



Instituto Superior de Economia e Gestão

UNIVERSIDADE TÉCNICA DE LISBOA

DESDE 1911

# **MASTER IN FINANCE**

## **MASTER'S FINAL WORK DISSERTATION**

**PERFORMANCE OF RETURN MODELS: A PORTFOLIO  
THEORETICAL APPROACH**

**CARLOS AUGUSTO ZERPA FRADE**

**OCTOBER - 2017**



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**SUPERVISION:**

**RAQUEL MEDEIROS GASPAR**

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## ABSTRACT

The objective of this research is evaluating the impact of the return generating models assumptions in the efficient frontier and its portfolios. This is accomplished by working with in-sample data (eliminating the estimation risk and focus on the model risk) looking at both the European and American stock markets for the past 7 years, and considering both, the case when shortselling is allowed and the case when it is forbidden. The process includes the calculation of efficient frontier under the assumptions of return generating models. In particular, we look at the Constant Correlation Model (CCM), the Single-Index Model (SIM) and the three factors Fama and French (1993) Multi-Factor Model (MFM). For both markets we compared the true efficient frontier generated from the in-sample MVT with the corresponding efficient frontiers from the return generating models. We show model risk is an important issue when applying MVT. The errors in all model are considerable. In addition, considering model risk for cases when the short-sell is not allowed, the CCM is more accurate than more sophisticated models. On the other hand, under conditions of short-sell allowed, the SIM seems to be more accurate.

## RESUMO

O objetivo desta investigação é avaliar o impacto das suposições dos modelos geradores de retornos na fronteira eficiente e seus portfólios. Isto foi conseguido mediante o trabalho in-sample (assim eliminando o risco de estimação, focando a investigação no risco de modelo) em ambos mercados de ativos financeiros, o Europeu e Americano, nos 7 anos anteriores, considerando ambos, o caso em que shortselling está permitido como também o caso em que está proibido. O processo inclui o cálculo da fronteira eficiente seguindo as suposições dos modelos geradores de retornos. Em particular o Constant Correlation Model (CCM), o Single-Index Model (SIM) e o modelo de três fatores de Fama e French (1993) Multi-Factor Model (MFM). Para os dois mercados de investimentos, comparamos a fronteira eficiente gerada aplicando MVT nos dados in-sample com a fronteira eficiente dos modelos de retorno selecionados. Mostramos que o risco do modelo é importante na aplicação do MVT. Sendo os erros encontrados em todos os modelos consideráveis. Também, considerando o risco de modelo para o caso de shortselling proibido, o CCM mostra melhor desempenho que os modelos mais sofisticados. Por outra parte, em condições de shortselling permitido, o SIM mostra melhor desempenho.

## KEYWORDS

Model risk, estimation risk, mean variance theory, constant correlation model, single index model, multi-factor model, efficient frontier, optimal portfolio, tangent portfolio, minimum variance portfolio, portfolio composition.

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## List of Abbreviations

MVT: Mean Variance Theory

CCM: Constant Correlation Model

SIM: Single Index Model

MFM: Multi Factor Model

SSNA: Short-Sell is not allowed

SSA: Short-sell is allowed

## 1. Introduction

Popularity of the return models stems primarily from the intuitive appeal of the dichotomy into risk and return. The most well-known two-parameter portfolio models have the following main assumptions: A perfect capital market, investor risk aversion and two-parameters return distribution implies the important “Efficient set theorem”: The optimal portfolio for any investor must be efficient in the sense no other portfolio with the same or higher expected return has lower dispersion of return. In the search for the optimal portfolio, Markowitz (1952) developed Mean Variance Theory that has been the basis of the modern portfolio analysis. Introducing the concept of diversification into the investment and security analysis world, the theory was widely accepted and welcomed by those who had the means to use it. MVT bases itself in the assumption that analysts are able to estimate expected returns, future returns reliabilities and correlation across any pair of securities. However, our ability to efficiently estimate MVT inputs is limited. For that reason, there are return generating models that could easily and practically describe and help to forecast correlations. Using such models, we face two type of risk, estimation risk and model risk. The estimation risk comes from the process of obtaining the inputs that each model requires, while the model risk comes from the assumption that each model have on its application. The subject of estimation risk has been widely tested and studied in recent years even though model risk research it’s not a common topic. The present work focus on the model risk, evidencing the effect that model assumption have in the efficient frontier and relevant portfolios, performing an in-sample analysis of real-life portfolios in the European and American stock market and measuring the biases introduced by well-known return generation models: The Constant Correlation Model (CCM), The Single-Index Model (SIM), and the Fama and French (1993) 3 factors model, as our Multi-Factor Model (MFM).



## 2. Literature Review

The two-attribute risk and return models are very popular in the economics and finance world for analyzing decisions under uncertainty. Holthausen (1981) denotes that the popularity of the return models stems primarily from the intuitive appeal of the dichotomy into risk and return.

The most well-known two-parameter portfolio model were researched by Tobin (1958) and Markowitz (1959). The main assumptions: A perfect capital market, investor risk aversion and two-parameters return distribution implies the important “Efficient set theorem”: The optimal portfolio for any investor must be efficient in the sense no other portfolio with the same or higher expected return has lower dispersion of return.

In the search for the optimal portfolio, its proportions and characteristics, Markowitz (1952, 1959) developed MVT. For over 65 years MVT has been the basis of the modern portfolio analysis. Introducing the concept of diversification into the investment and security analysis world, the theory was widely accepted and welcomed by those who had the means to use it<sup>1</sup>.

MVT bases itself in the assumption that analysts are able to estimate expected returns, future returns reliabilities and correlation across any pair of securities. However, our ability to efficiently estimate MVT inputs is limited, mostly because of the nature of the correlation structure and the large amount of correlation coefficients to be estimated, see Epps (1981).

Elton and Gruber (1973), Elton, Gruber and Urich (1978), Sharpe (1963) and King (1966), acknowledging the situation and used it as a motivation. Developing return generating models that could easily and practically describe and help to forecast correlations.

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<sup>1</sup> Considering it required highly advance technological resources allocation to perform computations that were non trivial at the time.

When using such models, we face two type of risk, estimation risk and model risk. The estimation risk comes from the process of obtaining the inputs that each model requires, while the model risk comes from the assumption that each model have on its application.

The subject of estimation risk has been widely tested and studied in recent years by Bignozzi and Tsanakas (2015), Cardoso and Gaspar (2016), Jegadeesh, Noh, Pukthuanthong, Roll and Wang (2015) and Siegal and Woodgate (2016) even though model risk research it's not a common topic.

The present work focus on the model risk, evidencing the effect that model assumption have in the efficient frontier and relevant portfolios, like minimum variance and tangent portfolio, performing an in-sample analysis of real-life portfolios in the European and American stock market and measuring the biases introduced by well-known return generation models: The Constant Correlation Model (CCM), The Single-Index Model (SIM), and the Fama and French (1993) 3 factors model, as our Multi-Factor Model (MFM).

Section 2.1 deepens into MVT and the computation of the correlation structures. The subsequent sections review the return generating models considered for the analysis. Section 2.2 reviews the Constant Correlation Model, Section 2.3 approach the analysis of the Single Index Model and, finally, Section 2.4 aboard the research developments on the Multi Index Model.

## **2.1 MVT, Breakthroughs and limitations.**

As stated before, MVT is the basis of modern portfolio analysis. Its introduction in 1952 marked a step stone in the history of the financial analysis world formally defining, for the first time, the investment opportunity set and the notion of efficient frontier. Investors could, then, choose from the efficient frontier according to his return and risk preferences.

Moreover, MVT served as foundation for several further developments that improved the performance of financial investments through portfolio selection. Among them, we recall MVT as the foundation of equilibrium models, such as CAPM, developed by Lintner (1965), Mossin (1966) and Sharpe (1964).

When applying MVT, there are two important steps. The first is the inputs estimation of the model. The Second, consists in using those inputs to determine the investment opportunity set and the associate efficient frontier. The second part thus, depends strongly on part one.

## **2.2 Constant Correlation Model**

The most common correlation assumption is that past correlation structure hold information about the future average correlation, but do not contain information about individual differences of securities contained in the correlation matrix.

Elton and Gruber (1973) and Elton, Gruber and Urich (1978) tested widely the forecast of future correlations by smoothing the historical correlation matrix data with averaging. They tested both aggregate and disaggregate type of averaging techniques. The Aggregate averaging assumes future pairwise correlation coefficients as the average of all correlation coefficients in the past correlation structure. The disaggregate type of averaging is assumes that an average correlation can be found among a group of securities. Their conclusion is that, even when they found differences in the forecast technique, the differences were small when compared to the forecasting error.

## **2.3 Single Index Model**

The SIM of Sharpe (1963), is by far the most popular model when implementing MVT. The primary assumption of Sharpe's model is that there exists one and only one common factor able to explain (systematic) co-movements in returns. This factor, called index, can describe perfectly the co-movement between securities. The previous statement is based on common observation of the behavior of securities relate to that of the market as a

whole. The correlation structure across securities is then assumed as the correlation between the securities' return and the index. Implicitly, the assumption is that there are no specific correlations across securities' returns.

## **2.4 Multi Factor Model**

Multi Factor Models emerged when King (1966) analyzed the impact in co-movement among securities beyond the market's impact, and found the co-movements within industries.

The efficiency of the MFM in performing accurately forecast directly depends on the definition of the factors used. Elton and Gruber (1973) state that one of the most common approaches in finance is let the data define the factors and to obtain the series of factors that best describes the historical variance-covariance matrix we should analyze past correlation structures. However, they found that even when adding factors to the single-index model resulted in a better description of the historical correlation structure, it also led to a lower prediction accuracy and a decrease in the models performance.

Roll and Ross (1980) found that a MFM needs at least three factors to describe the historical correlation structure. While Dhrymes, Friend and Gultekin (1984) found that, in most cases, more than just three factors are needed and that the exact number depends on the overall number of firms under analysis. Gibbons (1982) found that the number of factors needed is between six and seven.

Several authors have dedicated their time to the research of optimal processes that can allow the MFM to show it's truly potential. Chen, Roll and Ross (1986) as Burmeister, and Wall (1986) and McElroy (1988), experimented successfully with MFM based on a set of macroeconomic variables, while Fama and French (1993) propose factors based upon firm characteristics. Both types of MFM being extremely important with remarkable potential applications in the finance world.

