similar across both sets of data, but there are some relevant discrepancies to point out. The two most likely scenarios for both intervals are for the product to terminate in the 2nd year or the 5th year, even though that, for the Historical Data, the probability of the product terminating in Year 2 is much higher than the probability of terminating in Year 5, contrasting with the equilibrium we find between these 2 scenarios for the Monte Carlo Simulation. Another similarity between both datasets is that it is relatively unlikely for the product to terminate in either the 3rd or the 4th year, with the probabilities of both scenarios being slightly higher for the Historical Data when compared with the Simulation.

However, a familiar difference arises when considering only the cases where the product survives until Year 5:

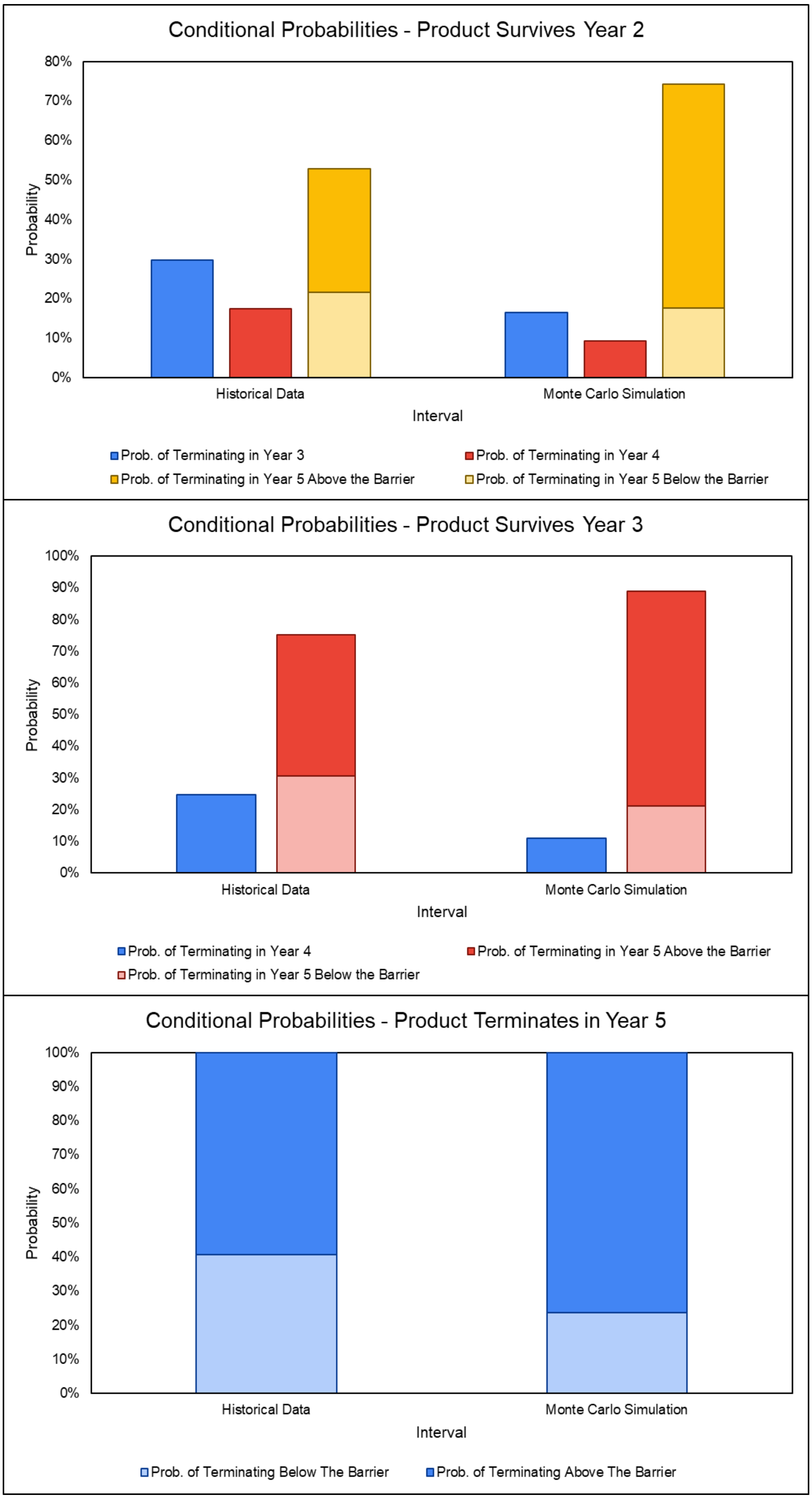
- Regarding the Historical Data, the probability of the product terminating above the barrier is 59.29%, computed through the division of the unconditional probability of the product terminating above the barrier in Year 5 (13.89%) by the unconditional probability of the product terminating in Year 5 (23.43%), and the probability of the product terminating below the barrier is 40.71%.

- Regarding the Monte Carlo Simulation, the probability of the product terminating above the barrier is 76.31% and the probability of the product terminating below the barrier is 23.69%.

This implies that the Monte Carlo Simulation is perhaps too optimistic when dealing with the scenario where the product terminates in the 5^{th} year, returning a significantly higher probability of termination above the barrier and a significantly lower probability of termination below the barrier when compared to the Historical Data. All relevant values regarding this analysis can be consulted in Table A.5 in the Appendix.