In terms of the comparison with the SIM, Brennam and Schwartz (1983), Nelson and Schaefer (1983), Elton, Gruber and Naber (1988), and Elton, Gruber and Michaely (1990) refer that MFM tend to be preferable because they might be more relevant. The reasons they suggest the last statement is true is because: The model have an better measure in the impact of changes in the interest rates, reflects the effect introduced by the differences in yield spread among government bonds and singular risk class, reflects the effect introduced by the differences in yield spread among government bonds, corporate bonds, and financial bonds, reflects the effect introduced by the differences in a call's value and reflects the effect introduced by the differences in tax rates

## **2.5 Historical comparison between models**

It is also important to test how well these models perform using parameters that are estimated based on historical data. At this point, it would be wrong to assume that adding more indexes to the MFM would result in a better performance than the single-index model or the constant correlation model. Having a more complex multi-index model only means that it is possible to reproduce more accurately the historical correlation structure, not that the model will forecast more accurately the future correlation structure.

The constant correlation model was tested extensively with respect to the single-index model, the multi-index model and the very historical correlation structures. In the three previous cases, the use of the constant correlation model, both with aggregate and disaggregate type of averaging, had a better performance. Elton, Gruber and Spitzer (2006) found forecast of future correlation structures with differences that were almost always statistically significant at a 0.05 level. Furthermore, differences in the performance of the portfolio in the four cases were significant enough to have an economic impact. Using the constant correlation model often led to an increase in returns of about 25%.

While testing the ability of producing future correlation structures of the Fama-French model against the constant correlation model, Chan, Karceski, and Lakonishok (1999) found that the model that produce lower forecasting errors is the constant correlation model. In the same line of investigation, Elton, Gruber and Spitzer (2006) find evidence

that suggest that simpler models appears to be better than more complex models, stating the preference as constant correlation model and Sharpe single-index model in that order. Cohen and Pogue (1967) tested the economic significance of the multi-index model (specialized form) against a single-index model. The authors divided the securities in the sample by the standard industrial classification. Both models were run with a market index and an industry index, concluding that the single-index model have more desirable properties since the model is simpler to use and led to lower expected risk than the multi-factor model.

Ledoit and Wolf (2003), observing that the increase in the complexity of the models tends to also increase the random noise that the model picks up, asked them self if combining models wouldn't improve the accuracy of the estimations. Deriving rules for combining results of forecasting from two different techniques, they found that combining the historical correlation structure with the Sharpe single-index model outperforms the individual models. However, in a later study in 2004, the authors found that combining the historical correlation structure with the constant correlation model works even better than their previous study. With the same approach as Ledoit and Wolf, a work performed by Elton, Gruber and Spitzer (2006) researched the use of a two-step procedure to find forecast. The first step is forecasting future average correlation between securities, while the second step is forecasting future difference from the mean. Concluding one, that an exponential smooth or a rolling average of past correlation structure works better in predicting the inputs of the model. And second, that organizing the population into industry groups or characteristics as size and assuming the correlation between securities as the average correlation between each group improves the forecasting results.

Once again, it is important to remember that, though there is extensive literature review in the estimation risk, there is no available literature where the estimation risk is segregated from the model risk so that it can be measured and analyzed. Most probably, the reason for the lack of research in the area is because it is hard to disaggregate one risk from another. Our empirical method uses in-sample analysis, so that there is no need to estimate any returns. Therefore, allowing us to focus only on measuring the effect of each model assumptions.

### **3. Data and Methodology**

In order to evidence the impact that return generating models assumptions have in the investment opportunity set, the efficient frontier obtained, and the associated portfolios, we propose an in-sample empirical analysis.

Out-of-sample results on the MVT are generally composed of inputs (estimation errors) and model risk resulting from the chosen return generating model (assumptions). In order to mitigate the estimation error and only quantify the effect caused by the return model itself, we have chosen to work in-sample, which means, we worked over realized data. Setting our starting investment period in a past date.

Under this control scenario, we know the “true” in-sample MVT inputs. Since all the returns are realized, we compute the realized expected returns, volatilities and correlations for all securities and built the correlation matrix as the original MVT suggest and consider it the realized scenario.

The reviewing of this section is organized as follows. Section 3.1 presents the data gathered and the methodology used to select it. Section 3.2 focus on the computation of the inputs for the MVT and each of the return generating model considered. Having as a division Subsection 3.2.1 for the Constant Correlation Model, Subsection 3.2.2 for the Single Index Model, Subsection 3.2.3 for the Multi Factor Model inputs. Subsequently, Section 3.3 details the calculation process of the set of portfolios under each method and Section 3.4 the methodology used for the comparison process.

#### **3.1 Data**

For this analysis, we look into two different investment opportunity sets: European stocks and American stocks. The aim of having two investment opportunity sets is to identify patterns in our results common for both datasets. To test if pattern arises due to the use of

restrictions in the portfolios, two scenarios are tested for each model considered. The first, short-sell is not allowed (SSNA) and second, short-sell allowed (SSA) for each model.

The components of each investment opportunity set represent the appropriate industries or sectors significantly involved in each market place so that each is a fair representation of the whole market. The number of stocks chosen was set in order to generate well enough diversified portfolios, thus, mitigating in its most efficient way the idiosyncratic risk associated. As Reilly (1985) suggests in his research, considering between 12 to 18 stocks should be enough. Mao (1970) and Sharpe (1972) also researched the optimal diversification of portfolios, reaching the conclusion that, in general, the benefits derived from diversification increase with the number of securities but at a decreasing rate. While the cost of diversification increase with the number of securities at an increasing rate, acknowledging that after the 20<sup>th</sup> stock, the benefit from increasing the diversification does not compensate the increase in the cost of diversification. The data gathered was the daily securities' price and the risk free rate both from the 01/01/2010 to the 31/12/2016, a seven year investment period.

Europe's opportunity set is built of 19 EuroStoxx 50's stocks, each representing one Stoxx All Europe 800 super sector. The index selected for the single-index model is the EuroStoxx 50<sup>2</sup>. America's opportunity set is built of 16 S&P500's stocks, each representing one Global Industry Classification Standard (GICS) super sector. The index selected for the single-index model is the S&P500 itself. See Table A.1 and A.2 for details on the exact shares considered for the each investment opportunity set.

Common daily stock prices are retrieved from Yahoo Finance. The risk free rates comes from the European Central Bank and the Federal Reserves for Europe and America, respectively. Book to market ratio and market capitalizations used in the Fama and French 3 factors MFM are retrieved from Bloomberg.

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<sup>2</sup> Selected because represents the performance of the 50 largest companies among the 19 supersectors in terms of free-float market cap in 11 Eurozone countries. The index captures about 60% of the free-float market cap of the EURO STOXX Total Market Index (TMI) and serves as one of the most popular underlying for financial products (options, futures, ETFs) and for benchmarking purposes.



## 3.2 Computation of models inputs

The inputs to generate the efficient frontiers and the set of portfolios which are compared in this work are the securities returns and the correlation structure. Each model specifies the calculation of the inputs under different specification, as stated in the literature review. At this point, it is important to remember that the main goal of this work is to analyze the impact of using return models to obtain MVT inputs.

### 3.2.1 The true MVT Inputs

We focus our study on the expected returns and correlation matrix resulting from the in-sample data. Creating a vector  $\bar{R}^{MVT}$  composed of the annual return of each of the stocks for each opportunity set and a matrix  $V^{MVT}$  composed with the correlations between the same returns. The above values are the inputs of the true MVT scenario.

This data serves to achieve the MVT efficient frontier, used as benchmark throughout this study, being compared with the other models efficient frontiers. Having this in consideration, these parameters are referred to as true expected returns and true correlation structure. Anything else referred to as true is assumed to be related with these same parameters or model.  $\bar{R}^{MVT}$  and  $V^{MVT}$  are available in the appendix under Table A.3 and Table A.4, respectively, for the European opportunity set and Table A.5 and Table A.6, respectively, for the American opportunity set.

### 3.2.2 Constant Correlation Model Inputs

For the Constant Correlation Model (CCM), the inputs were computed following Elton and Gruber (1973) and Elton, Gruber and Urich (1978) researches. Using the data gathered, the annual mean return  $\bar{R}^{CCM}$  (for this case, equals to  $\bar{R}^{MVT}$ , see Table A.3 in the appendix) of each security was computed. The Correlation Matrix  $V^{CCM}$  (available in the appendix under Table A.7 for the European opportunity set and Table A.8 for the American opportunity set. Noting that the diagonal of the matrix of MVT and CCM are

equal due to the fact that the input is the same, the standard deviation) was computed using an aggregate averaging technique, following the below formula, as the replacement of the pairwise correlation between each securities return.

$$\rho = \frac{\sum_{i=1}^N \sum_{j=1}^N \rho_{ij}}{\frac{N*(N-1)}{2}} \quad (1)$$

### 3.2.3 Single Index Model Inputs

For the Single Index Model (SIM), the inputs were computed following William Sharpe (1963) research. Using the data gathered, an auto-regression model was developed between the return of each securities and the return of the index chosen. With such auto-regression model, we found the parameters  $\alpha$  and  $\beta$  for each security. The before mentioned parameters represent the independent and dependent, respectively, part on the securities return on the market. The annual mean return  $\bar{R}^{SIM}$  (available in the appendix under Table A.9 for the European opportunity set and Table A.10 for the American opportunity set) of each security was computed using the following formula:

$$R_i = \alpha_i + \beta_i * R_M \quad (2)$$

The Correlation Matrix  $V^{SIM}$  (available in the appendix under Table A.11 for the European opportunity set and Table A.12 for the American opportunity set) was computed according to the model's specifications, following the below formulas:

$$\sigma_i^2 = \beta_i^2 * \sigma_{RM}^2 \quad (3)$$

$$\sigma_{ij}^2 = \beta_i * \beta_j * \sigma_{RM}^2 \quad (4)$$

### 3.2.4 Multi-Factor Model Inputs

For the Multi-Factor Model (MFM), the inputs were computed following Fama and French (1993) research. Using the data gathered, an auto-regression model was developed between the return of each securities and the models parameters. Such parameters include the markets excess return, which is assumed to be the return on the Index chosen minus the risk-free rate, the size premium and the value premium.

To compute the size and value premium, the model suggests the construction of six portfolios considering the market capitalization and the book to market value of each

asset. This portfolios are: Small size companies with low book to market value of its assets (S/L), Small size companies with medium book to market value of its assets (S/M), Small size companies with high book to market value of its assets (S/H), Big size companies with low book to market value of its assets (B/L), Big size companies with medium book to market value of its assets (B/M) and Big size companies with high book to market value of its assets (B/H).