The conditional probability analysis shown in Figure 9 leads to analogous conclusions: given that the product does not terminate in Year 2, the most likely scenario is for it to terminate in Year 5, and the same holds for Year 3. The main divergence between the Historical Data and the Monte Carlo Simulation arises once again when looking further into the cases where the product survives until Year 5: the Monte Carlo Simulation results in a higher likelihood of the Express Certificate terminating above the barrier in the final year when compared to the Historical Data. All relevant values regarding this analysis can be consulted in Table A.6 in the Appendix.

Figure 9. Conditional Probabilities – Historical Data vs. Monte Carlo Simulation



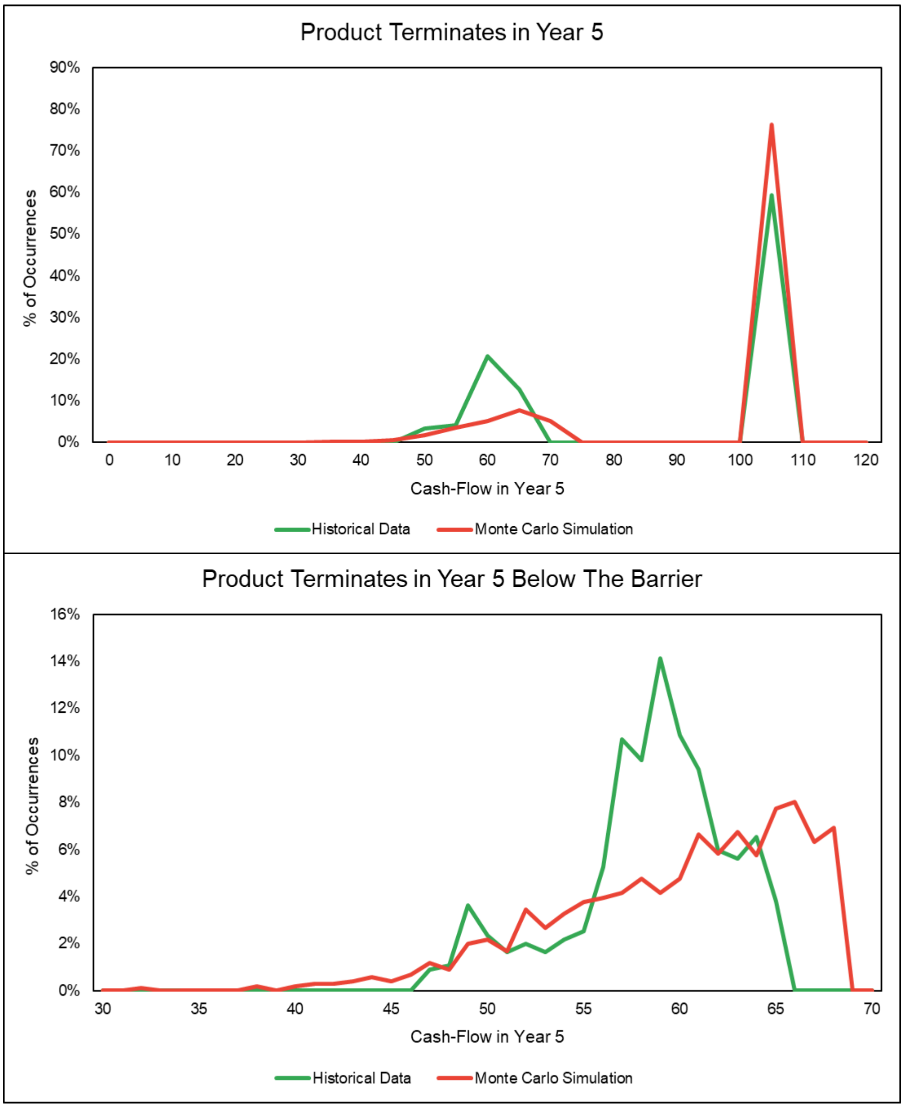
Expanding on this probability analysis framework, the following point to address is the product's average lifespan, also known as fugit (Table 6). Fugit's calculation method is explained in Section 4.1. For the Historical Data, the Certificate has a fugit of 2.989 years, whereas, for the Monte Carlo Simulation, the Certificate has a fugit of 3.479 years. This difference is explained by the higher unconditional probabilities of early termination, i.e., termination in Years 2, 3 and 4, and lower unconditional probability of termination at maturity (Year 5) derived from the Historical Data when compared to the Monte Carlo Simulation. As the product is more likely to terminate earlier, its average lifespan is consequently shorter.

Table 6. Fugit - Historical Data vs. Monte Carlo Simulation

Interval	Fugit (years)
Historical Data	2.989
Monte Carlo Simulation	3.479

We continue the analysis by looking further into the cash-flow that the product generates when it reaches the 5th year. If the product survives until Year 5 and the underlying price on the final autocall observation date (S_T) is equal to or higher than 65% of the initial underlying price (S_0), the holder receives 102.65€ in Year 5. If the product survives until Year 5 and S_T is lower than 65% of S_0, the holder receives 2.65€ plus $100€ \times \frac{S_T}{S_0}$ in Year 5. Figure 10 depicts the fifth-year cash-flows generated by the Certificate in the cases where it survives until Year 5.