Once the six portfolios are computed, the Size premium SMB (small minus big) and the HML (high minus low) components required by the model were computed as follows:

$$SMB = \frac{1}{3} * (S/L + S/M + S/H) - \frac{1}{3} * (B/L + B/M + B/H) \quad (5)$$

$$HML = \frac{1}{2} * (S/H + B/H) - \frac{1}{2} * (S/L + B/L) \quad (6)$$

With such auto-regression model, we found the parameters  $\alpha$ ,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  for each security. The annual mean return  $\bar{R}^{MFM}$  (available in the appendix under Table A.13 for the European opportunity set and Table A.14 for the American opportunity set) of each security was computed using the following formula:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_{i1} * (R_{Mt} - R_{ft}) + \beta_{i2} * SMB + \beta_{i3} * HML \quad (7)$$

The Correlation Matrix  $V^{MFM}$  (available in the appendix under Table A.15 for the European opportunity set and Table A.16 for the American opportunity set) was computed according to the model's specifications, following the below formulas:

$$\sigma_i^2 = \beta_{i1}^2 * \sigma_{(RM-Rf)}^2 + \beta_{i2}^2 * \sigma_{(SMB)}^2 + \beta_{i3}^2 * \sigma_{(HML)}^2 \quad (8)$$

$$\sigma_{ij}^2 = \beta_{i1} * \beta_{j1} * \sigma_{(RM-Rf)}^2 + \beta_{i2} * \beta_{j2} * \sigma_{(SMB)}^2 + \beta_{i3} * \beta_{j3} * \sigma_{(HML)}^2 \quad (9)$$

### 3.3 Efficient frontier and set of portfolios computation

Once the inputs of each model were computed accordingly, the method used to obtain the efficient frontier and set of portfolios to analyze (naïve portfolio, minimum variance portfolio and maximum Sharpe ratio portfolio) was the same, indifferently of the model used to compute the inputs. Using the formulas as follows:

$$\bar{R}_p = X^T * \bar{R} \quad (10)$$

$$\sigma_p^2 = X^T * V * X \quad (11)$$

Where:

$\bar{R}$ : is the returns matrix

X: is the proportion of each security matrix

V: is the correlation matrix between securities

The minimum variance portfolio was obtain such that the variance of the portfolio  $\sigma_p$  was minimum:

$$\begin{aligned} \text{Min}_{x_a, x_b, x_c, \dots, x_n} \sigma_{p,x}^2 = & x_a^2 \sigma_a^2 + x_b^2 \sigma_b^2 + x_c^2 \sigma_c^2 + \dots + x_n^2 \sigma_n^2 + 2x_a x_b \sigma_{ab} + \\ & 2x_a x_c \sigma_{ac} + 2x_b x_c \sigma_{bc} + \dots + 2x_{n-1} x_n \sigma_{n-1,n} \quad \text{s.t. } x_a + x_b + x_c = 1 \end{aligned} \quad (12)$$

In matrix expression:

$$\text{Min}_x \sigma_{p,x}^2 = x' \Sigma x \quad \text{s.t. } x' \mathbf{1} = 1 \quad (13)$$

The tangent portfolio was obtain such that the Sharpe ratio was maximized.

$$\text{Max}_x R_p = x' R \quad \text{s.t. } \sigma_p^2 = x' \Sigma x = \sigma_{p,0}^2 \quad \text{and } x' \mathbf{1} = 1 \quad (14)$$

To compute the portfolio frontier in  $(R, \sigma)$  space (Markowitz bullet) we only need to find two efficient portfolios. The remaining frontier portfolios can then be expressed as convex combinations of these two portfolios. The following proposition describes the process for the three risky asset case using matrix algebra.

Let  $x = (x_a, x_b, x_c)'$  and  $y = (y_a, y_b, y_c)'$  be any two minimum variance portfolios with different target expected returns  $x' R = R_{p,0} \neq y' R = R_{p,1}$ .

That is, portfolio  $x$  solves

$$\text{Min}_x \sigma_{p,x}^2 = x' \Sigma x \quad \text{s.t. } x' R = R_{p,0} \quad \text{and } x' \mathbf{1} = 1,$$

And portfolio  $y$  solves

$$\text{Min}_y \sigma_{p,y}^2 = y' \Sigma y \quad \text{s.t. } y' R = R_{p,1} \quad \text{and } y' \mathbf{1} = 1.$$

Let  $\alpha$  be any constant and define the portfolio  $z$  as a linear combination of portfolios  $x$  and  $y$ :

$$z = \alpha \cdot x + (1 - \alpha)y = \begin{bmatrix} \alpha x_a + (1 - \alpha)y_a \\ \alpha x_b + (1 - \alpha)y_b \\ \alpha x_c + (1 - \alpha)y_c \end{bmatrix} \quad (15)$$

Then the portfolio  $z$  is a minimum variance portfolio with expected return and variance given by:

$$R_{p,z} = z'R = \alpha \cdot R_{p,x} + (1 - \alpha)R_{p,y}, \quad (16)$$

$$\sigma^2_{p,z} = z'\Sigma z = \alpha^2 \sigma^2_{p,x} + (1 - \alpha)^2 \sigma^2_{p,y} + 2\alpha(1 - \alpha)\sigma_{xy}, \quad (17)$$

Where

$$\sigma^2_{p,x} = x'\Sigma x, \sigma^2_{p,y} = y'\Sigma y, \sigma^2_{x,y} = x'\Sigma y.$$

### 3.4 Comparison Process

In order to perform a standardized and rational comparison between models, three methods were selected. The first method includes to compute the difference in risk (volatility) for each level of return for the expected and realized efficient frontiers of each model. The lower this difference is, the closest is the expected from the realized efficient frontier. Second method includes to compute the difference in risk (volatility) for each level of return for the control MVT and realized efficient frontier of each model. The lower this difference is, the closest is the expected from the realized efficient frontier. Lastly, the third method was developed specially for the purpose of measuring the difference between the compositions of portfolios. The difference ratio takes the form of the following formula:

$$Difference\ ratio_{x,y} = \frac{\sum_{i=1}^n (x_i^y - x_i^z)^2}{\sum_{i=1}^n (x_i^y - x_i^{naive})^2} \quad (18)$$

Where:

$n$ : Is the number of assets

$y$  and  $z$  stand for the MVT portfolio and the portfolio from the respective model being analyzed, respectively.

The lower this index is, the closest is the composition of the model's portfolios from the control MVT portfolio's composition compared to the naïve portfolio.

The above mentioned ratio was set up to test the intrinsic fact that, even when two portfolios can be a perfect match when being compared considering the return and volatility, that does not mean they have the same combination of assets and composition.

In order to perform an accurate comparison between the portfolios composition, the ratio tests the difference between the in-sample MVT portfolio composition and the selected model portfolio composition, divided by the difference between the in-sample MVT portfolio composition and the naïve portfolio composition. Both parts are squared so the difference can be always a positive number.

## 4. Results

The present chapter aboard the results obtained by applying the methodology explained in the last chapter to the data gathered. It is divided into two main sections, first, Section 4.1 covers the results obtained for the European opportunity set. The same section is subdivided in six subsection, Subsection 4.1.1 for MVT results, 4.1.2 for CCM results, 4.1.3 for SIM results, 4.1.4 for MFM results, 4.1.5 for the overall differences between the results obtained from each model and, finally, Subsection 4.1.6 for the combination results regarding each models relevant portfolios. The second part, Section 4.2 covers the results obtained for the American opportunity set. Also following the same standards as the previous section, it is subdivided into Subsection 4.2.1 for MVT results, 4.2.2 for CCM results, 4.2.3 for SIM results and finally, Subsection 4.2.4 for MFM results, 4.2.5 for the overall differences between the results obtained from each model and, finally, Subsection 4.2.6 for the combination results and comparison regarding each models relevant portfolios.

### 4.1 True Efficient Frontiers and Optimal Portfolios

In the following section, the results obtained by applying MVT to the European and American opportunity sets are shown. The graphs presents the efficient frontiers with their respective minimum variance and tangent portfolio. Also, it provides the position of the naïve portfolio.

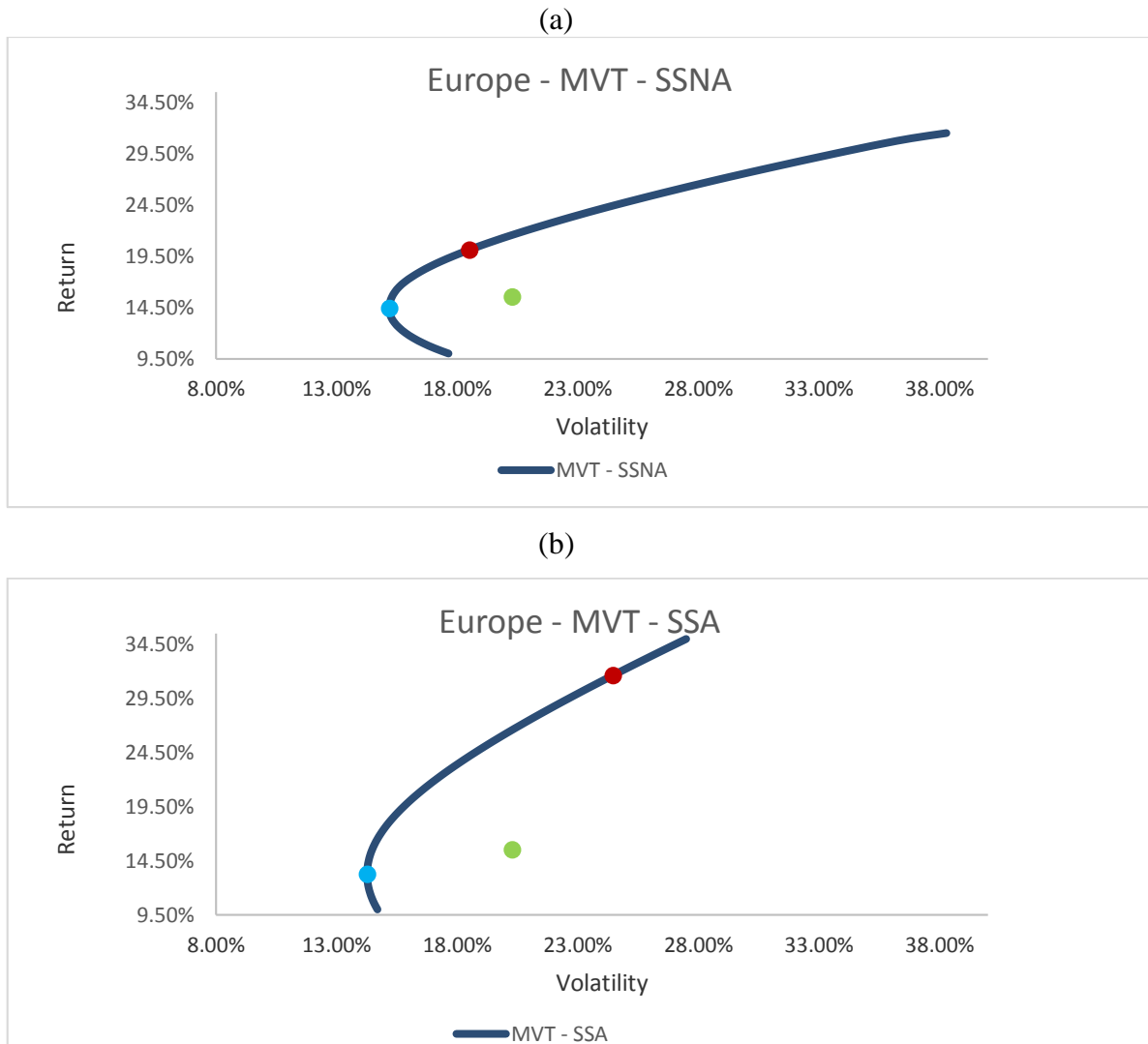
#### 4.1.1 European MVT Efficient Frontier

The below Figure 1 (a) presents the efficient frontier obtained by applying the MVT to the European dataset with short-sell not allowed. The tangent portfolio shows a return of 20.13% and a standard deviation of 18.52% while the minimum variance portfolio shows a return of 14.41% and a standard deviation of 15.19%

Figure 1 (b) presents the efficient frontier obtained by applying the MVT to the European dataset with short-sell allowed. The tangent portfolio shows a return of 31.64% and a

standard deviation of 24.46% while the minimum variance portfolio shows a return of 13.26% and a standard deviation of 14.27%

Figure 1- In-Sample MVT Efficient Frontier for Europe



Efficient frontier for the in-sample MVT in the European investment market, both for (a) short-sell restricted and (b) short-sell allowed

#### 4.1.2 American MVT Efficient Frontier

The below Figure 2 (a) presents the efficient frontier obtained by applying the MVT to the American dataset with short-sell not allowed. The tangent portfolio shows a return of



21.71% and a standard deviation of 14.83% while the minimum variance portfolio shows a return of 14.98% and a standard deviation of 12.13%

Figure 2 (b) presents the efficient frontier obtained by applying the MVT to the American dataset with short-sell allowed. The tangent portfolio shows a return of 25.21% and a standard deviation of 15.91% while the minimum variance portfolio shows a return of 15.30% and a standard deviation of 11.85%

Figure 2 - In-Sample MVT Efficient Frontier for America



Efficient frontier for the in-sample MVT in the American investment market, both for (a) short-sell restricted and (b) short-sell allowed

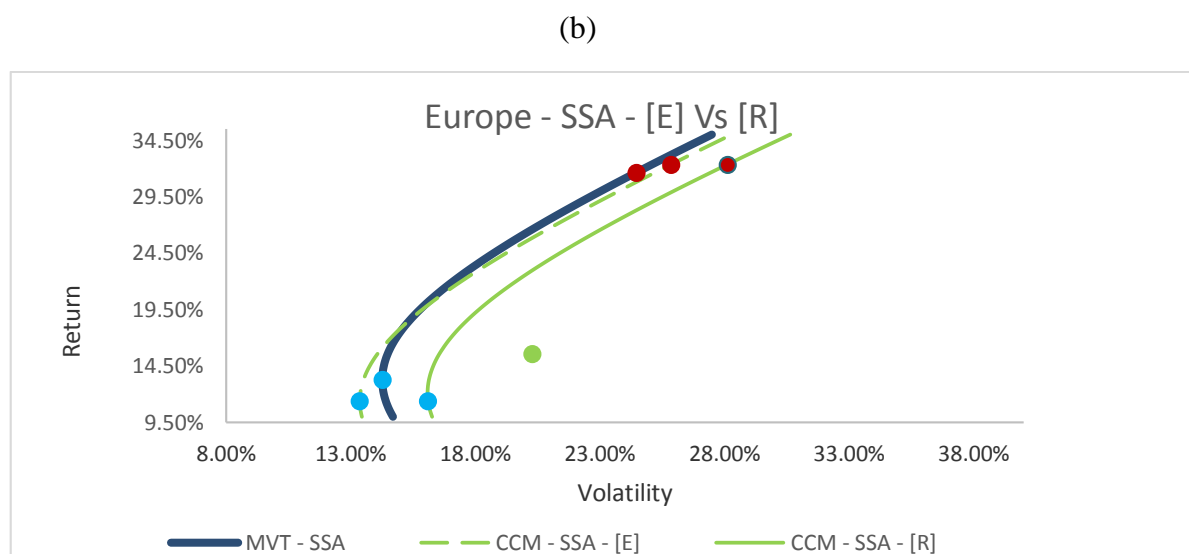
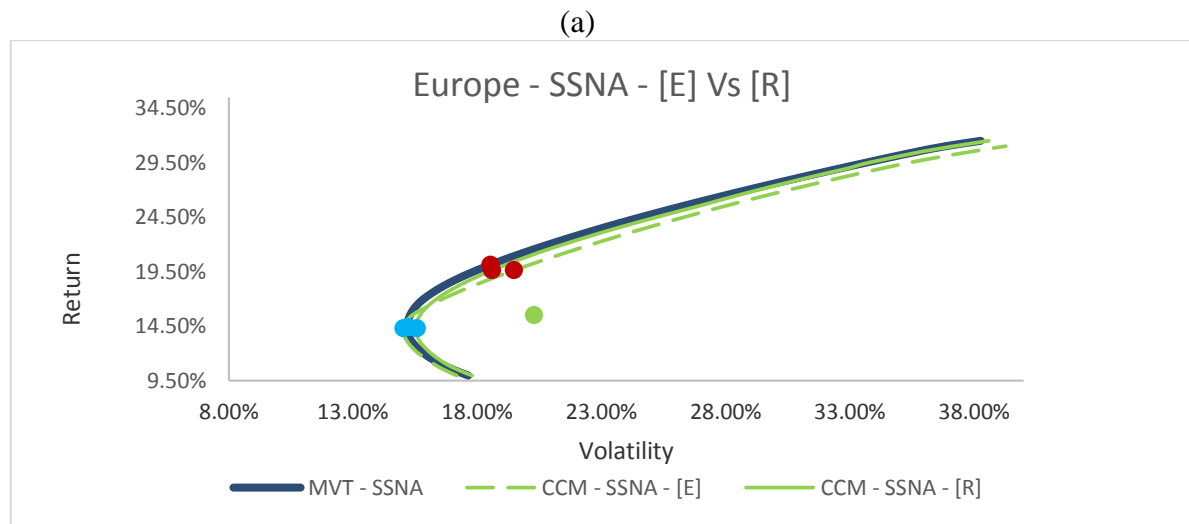
## 4.2 Using Constant Correlation Model

Each subsection presents the graphs of both the expected [E] and realized [R] efficient frontiers with their respective minimum variance and tangent portfolio for each opportunity set. Moreover, each subsection presents the results for both restriction cases, short-sell is allowed and short-sell is not allowed.

**4.2.1 European CCM Efficient Frontier**

Figure 3 (a) presents the efficient frontier obtained by applying the CCM to the European dataset with short-sell not allowed. The expected tangent portfolio shows a return of 19.62% and a standard deviation of 19.47% while the expected minimum variance

Figure 3 - CCM Efficient Frontier for Europe



Efficient frontier for the CCM in the European investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized CCM efficient frontier.

shows a return of 14.29% and a standard deviation of 15.02%. The realized tangent portfolio shows a return of 19.62% and a standard deviation of 18.58% while the expected minimum variance portfolio shows a return of 14.29% and a standard deviation of 15.56%.

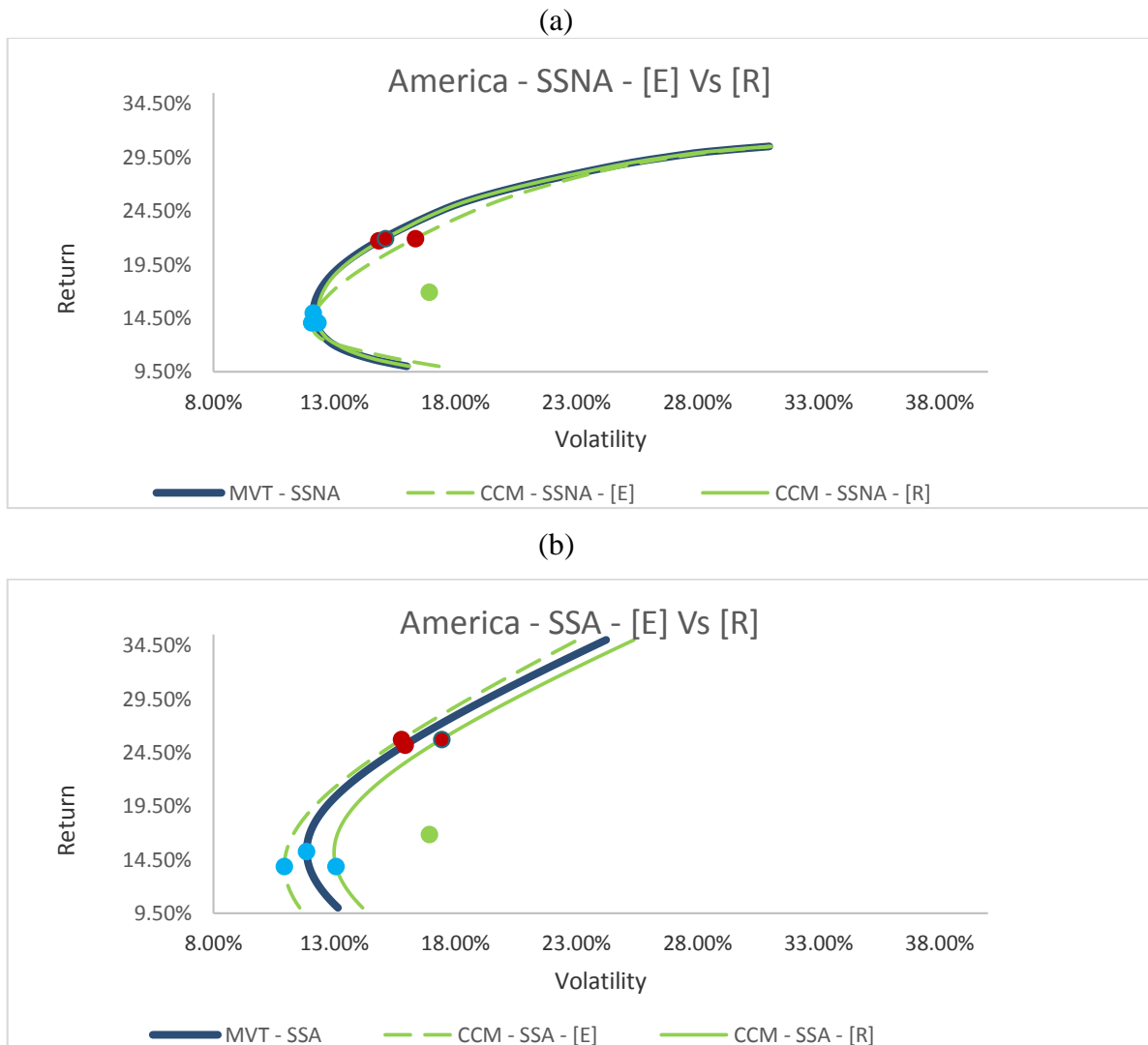
Figure 3 (b) presents the efficient frontier obtained by applying the CCM to the European dataset with short-sell allowed. The expected tangent portfolio shows a return of 32.36% and a standard deviation of 25.85% while the expected minimum variance portfolio shows a return of 11.40% and a standard deviation of 13.36%. The realized tangent portfolio shows a return of 32.36% and a standard deviation of 28.11% while the expected minimum variance portfolio shows a return of 11.40% and a standard deviation of 16.10%.