Figure 10. Cash-Flow in Year 5 – Historical Data vs. Monte Carlo Simulation

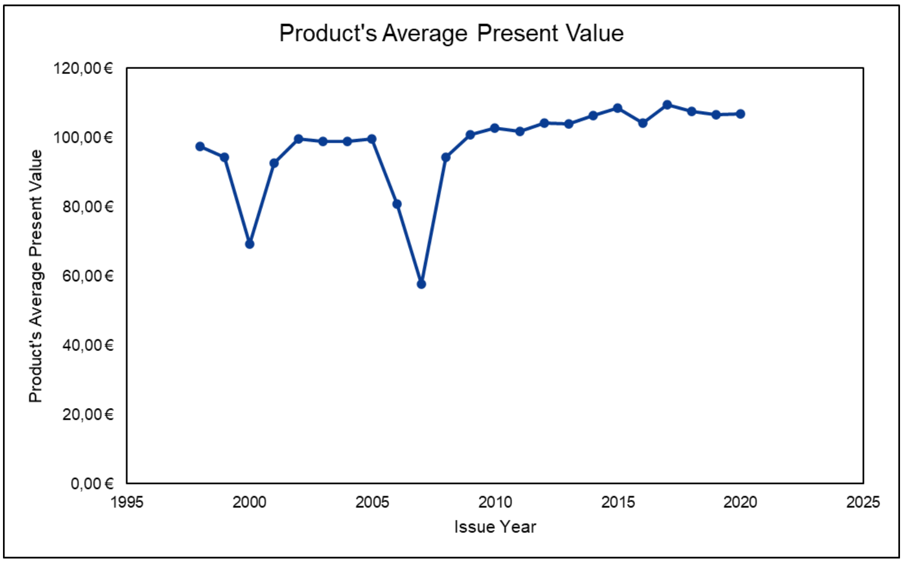


The first graph in Figure 10 shows all the cases where the product terminates in Year 5, whilst the second graph shows only the cases where the product terminates in Year 5 and S_T is below the barrier level. Looking further into the cash-flows, we observe that the Monte Carlo Simulation returns a higher average cash-flow in Year 5 and a wider range of cash-flows if the product terminates below the barrier in that same year when compared to Historical Data.

For the Historical Data, the average cash-flow in Year 5 is 84.43€, and if the Certificate terminates below the barrier, cash-flows range from 46.23€ to 64.80€. For the Monte Carlo Simulation, the average cash-flow in Year 5 is 92.38€, and if the Certificate terminates below the barrier, cash-flows range from 31.50€ to 67.65€. All relevant values regarding this analysis can be consulted in Table A.7 in the Appendix.

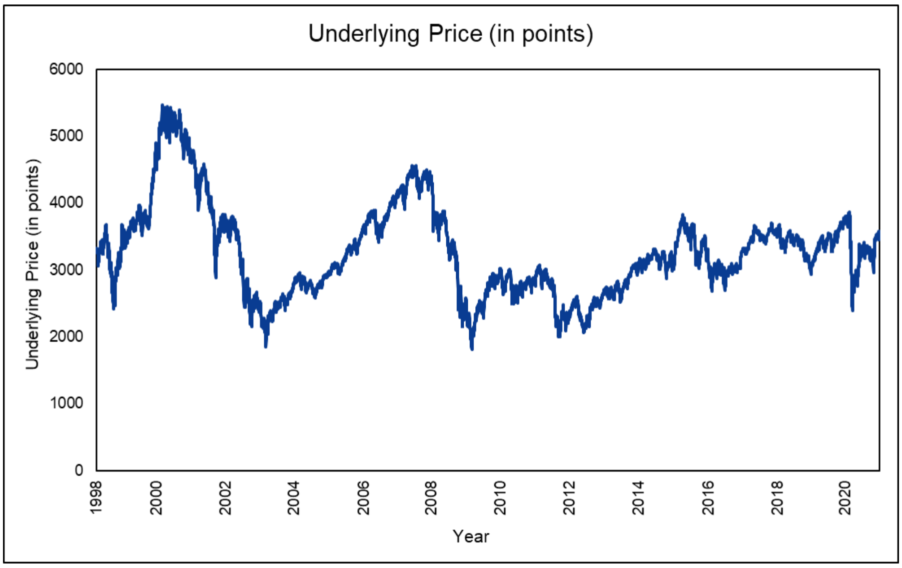
The next topic to cover is the Certificate's present value. Before establishing a comparison with the Monte Carlo Simulation, Figure 11 displays the historical distribution of the product's average present value by issue year. If the Certificate had been issued in any year before 2009, its average present value would always be lower than 100€, the amount paid by the holder of the product in its purchase, reaching a minimum of 57.62€ if it had been issued in 2007. However, if the Certificate had been issued from 2009 onwards, the product's average present value would always be higher than 100€, reaching a maximum of 109.32€ if it had been issued in 2017. Taking all the observations into account, the average product present value according to the Historical Data is 97.57€. All relevant values regarding this analysis can be consulted in Table A.8 in the Appendix.