#### **4.2.2 American CCM Efficient Frontier**

Figure 4 (a) presents the efficient frontier obtained by applying the CCM to the American dataset with short-sell not allowed. The expected tangent portfolio shows a return of 21.93% and a standard deviation of 16.35% while the expected minimum variance portfolio shows a return of 14.06% and a standard deviation of 12.05%. The realized tangent portfolio shows a return of 21.93% and a standard deviation of 15.11% while the expected minimum variance portfolio shows a return of 14.06% and a standard deviation of 12.31%.

Figure 4 (b) presents the efficient frontier obtained by applying the CCM to the American dataset with short-sell allowed. The expected tangent portfolio shows a return of 25.73% and a standard deviation of 15.77% while the expected minimum variance portfolio shows a return of 13.91% and a standard deviation of 10.93%. The realized tangent portfolio shows a return of 25.73% and a standard deviation of 17.43% while the expected minimum variance portfolio shows a return of 13.91% and a standard deviation of 13.07%.

Figure 4 - CCM Efficient Frontier for America



Efficient frontier for the CCM in the America investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized CCM efficient frontier.

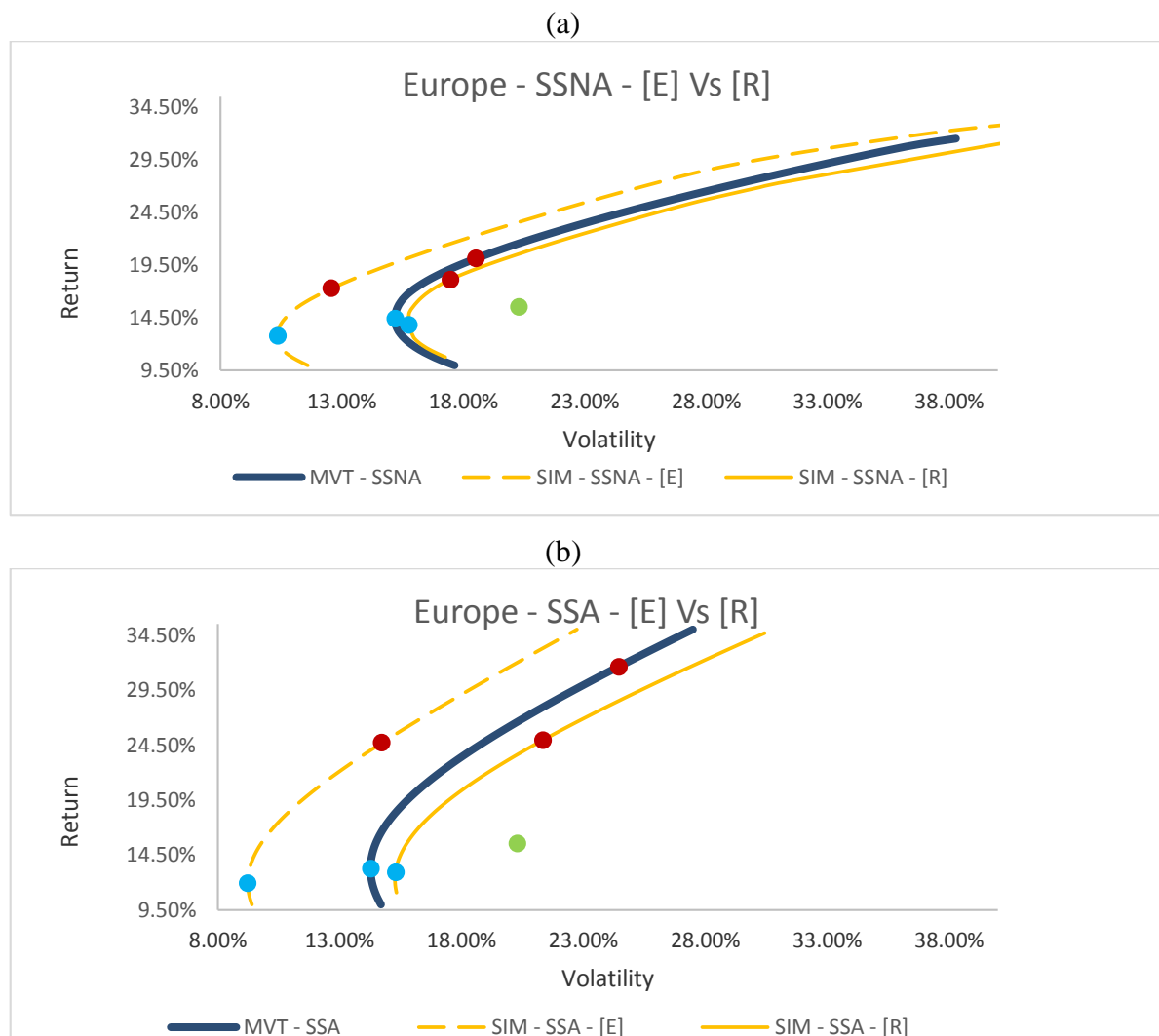
### 4.3 Using Single Index Model

Each subsection presents the graphs of both the expected [E] and realized [R] efficient frontiers with their respective minimum variance and tangent portfolio for each opportunity set. Moreover, each subsection presents the results for both restriction cases, short-sell is allowed and short-sell is not allowed.

### 4.3.1 European SIM Efficient Frontier

Figure 5 (a) presents the efficient frontier obtained by applying the SIM to the European dataset with short-sell not allowed. The expected tangent portfolio shows a return of 17.28% and a standard deviation of 12.57% while the expected minimum variance portfolio shows a return of 12.75% and a standard deviation of 10.35%. The realized tangent portfolio shows a return of 18.12% and a standard deviation of 17.46% while the expected minimum variance portfolio shows a return of 13.79% and a standard deviation of 15.75%.

Figure 5 - SIM Efficient Frontier for Europe

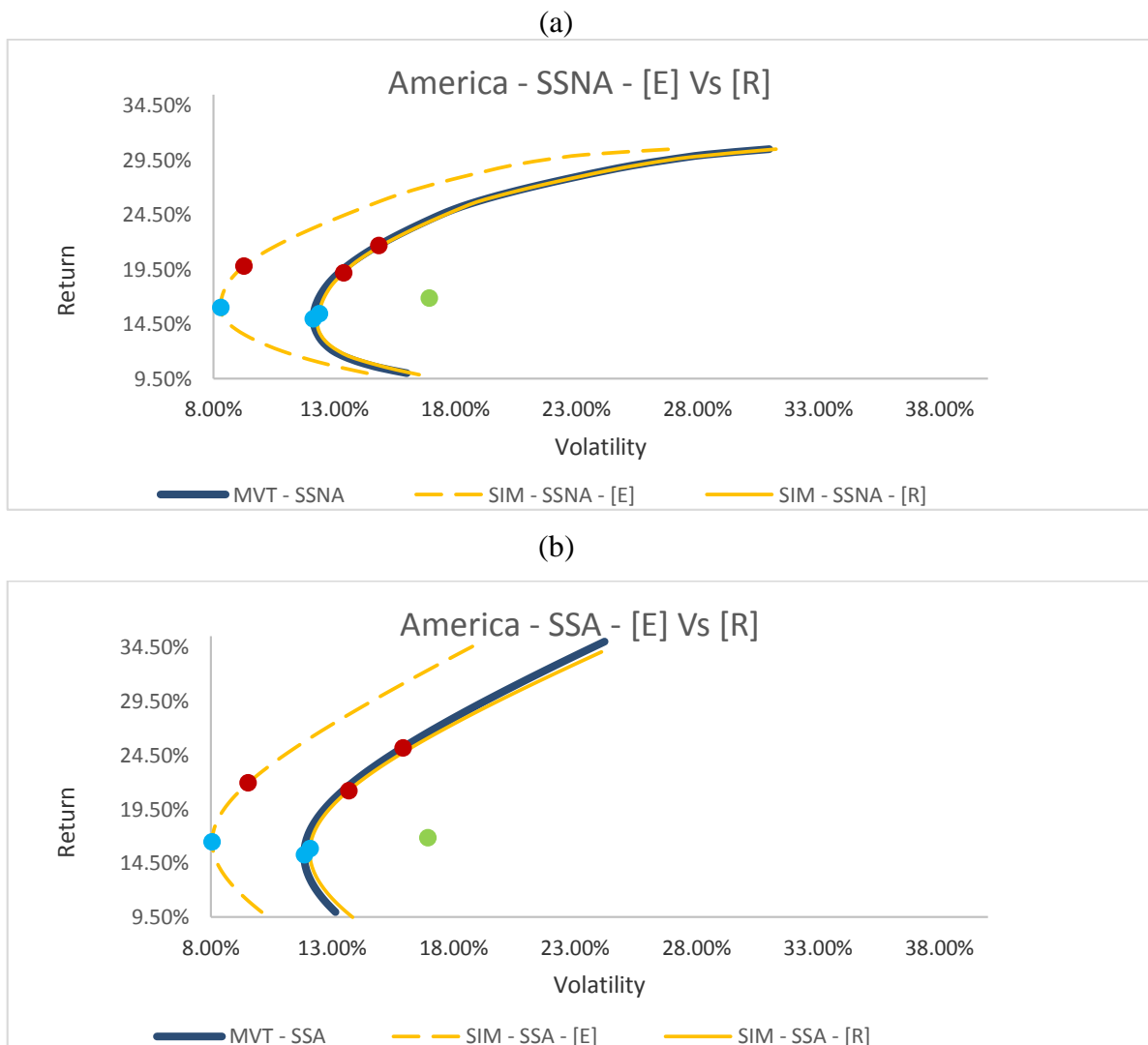


Efficient frontier for the SIM in the European investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized SIM efficient frontier.