Figure 11. Product's Average Present Value by Issue Year – Historical Data



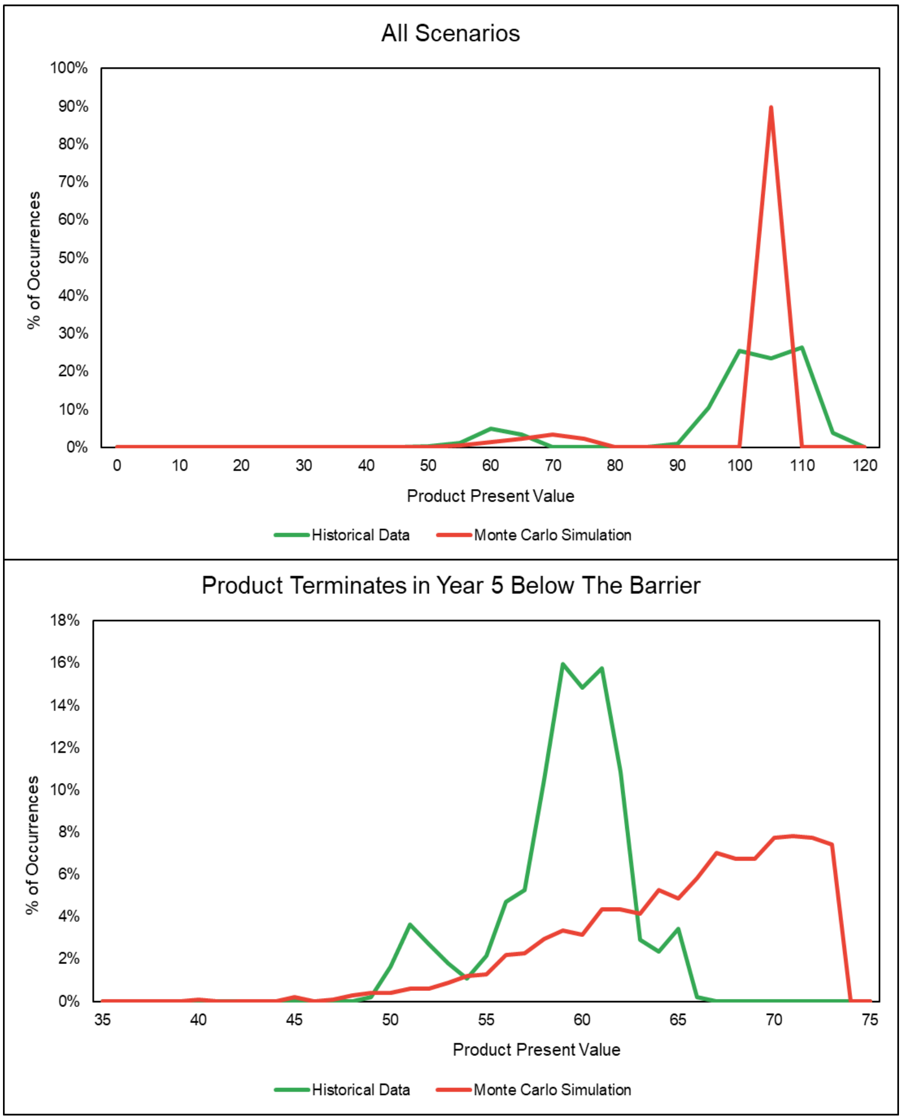
The two major drops in the product's present value correspond to the years 2000 and 2007. This implies that if the product had been issued in one of those years, the most likely scenario would be for it to survive until the fifth year and terminate below the barrier, resulting in a lower average present value. This is justified by the drastic decrease registered in the underlying price in the years that followed both 2000 and 2007 (Figure 12), which can be attributed to the burst of the dotcom bubble and the subprime mortgage crisis, respectively. These prolonged decreases mean the index price falls below the barrier for the vast majority of the autocall observation dates correspondent to issuances in 2000 or 2007, contributing to the increased probability of the product terminating at its full maturity below the barrier and returning a lower average present value.

Figure 12. Underlying Price Historical Evolution



When establishing the product's present value comparison between both sets of data, we must keep in mind the difference between the discount rates applied in each situation. For the Monte Carlo Simulation, we use Deutsche Bank's cost of funding of 1.6021% to discount all the cash-flows generated by the Certificate to their present value at the time of issuance. However, for the Historical Data, the discount rate is equal to the yield of a five-year Germany government bond issued on the day prior to the issuance of the product. Therefore, as the issuance date varies with each observation considered, the discount rate also varies and does not take on a fixed value, as seen previously in Figure 3. Figure 13 shows the results of this analysis.

Figure 13. Product's Present Value – Historical Data vs. Monte Carlo Simulation



The first graph in Figure 13 represents the present value of the Express Certificate taking into account every termination scenario. The second graph represents the present value of the Express Certificate considering only the cases where the product terminates in Year 5 below the barrier level. This specific scenario is displayed due to being the only one where the present value is uncertain for both sets of data. For all other termination scenarios, the payoffs are linear and not dependent on the underlying's performance. However, for Historical Data, there is uncertainty in the other scenarios regarding the present value, but this arises solely from the uncertainty in the discount rate's value. In nominal terms, the payoffs are also pre-determined.

Considering all scenarios, for the Historical Data, the product's average present value is 97.57€ and the probability of the present value being higher than the product's notional amount of 100€ is 53.65%. This means that, in 53.65% of the cases, the holder of the Certificate earns back more than what he or she paid for the product. For the Monte Carlo Simulation, the product's average present value is 99.46€ and the probability of the present value being higher than the notional is 89.90%.

Considering only the observations where the product terminates in Year 5 below the barrier, for the Historical Data, the product's average present value is 58.50€, whereas, for the Monte Carlo Simulation, it is 64.98€. Naturally, there is no possibility of the holder of the Certificate earning back more than what he or she paid for in this scenario. All relevant values regarding this analysis can be consulted in Table A.9 in the Appendix.

The main takeaway from this assessment is that the Monte Carlo Simulation returns a higher average present value and a higher probability of the product's holder earning back more than what he or she paid for the product, when compared to the Historical Data.

Finally, after exploring the product's valuation as whole, we look into the valuation of its individual components, shown in Table 7. We conclude that the present value of both components is higher for the Monte Carlo Simulation than for the Historical Data. The higher product's fugit for the Monte Carlo Simulation justifies the higher valuation of the callable bond when compared to the Historical Data – the longer the product lives, the higher the cash-flow in the form of coupon payments. However, the higher valuation for the put option is not exclusively determined by the marginally higher probability of the product terminating in Year 5 below the barrier for the Monte Carlo Simulation. In fact, the Monte Carlo Simulation process itself leads to a higher cash-flow and, consequently, a higher product's average present value for that scenario than the Historical Data (as shown in Figure 13), which results in a higher valuation for the put option since its payoff is related to the cash-flow produced if the product terminates in Year 5 below the barrier.

Table 7. Present Value Decomposition – Historical Data vs. Monte Carlo Simulation

Component	Average Present Value	
	Historical Data	Monte Carlo Simulation
Callable Bond	92.90€	94.17€
Put Option	4.67€	5.29€
Structured Product	97.57€	99.46€

5. Conclusion

The aim of this project is to analyse the performance of a Fixed Coupon Express Certificate linked to the EURO STOXX 50 Index, issued by Deutsche Bank on the 4th of February 2020. In order to follow this line of thought, we performed a Monte Carlo Simulation with 10,000 trials, using the Geometric Brownian Motion to generate the possible prospective prices for the underlying, with the purpose of obtaining an adequate valuation estimate for our Express Certificate. Furthermore, in the individual part of this study, I developed a backtesting and performance assessment on the product, establishing a comparison between historical data gathered from the index since 1998 and the results from the Monte Carlo Simulation.

Looking further into the first set of results, derived from the Monte Carlo Simulation process considering three different volatilities, $\sigma = 7.01\%$, $\sigma = 12.78\%$ and $\sigma = 29.25\%$, our analysis offers two key takeaways. Firstly, we observe that the higher the volatility considered, the higher the probability of the Express Certificate terminating below the barrier level in 2025, which consequently results in a lower product's average present value and a lower probability of the product's holder earning back more than he or she paid for the product. The second takeaway is that the higher the volatility considered, the lower the value of the callable bond and the higher the value of the put option, the two components embedded in the structured product.

Moving to the second set of results, obtained from the comparison between the central Monte Carlo Simulation, with $\sigma = 12.78\%$, and Historical Data on the performance of the index since its inception, the findings can be summarized in two major categories.

Regarding the probability analysis, we observe that the Monte Carlo Simulation is perhaps too optimistic when addressing the possibility of the product terminating in Year 5, returning a significantly higher probability of termination above the barrier and a significantly lower probability of termination below the barrier when compared to the Historical Data. Furthermore, lower unconditional probabilities of early termination and a higher probability of termination at full maturity (Year 5) for the Monte Carlo Simulation results in a reasonably higher fugit, the product's average lifespan, when compared to the Historical Data.

Regarding the valuation analysis, the Monte Carlo Simulation returns a higher product's average present value and a higher probability of the product's holder earning back more than he or she paid for the product. Looking into the present value of the individual components of the product, we conclude that it is higher in the Monte Carlo Simulation for both of them.

As a final concluding remark, taking into account both sets of results, the product seems slightly overpriced.

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Appendix

Figure A.1 Product's Key Information Document (Page 1)

Key Information Document

Purpose

This document provides you with key information about this investment product. It is not marketing material. The information is required by law to help you understand the nature, risks, costs, potential gains and losses of this product and to help you compare it with other products.

Product

Product name	Fixed Coupon Express Certificate linked to EURO STOXX 50® Index (Price Index)
Product identifiers	ISIN: DE000DB9U0V9 | WKN: DB9U0V
PRIIP manufacturer	Deutsche Bank AG. The product issuer is Deutsche Bank AG, Frankfurt.
Website	www.db.com/contact
Telephone number	Call +49-69-910-00 for more information.
Competent authority of the PRIIP manufacturer	German Federal Financial Supervisory Authority (BaFin)
Date of production	7 December 2021

You are about to purchase a product that is not simple and may be difficult to understand.

1. What is this product?

Type

German law governed certificates

Objectives

(Terms that appear in **bold** in this section are described in more detail in the table(s) below.)

The product is designed to provide a return in the form of (1) regular fixed coupon payments and (2) a cash payment on termination of the product. The timing and amount of this payment will depend on the performance of the **underlying**.

Early termination following an autocall: The product will terminate prior to the **maturity date** if, on any **autocall observation date**, the **reference level** is at or above the relevant **autocall barrier level**. On any such early termination, you will on the immediately following **autocall payment date** receive, in addition to a final coupon payment, a cash payment equal to the autocall payment of EUR 100. No coupon payments will be made on any date after such **autocall payment date**. The relevant dates and **autocall barrier levels** are shown in the table(s) below.

Autocall observation dates	*Autocall barrier levels*	*Autocall payment dates*
2 February 2022	3,732.28	7 February 2022
1 February 2023	3,732.28	6 February 2023
1 February 2024	3,732.28	6 February 2024
3 February 2025	2,425.982	Maturity date

Coupon: If the product has not terminated early, on each **coupon payment date** you will receive a coupon payment of EUR 2.65. The coupon payments are not linked to the performance of the **underlying**. The relevant dates are shown in the table(s) below.

Coupon payment dates
8 February 2021
7 February 2022
6 February 2023
6 February 2024
Maturity date

Termination on the maturity date: If the product has not terminated early, on the **maturity date** you will receive:

1. if the **final reference level** is at or above the **barrier level**, a cash payment equal to EUR 100; or
2. if the **final reference level** is below the **barrier level**, a cash payment directly linked to the performance of the **underlying**. The cash payment will equal (i) the **product notional amount** multiplied by (ii) (A) the **final reference level** divided by (B) the **strike level**.