Figure 5 (b) presents the efficient frontier obtained by applying the SIM to the European dataset with short-sell allowed. The expected tangent portfolio shows a return of 24.70% and a standard deviation of 14.74% while the expected minimum variance portfolio shows a return of 11.96% and a standard deviation of 9.22%. The realized tangent portfolio shows a return of 24.97% and a standard deviation of 21.54% while the expected minimum variance portfolio shows a return of 12.93% and a standard deviation of 15.29%.

### 4.3.2 American SIM Efficient Frontier

Figure 6 - SIM Efficient Frontier for America



Efficient frontier for the SIM in the America investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized SIM efficient frontier.

Figure 6 (a) presents the efficient frontier obtained by applying the SIM to the American dataset with short-sell not allowed. The expected tangent portfolio shows a return of 19.79% and a standard deviation of 9.26% while the expected minimum variance portfolio shows a return of 16.06% and a standard deviation of 8.30%. The realized tangent portfolio shows a return of 19.21% and a standard deviation of 13.37% while the expected minimum variance portfolio shows a return of 15.49% and a standard deviation of 12.38%.

Figure 6 (b) presents the efficient frontier obtained by applying the SIM to the American dataset with short-sell allowed. The expected tangent portfolio shows a return of 21.93% and a standard deviation of 9.54% while the expected minimum variance portfolio shows a return of 16.48% and a standard deviation of 8.05%. The realized tangent portfolio shows a return of 21.24% and a standard deviation of 13.68% while the expected minimum variance portfolio shows a return of 15.89% and a standard deviation of 12.09%.

## **4.4 Using Fama – French 3 Factors Model**

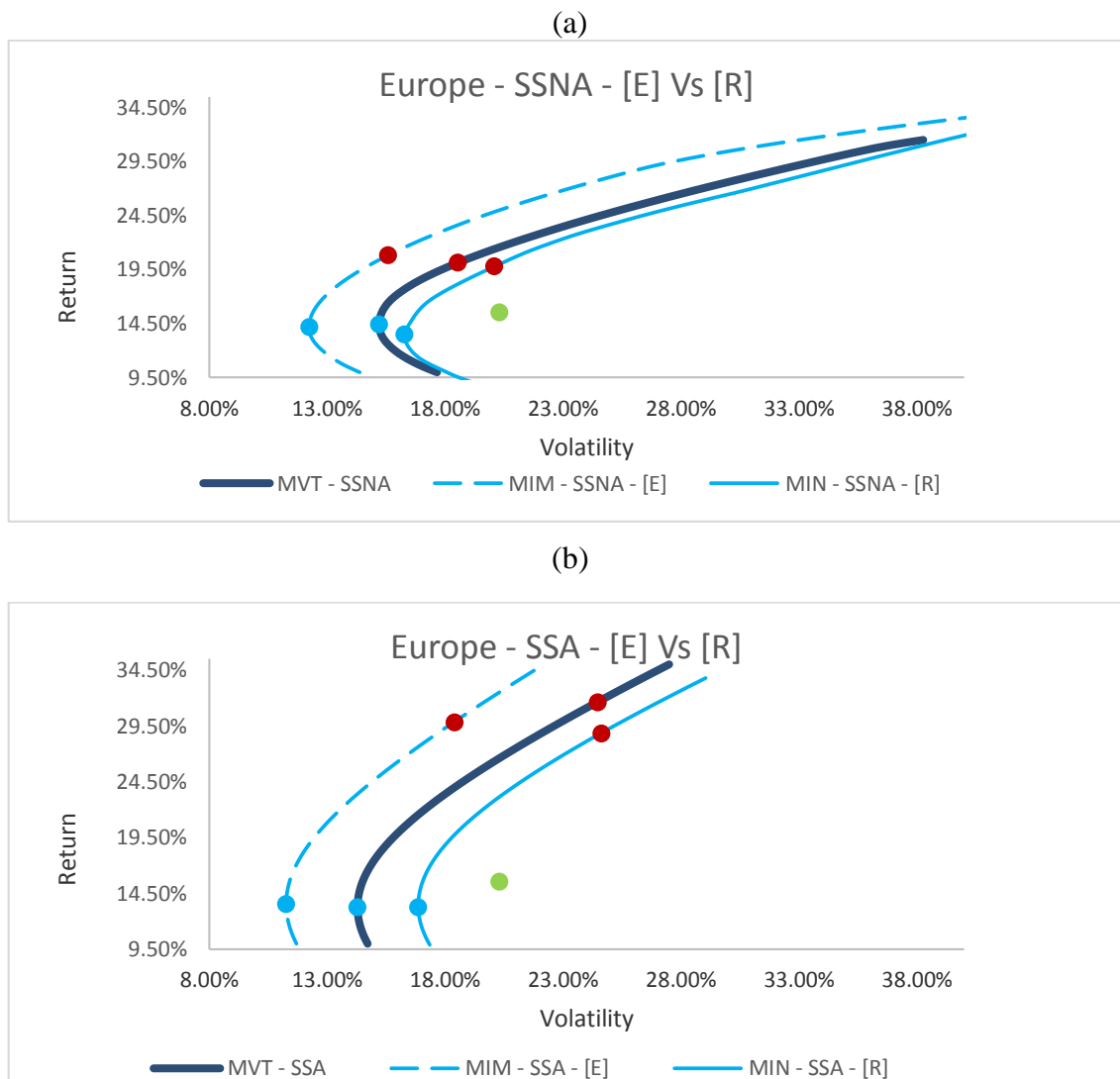
Each subsection presents the graphs of both the expected [E] and realized [R] efficient frontiers with their respective minimum variance and tangent portfolio for each opportunity set. Moreover, each subsection presents the results for both restriction cases, short-sell is allowed and short-sell is not allowed.

### **4.4.1 European MFM Efficient Frontier**

Figure 7 (a) presents the efficient frontier obtained by applying the MFM to the European dataset with short-sell not allowed. The expected tangent portfolio shows a return of 20.82% and a standard deviation of 15.58% while the expected minimum variance portfolio shows a return of 14.15% and a standard deviation of 12.22%. The realized tangent portfolio shows a return of 19.76% and a standard deviation of 20.07% while the expected minimum variance portfolio shows a return of 13.49% and a standard deviation of 16.27%.

Figure 7 (b) presents the efficient frontier obtained by applying the SIM to the European dataset with short-sell allowed. The expected tangent portfolio shows a return of 29.85% and a standard deviation of 18.39% while the expected minimum variance portfolio shows a return of 13.55% and a standard deviation of 11.26%. The realized tangent portfolio shows a return of 28.86% and a standard deviation of 24.62% while the expected minimum variance portfolio shows a return of 13.27% and a standard deviation of 16.85%.

Figure 7 - MFM Efficient Frontier for Europe



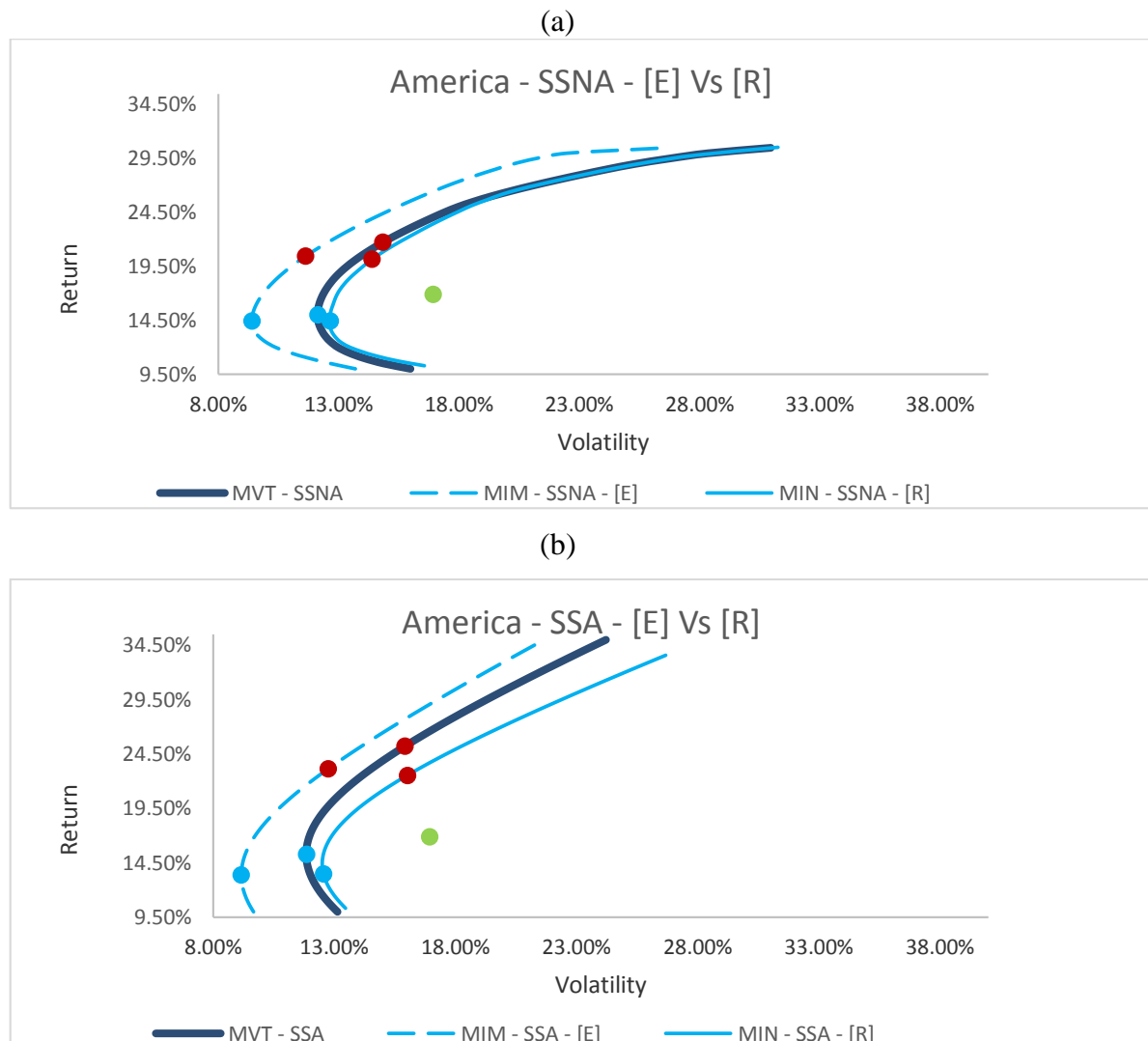
Efficient frontier for the MFM in the European investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized MFM efficient frontier.