Under the product terms, certain dates specified above and below will be adjusted if the respective date is either not a business day or not a trading day (as applicable). Any adjustments may affect the return, if any, you receive.

When purchasing this product during its lifetime, the purchase price may include accrued coupon on a pro rata basis.

You do not have any entitlement to a dividend from the **underlying** and you have no right to any further entitlement resulting from the **underlying** (e.g., voting rights).

Underlying	EURO STOXX 50 (Price return index) (ISIN: EU0009658145)	**Strike level**	3,732.28
Underlying market	Equity	**Barrier level**	2,425.982
Product notional amount	EUR 100	**Reference level**	The closing level of the **underlying** as per the **reference source**
Product currency	Euro (EUR)	**Reference source**	STOXX Limited, Zurich
Underlying currency	Euro (EUR)	**Final reference level**	The **reference level** on the **valuation date**
Issue date	4 February 2020	**Valuation date**	3 February 2025
Initial reference level	3,732.28	**Maturity date / term**	6 February 2025

The issuer may terminate the product with immediate effect in the event of obvious written or mathematical errors in the terms and conditions or if certain extraordinary events provided in the terms and conditions occur. Examples of extraordinary events include (1) material changes, particularly in connection with the **underlying**, including where an index ceases to be calculated, and (2) events, in

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Figure A.2 Product's Key Information Document (Page 2)

particular due to changes in certain external conditions that hinder the issuer in meeting its obligations in connection with the product or – depending on the terms and conditions of the security – otherwise affect the product and/or the issuer. In case of immediate termination, the return (if any) may be significantly lower than the purchase price, but will reflect the product's market value and, if higher, any minimum redemption (alternatively, in some cases the corresponding compounded amount may be paid out at the product's scheduled maturity). Instead of immediate termination, the issuer may also amend the terms and conditions.

Provided that in the event of any inconsistency and/or conflict between the foregoing paragraph and any applicable law, order, rule or other legal requirement of any governmental or regulatory authority in a territory in which this product is offered, such national requirements shall prevail.

Intended retail investor

The product is intended for private clients who pursue the objective of general capital formation/asset optimization and have a medium-term investment horizon. This product is a product for clients who have sufficient knowledge and / or experience to make an informed investment decision. The investor can bear losses up to the total loss of the capital invested and attaches no importance to capital protection.

2. What are the risks and what could I get in return?

Risk indicator

1 | 2 | 3 | 4 | 5 | 6 | 7

Lower risk — **Higher risk**

The risk indicator assumes you keep the product for 3 years and 2 months. The actual risk can vary significantly if you cash in at an early stage and you may get back less.

The summary risk indicator is a guide to the level of risk of this product compared to other products. It shows how likely it is that the product will lose money because of movements in the markets or because we are not able to pay you.

We have classified this product as 1 out of 7, which is the lowest risk class. This rates the potential losses from future performance at a very low level, and poor market conditions are very unlikely to impact our capacity to pay you.

This product does not include any protection from future market performance so you could lose some or all of your investment.

If we are not able to pay you what is owed, you could lose your entire investment.

Performance scenarios

Market developments in the future cannot be accurately predicted. The scenarios shown are only an indication of some of the possible outcomes based on recent returns. Actual returns could be lower.

Investment: EUR 10,000

Scenarios		*1 year*	*2 years*	*3 years and 2 months (Recommended holding period)*
Stress scenario	**What you might get back after costs**	EUR 8,339.17	EUR 9,058.79	EUR 10,161.35
	Average return each year	-16.61%	-4.82%	0.51%
Unfavourable scenario	**What you might get back after costs**	EUR 9,977.56	EUR 9,962.67	EUR 10,161.35
	Average return each year	-0.22%	-0.19%	0.51%
Moderate scenario	**What you might get back after costs**	EUR 10,171.53	EUR 10,182.25	EUR 10,161.35
	Average return each year	1.72%	0.91%	0.51%
Favourable scenario	**What you might get back after costs**	EUR 10,174.80	EUR 10,208.30	EUR 10,161.35
	Average return each year	1.75%	1.04%	0.51%

This table shows the money you could get back over the next 3 years and 2 months under different scenarios, assuming that you invest EUR 10,000.

The scenarios shown illustrate how your investment could perform. You can compare them with the scenarios of other products. The scenarios presented are an estimate of future performance based on evidence from the past on how the value of this investment varies, and are not an exact indicator. What you get will vary depending on how the market performs and how long you keep the product. The stress scenario shows what you might get back in extreme market circumstances, and it does not take into account the situation where we are not able to pay you.

The figures shown include all the costs of the product itself, but may not include all the costs that you pay to your advisor or distributor. The figures do not take into account your personal tax situation, which may also affect how much you get back.

3. What happens if Deutsche Bank AG, Frankfurt is unable to pay out?

You are exposed to the risk that the issuer might be unable to fulfil its obligations in respect of the product – e.g. in the event of insolvency (inability to pay / over-indebtedness) or an administrative order of resolution measures. In case of a crisis of the issuer such an order can also be issued by a resolution authority in the run-up of an insolvency proceeding. In doing so, the resolution authority has extensive intervention powers. Among other things, it can reduce rights of the investors to zero, terminate the product or convert it into shares of the issuer and suspend rights of the investors. With regard to the basic ranking of the issuer's obligations in the event of action by the resolution authority, please see www.bafin.de and search for the keyword "Haftungskaskade". A total loss of your capital invested is possible. The product is a debt instrument and as such is not covered by any deposit protection scheme.