### 4.2.4 American MFM Efficient Frontier

Figure 8 (a) presents the efficient frontier obtained by applying the MFM to the American dataset with short-sell not allowed. The expected tangent portfolio shows a return of 20.43% and a standard deviation of 11.62% while the expected minimum variance portfolio shows a return of 14.40% and a standard deviation of 9.39%. The realized tangent portfolio shows a return of 20.13% and a standard deviation of 14.38% while the expected minimum variance portfolio shows a return of 14.40% and a standard deviation of 12.64%.

Figure 8 - MFM Efficient Frontier for America



Efficient frontier for the MFM in the America investment market, both for (a) short-sell restricted and (b) short-sell allowed. Comparing In-Sample MVT, expected and realized MFM efficient frontier.

Figure 8 (b) presents the efficient frontier obtained by applying the SIM to the American dataset with short-sell allowed. The expected tangent portfolio shows a return of 23.14% and a standard deviation of 12.74% while the expected minimum variance portfolio shows a return of 13.39% and a standard deviation of 9.16%. The realized tangent portfolio shows a return of 22.53% and a standard deviation of 16.02% while the expected minimum variance portfolio shows a return of 13.48% and a standard deviation of 12.55%.

## 4.5 Discussion

### 4.5.2 Efficient frontier comparison for Europe

Figure 9 shows the results obtained by computing the difference in risk (volatility) between expected and realized results in the efficient frontiers for each fixed level of returns, for both SSNA and SSA cases. Figure 10 shows the results obtained by computing the difference in risk (volatility) between MVT and realized results in the efficient frontiers for each fixed level of returns, for both SSNA and SSA cases.

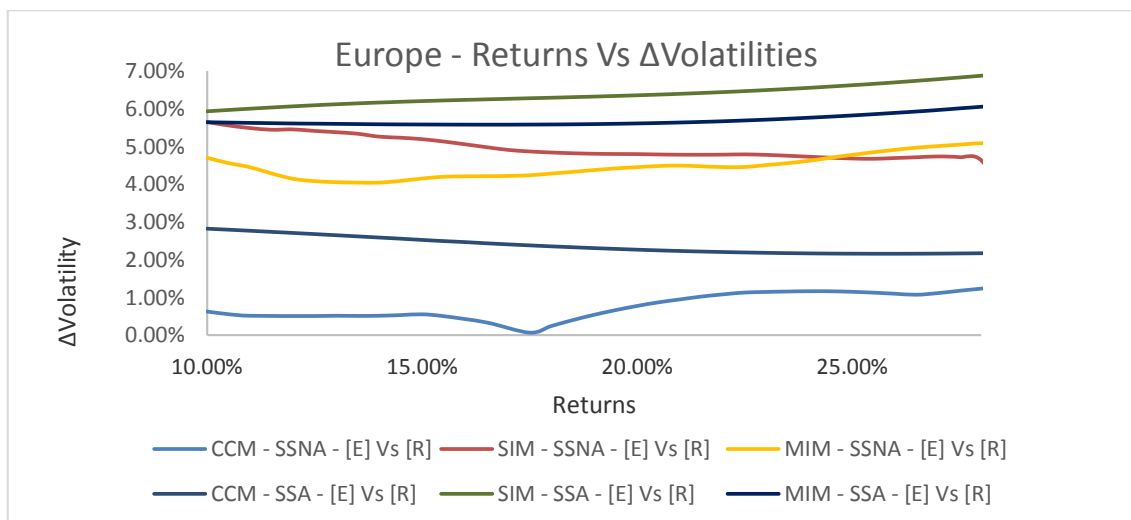
Table 1 and 2 show the composition of both, the minimum variance and the tangent portfolio, for each model used during this research, for both SSNA and SSA cases. At the end of each table, its shown difference in composition ratio. The lower the index is, the closer is the composition of the portfolio to the composition of our control, the MVT.

### 4.5.2 Efficient frontier comparison for America

Figure 11 shows the results obtained by computing the difference in risk (volatility) between expected and realized results in the efficient frontiers for each fixed level of returns, for both SSNA and SSA cases. Figure 12 shows the results obtained by computing the difference in risk (volatility) between MVT and realized results in the efficient frontiers for each fixed level of returns, for both SSNA and SSA cases.

Table 3 and 4 show the composition of both, the minimum variance and the tangent portfolio, for each model used during this research, for both SSNA and SSA cases.

Figure 9- Difference between [E] and [R] on Europe



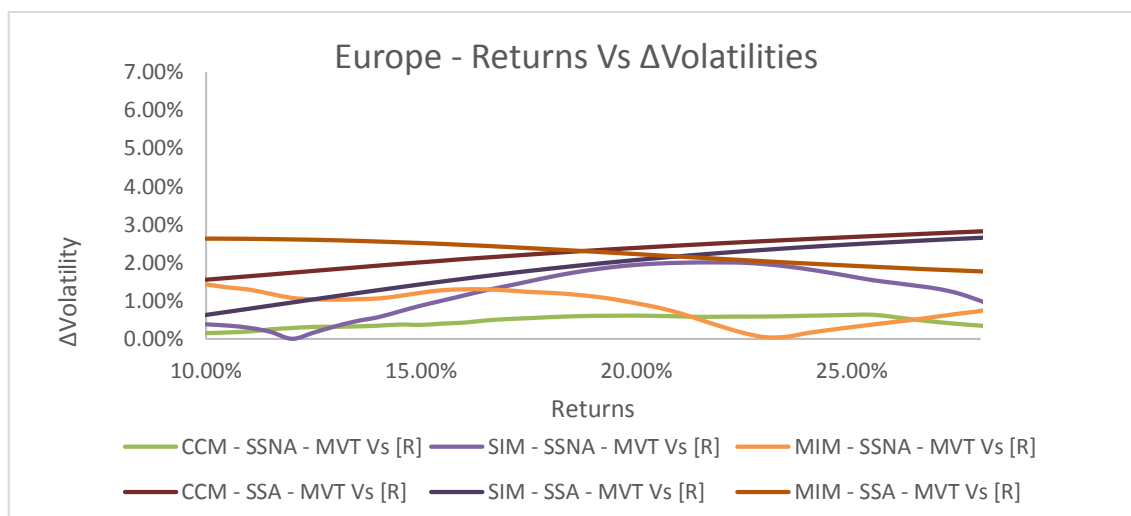
Difference in the level of risk (volatility) between expected and realized portfolios for the same level of returns in the European market.

Table 1 - Composition of Portfolios for Europe – SSNA

Securities	Portfolio Composition - Europe - SSNA							
	Minimum Variance				Tangent Portfolio			
	MVT	CCM	SIM	MIM	MVT	CCM	SIM	MIM
ABE	1.36%	3.34%	3.98%	0.39%	0.00%	0.00%	0.00%	0.00%
AC	0.00%	0.00%	0.00%	1.65%	0.00%	0.00%	0.00%	0.00%
AGS	0.00%	0.00%	0.00%	5.33%	0.00%	0.00%	0.00%	2.60%
AD	23.56%	21.50%	16.02%	10.06%	11.24%	7.19%	11.54%	0.00%
APAM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.51%
BAYN	0.00%	0.48%	2.80%	0.11%	0.00%	0.00%	0.00%	0.00%
BNP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
DTE	0.65%	0.00%	0.46%	0.16%	18.06%	11.67%	9.55%	11.11%
EDP	7.14%	3.56%	9.18%	9.79%	0.00%	0.00%	0.00%	0.00%
EXO	0.00%	0.00%	0.00%	0.00%	1.87%	12.30%	13.00%	21.03%
FCA	0.00%	0.00%	0.00%	0.00%	7.61%	6.46%	1.97%	4.21%
GFC	6.23%	4.58%	2.29%	5.27%	5.67%	4.45%	5.13%	10.74%
HEIA	16.20%	19.11%	13.01%	8.08%	16.69%	15.18%	6.48%	0.00%
NOS	2.73%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
REP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
SAP	13.20%	14.81%	4.44%	0.00%	21.37%	27.63%	11.29%	11.01%
UCB	12.22%	5.43%	14.98%	10.57%	17.49%	8.65%	15.41%	16.82%
UNA	12.71%	27.18%	32.84%	39.80%	0.00%	6.47%	25.64%	19.63%
WIE	4.01%	0.00%	0.00%	8.79%	0.00%	0.00%	0.00%	0.35%
Difference Ratio		23.0994	38.8913	37.0669		47.7878	47.1580	41.7697

Composition of the minimum variance and tangent portfolios for the European market under the short-sell restriction. Showing also, the computed Difference Ratio.