4. What are the costs?

The Reduction in Yield (RIY) shows what impact the total costs you pay will have on the investment return you might get. The total costs take into account one-off, ongoing and incidental costs.

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Figure A.3 Product's Key Information Document (Page 3)

The amounts shown here are the cumulative costs of the product itself, for three different holding periods. They include potential early exit penalties. The figures assume you invest EUR 10,000. The figures are estimates and may change in the future.

Costs over time

Investment: EUR 10,000			
Scenarios	*If you cash in after 1 year*	*If you cash in after 2 years*	*If you cash in at the end of the recommended holding period*
Total costs	EUR 0.00	EUR 0.00	EUR 0.00
Impact on return (RIY) per year	0.00%	0.00%	0.00%

The costs shown in the table above represent how much the expected costs of the product would affect your return, assuming the product performs in line with the moderate performance scenario.

The person selling you or advising you about this product may charge you other costs. If so, this person will provide you with information about these costs, and show you the impact that all costs will have on your investment over time.

Composition of costs

The table below shows:

- The impact each year of the different types of costs on the investment return you might get at the end of the recommended holding period.
- The meaning of the different cost categories.

The table shows the impact on return per year.			
One-off costs	*Entry costs*	0.00%	The impact of the costs already included in the price.
	Exit costs	0.00%	The impact of the costs of exiting your investment when it matures.
Ongoing costs	*Portfolio transaction costs per year*	0.00%	The impact of the costs of us buying and selling underlying investments for the product.
	Other ongoing costs	0.00%	The impact of the costs that we take each year for managing your investments.

5. How long should I hold it and can I take money out early?

Recommended holding period: 3 years and 2 months

The product aims to provide you with the return described under "1. What is this product?" above. However, this only applies if the product is held to maturity. It is therefore recommended that the product is held until 6 February 2025 (maturity).

The product does not guarantee the possibility to disinvest other than by selling the product either (1) through the exchange (where the product is listed) or (2) off-exchange. No fees or penalties will be charged by the issuer for any such transaction. However if you sell the product in the secondary market you will incur a bid/offer spread. By selling the product before its maturity, you may receive back less than you would have received if you had kept the product until maturity.

Exchange listing	Börse Stuttgart and Deutsche Börse AG	**Last exchange trading day**	31 January 2025 (Börse Stuttgart) and 31 January 2025 (Deutsche Börse AG)
Smallest tradable unit	1 unit	**Price quotation**	Units

In volatile or unusual market conditions, or in the event of technical faults/disruptions, the purchase and/or sale of the product can be temporarily hindered and/or suspended and may not be possible at all.

6. How can I complain?

Any complaint regarding the conduct of the person advising on, or selling, the product can be submitted directly to that person.

Any complaint regarding the product or the conduct of the manufacturer of this product can be submitted in writing at:

Jurisdiction	Postal address	Email address	Website
Germany	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Austria	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com
Luxembourg	Deutsche Bank AG, X-markets, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany	x-markets.team@db.com	www.xmarkets.db.com

7. Other relevant information

Any additional documentation in relation to the product and in particular the prospectus, any supplements thereto and the final terms are published on the manufacturer's website (www.xmarkets.db.com/DocumentSearch ; after entering of the respective ISIN or WKN), all in accordance with legal requirements. In order to obtain more detailed information - and in particular details of the structure and risks associated with an investment in the product - you should read these documents. These documents are also available free of charge from Deutsche Bank AG, Mainzer Landstrasse 11-17, 60329 Frankfurt am Main, Germany, in accordance with legal requirements.

Table A.1 Unconditional Probabilities – Monte Carlo Simulation

Volatility	Prob. of Terminating in 2022	Prob. of Terminating in 2023	Prob. of Terminating in 2024	Prob. of Terminating in 2025	Sum of All Probs.	Prob. of Terminating in 2025	
						Prob. of Terminating Above the Barrier	Prob. of Terminating Below the Barrier
σ = 7.01%	41.87%	9.29%	5.19%	43.65%	100.00%	43.00%	0.65%
σ = 12.78%	42.63%	9.47%	5.27%	42.63%	100.00%	32.53%	10.10%
σ = 29.25%	39.86%	8.69%	4.91%	46.54%	100.00%	14.65%	31.89%

Table A.2 Conditional Probabilities – Monte Carlo Simulation

Condition	Volatility	Prob. of Terminating in 2023	Prob. of Terminating in 2024	Prob. of Terminating in 2025	Sum of All Probs.	Prob. of Terminating in 2025	
						Prob. of Terminating Above the Barrier	Prob. of Terminating Below the Barrier
Product Survives 2022	σ = 7.01%	15.98%	8.93%	75.09%	100.00%	73.97%	1.12%
	σ = 12.78%	16.51%	9.19%	74.31%	100.00%	56.70%	17.61%
	σ = 29.25%	14.45%	8.16%	77.39%	100.00%	24.36%	53.03%
Product Survives 2023	σ = 7.01%		10.63%	89.37%	100.00%	88.04%	1.33%
	σ = 12.78%		11.00%	89.00%	100.00%	67.91%	21.09%
	σ = 29.25%		9.54%	90.46%	100.00%	28.47%	61.98%
Product Survives 2024	σ = 7.01%			100.00%	100.00%	98.51%	1.49%
	σ = 12.78%			100.00%	100.00%	76.31%	23.69%
	σ = 29.25%			100.00%	100.00%	31.48%	68.52%

Table A.3 Cash-Flow in 2025 – Monte Carlo Simulation

Condition	Volatility	Minimum	Median	Maximum	Average
Product Terminates in 2025	σ = 7.01%	52.99€	102.65€	102.65€	102.09€
	σ = 12.78%	31.50€	102.65€	102.65€	92.38€
	σ = 29.25%	8.02€	54.38€	102.65€	62.56€
Product Terminates in 2025 Below the Barrier	σ = 7.01%	52.99€	65.75€	67.65€	64.76€
	σ = 12.78%	31.50€	60.58€	67.65€	59.32€
	σ = 29.25%	8.02€	44.66€	67.59€	44.14€

Table A.4 Product's Present Value – Monte Carlo Simulation

Scenarios	Volatility	Minimum PV	Median PV	Maximum PV	Average PV	Prob. of Product PV > Notional (100€)	Prob. of Product PV ≤ Notional (100€)	Sum of Probs.
All Scenarios	σ = 7.01%	59.13€	103.05€	105.00€	103.30€	99.35%	0.65%	100.00%
	σ = 12.78%	39.28€	102.05€	105.00€	99.46€	89.90%	10.10%	100.00%
	σ = 29.25%	17.60€	102.05€	105.00€	86.37€	68.11%	31.89%	100.00%
Product Terminates in 2025 Below the Barrier	σ = 7.01%	59.13€	70.92€	72.67€	70.00€	0.00%	100.00%	100.00%
	σ = 12.78%	39.28€	66.14€	72.67€	64.98€	0.00%	100.00%	100.00%
	σ = 29.25%	17.60€	51.43€	72.62€	50.96€	0.00%	100.00%	100.00%

Table A.5 Unconditional Probabilities – Historical Data vs. Monte Carlo Simulation

Interval	Prob. of Terminating in Year 2	Prob. of Terminating in Year 3	Prob. of Terminating in Year 4	Prob. of Terminating in Year 5	Sum of All Probs.	Prob. of Terminating in Year 5	
						Prob. of Terminating Above the Barrier	Prob. of Terminating Below the Barrier
Historical Data	55.65%	13.20%	7.72%	23.43%	100.00%	13.89%	9.54%
Monte Carlo Simulation	42.63%	9.47%	5.27%	42.63%	100.00%	32.53%	10.10%

Table A.6 Conditional Probabilities – Historical Data vs. Monte Carlo Simulation

Condition	Interval	Prob. of Terminating in Year 3	Prob. of Terminating in Year 4	Prob. of Terminating in Year 5	Sum of All Probs.	Prob. of Terminating in Year 5	
						Prob. of Terminating Above the Barrier	Prob. of Terminating Below the Barrier
Product Survives Year 2	Historical Data	29.76%	17.41%	52.82%	100.00%	31.32%	21.50%
	Monte Carlo Simulation	16.51%	9.19%	74.31%	100.00%	56.70%	17.61%
Product Survives Year 3	Historical Data		24.79%	75.21%	100.00%	44.59%	30.62%
	Monte Carlo Simulation		11.00%	89.00%	100.00%	67.91%	21.09%
Product Survives Year 4	Historical Data			100.00%	100.00%	59.29%	40.71%
	Monte Carlo Simulation			100.00%	100.00%	76.31%	23.69%

Table A.7 Cash-Flow in Year 5 – Historical Data vs. Monte Carlo Simulation

Condition	Interval	Minimum	Median	Maximum	Average
Product Terminates in Year 5	Historical Data	46.23€	102.65€	102.65€	84.43€
	Monte Carlo Simulation	31.50€	102.65€	102.65€	92.38€
Product Terminates in Year 5 Below the Barrier	Historical Data	46.23€	58.50€	64.80€	57.90€
	Monte Carlo Simulation	31.50€	60.58€	67.65€	59.32€

Table A.8 Average Product's Present Value by Issue Year – Historical Data

Issue Year	Average Product Present Value	Issue Year	Average Product Present Value	Issue Year	Average Product Present Value
1998	97.31€	2006	80.84€	2014	106.34€
1999	94.29€	2007	57.62€	2015	108.52€
2000	69.09€	2008	94.33€	2016	104.12€
2001	92.59€	2009	100.70€	2017	109.32€
2002	99.51€	2010	102.76€	2018	107.60€
2003	98.78€	2011	101.75€	2019	106.50€
2004	98.74€	2012	104.12€	2020	106.73€
2005	99.65€	2013	104.00€	Total	97.57€

Table A.9 Product's Present Value – Historical Data vs. Monte Carlo Simulation

Scenarios	Interval	Minimum PV	Median PV	Maximum PV	Average PV	Prob. of Product PV > Notional (100€)	Prob. of Product PV ≤ Notional (100€)	Sum of Probs.
All Scenarios	Historical Data	48.99€	100.58€	114.08€	97.57€	53.65%	46.35%	100.00%
	Monte Carlo Simulation	39.28€	102.05€	105.00€	99.46€	89.90%	10.10%	100.00%
Product Terminates in 2025 Below the Barrier	Historical Data	48.99€	59.06€	65.60€	58.50€	0.00%	100.00%	100.00%
	Monte Carlo Simulation	39.28€	66.14€	72.67€	64.98€	0.00%	100.00%	100.00%