Figure 10-Difference between MVT and [R] on Europe



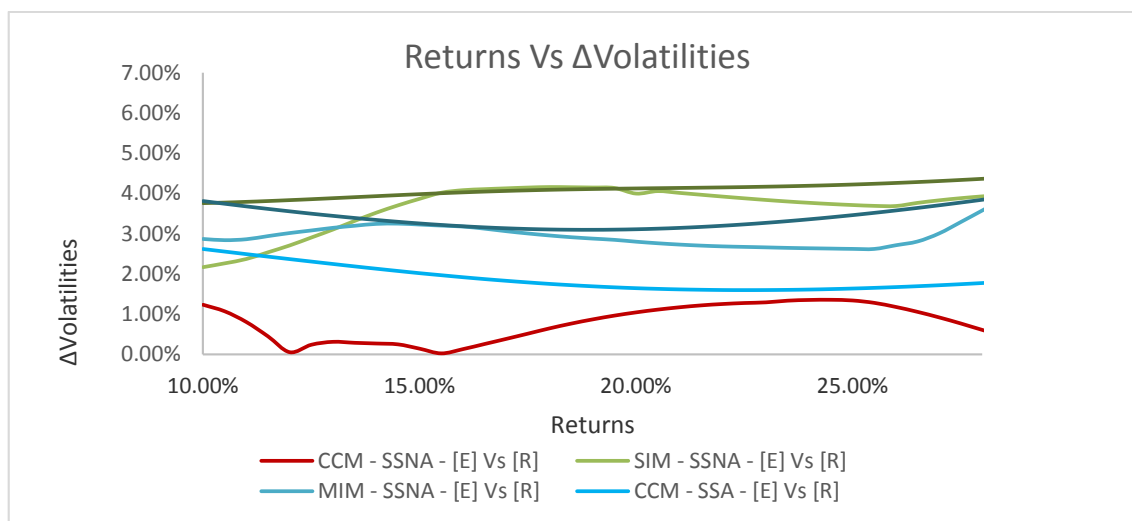
Difference in the level of risk (volatility) between in-sample MVT and realized portfolios for the same level of returns in the American market.

Table 2-Composition of portfolios for Europe - SSA

Portfolio Composition - Europe - SSA								
Securities	Minimum Variance				Tangent Portfolio			
	MVT	CCM	SIM	MIM	MVT	CCM	SIM	MIM
ABE	8.77%	8.42%	10.27%	7.59%	-32.19%	-34.10%	-20.15%	-37.90%
AC	-3.73%	-0.80%	2.61%	3.35%	-28.32%	-12.51%	-1.60%	0.21%
AGS	-0.88%	-4.45%	-2.70%	6.45%	7.34%	-3.74%	-2.29%	11.72%
AD	23.11%	24.16%	17.28%	12.75%	22.46%	27.50%	20.30%	15.08%
APAM	-2.73%	-6.60%	-5.43%	-1.14%	7.78%	-1.81%	0.59%	7.08%
BAYN	-8.14%	5.84%	7.82%	4.00%	-16.63%	5.15%	12.96%	1.78%
BNP	-7.63%	-4.99%	-10.99%	-1.05%	-14.68%	-12.51%	-22.99%	-13.71%
DTE	1.32%	-7.02%	0.95%	-0.68%	21.36%	19.90%	9.86%	11.70%
EDP	9.70%	8.63%	10.84%	12.63%	-10.36%	-24.99%	-2.55%	-4.84%
EXO	-4.69%	-2.81%	0.15%	1.61%	24.75%	24.05%	22.41%	28.08%
FCA	-2.54%	-6.21%	-4.64%	-3.68%	15.94%	15.84%	8.92%	9.66%
GFC	13.07%	9.54%	7.35%	7.12%	26.26%	21.13%	17.59%	21.46%
HEIA	15.15%	22.16%	15.42%	10.90%	27.12%	34.89%	17.71%	11.93%
NOS	5.64%	1.41%	0.82%	-18.34%	4.77%	-9.85%	0.31%	-13.91%
REP	0.55%	-1.36%	-1.69%	-5.96%	-27.59%	-33.64%	-26.49%	-38.55%
SAP	18.34%	18.45%	9.52%	1.71%	39.23%	46.20%	24.08%	28.38%
UCB	13.48%	10.25%	13.25%	12.60%	26.80%	25.48%	16.86%	20.14%
UNA	14.63%	29.07%	29.14%	40.48%	6.00%	27.76%	30.02%	34.33%
WIE	6.59%	-3.69%	0.05%	9.67%	-0.04%	-14.74%	-5.54%	7.36%
<b>Difference Ratio</b>		<b>197.0</b>	<b>195.8</b>	<b>4102.9</b>		<b>285.1</b>	<b>2278.5</b>	<b>415.9</b>

Composition of the minimum variance and tangent portfolios for the European market under the short-sell allowed case. Showing also, the computed Difference Ratio.

Figure 11-Difference between [E] and [R] on America



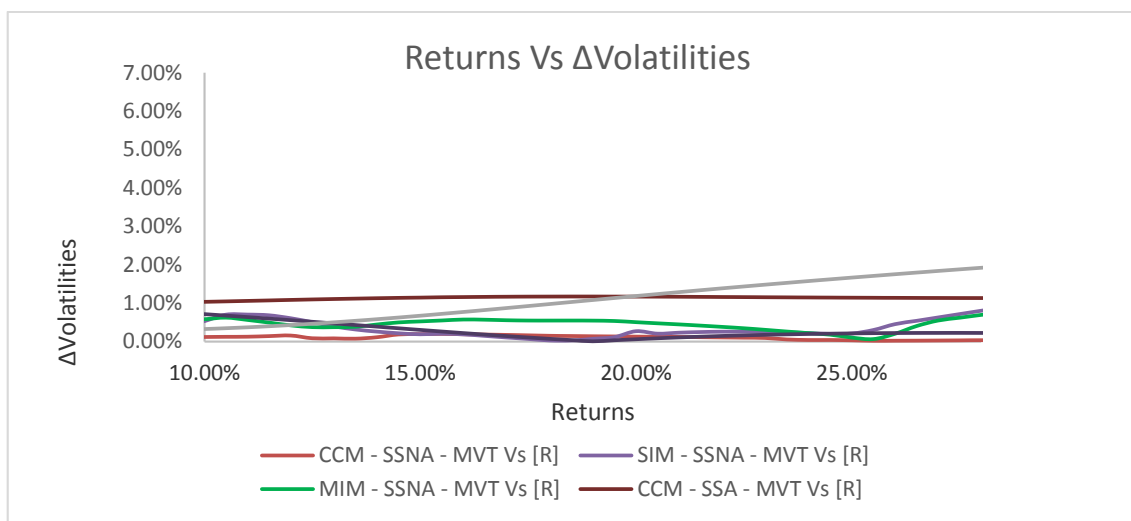
Difference in the level of risk (volatility) between expected and realized portfolios for the same level of returns in the American market.

Table 3-Composition of portfolios for America - SSNA

Portfolio Composition - American - SSNA								
Securities	Minimum Variance				Tangent Portfolio			
	MVT	CCM	SIM	MIM	MVT	CCM	SIM	MIM
AMZN	0.51%	0.00%	0.00%	0.00%	15.49%	16.34%	9.20%	14.69%
YUM	2.89%	0.00%	6.57%	0.16%	2.69%	3.68%	6.80%	0.00%
HSY	15.45%	11.65%	18.88%	15.24%	33.67%	33.31%	23.56%	22.73%
CHK	0.00%	0.00%	0.00%	0.53%	0.00%	0.00%	0.00%	0.00%
BAC	0.00%	0.00%	0.00%	6.72%	0.00%	0.00%	0.00%	5.43%
BLK	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
ABT	12.31%	12.90%	6.69%	6.76%	0.00%	0.00%	0.00%	0.00%
PFE	9.99%	13.13%	10.99%	12.72%	0.00%	0.00%	0.00%	0.00%
FDX	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
GE	0.00%	2.54%	0.00%	0.00%	0.00%	2.01%	0.00%	0.00%
AAPL	5.70%	0.00%	1.40%	0.77%	17.90%	18.19%	9.29%	12.82%
EA	0.00%	0.00%	1.07%	0.00%	8.22%	6.59%	6.32%	8.02%
DOW	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AVB	0.00%	1.91%	8.62%	2.59%	0.27%	2.82%	8.32%	3.91%
VZ	19.25%	24.71%	16.39%	21.78%	8.74%	12.34%	13.83%	7.76%
ED	33.91%	33.16%	29.36%	32.72%	13.03%	4.73%	22.68%	24.65%
Difference Ratio		108.5195	64.7379	82.8754		513.3331	537.3724	61.0495

Composition of the minimum variance and tangent portfolios for the American market under the short-sell restriction. Showing also, the computed Difference Ratio.

Figure 12-Difference between MVT and [R] on America



Difference in the level of risk (volatility) between in-sample MVT and realized portfolios for the same level of returns in the American market.

Table 4-Composition of portfolios for America – SSA

Securities	Minimum Variance				Tangent Portfolio			
	MVT	CCM	SIM	MIM	MVT	CCM	SIM	MIM
ABE	1.77%	-3.94%	1.54%	-1.58%	17.90%	17.18%	12.32%	19.33%
AC	4.90%	2.14%	8.48%	2.78%	8.93%	8.84%	11.29%	-5.57%
AGS	15.72%	14.70%	18.35%	15.91%	35.48%	33.61%	23.41%	25.85%
AD	0.17%	-6.42%	-0.56%	1.22%	-5.35%	-15.49%	-3.92%	-2.78%
APAM	-0.52%	-5.06%	-1.96%	7.83%	-7.30%	-11.04%	-4.14%	10.28%
BAYN	-9.78%	-0.51%	-4.33%	1.05%	-17.86%	-7.33%	-8.52%	0.46%
BNP	14.74%	15.83%	10.65%	9.87%	-14.24%	-10.95%	-3.33%	-15.92%
DTE	13.06%	16.27%	14.85%	15.57%	10.68%	7.27%	7.03%	3.74%
EDP	1.59%	1.91%	-1.31%	-3.17%	-2.34%	-0.52%	-1.44%	-2.33%
EXO	0.92%	6.51%	-1.13%	-1.71%	4.89%	8.22%	-2.08%	-6.75%
FCA	7.19%	0.13%	3.39%	2.30%	21.39%	19.70%	13.12%	17.09%
GFC	0.00%	-4.92%	1.91%	-3.30%	11.47%	9.26%	7.85%	11.99%
HEIA	-5.30%	-2.91%	-5.01%	-6.35%	1.10%	0.15%	-2.27%	-4.09%
NOS	2.28%	5.94%	10.12%	5.50%	8.01%	8.77%	12.10%	10.68%
REP	21.73%	26.30%	17.21%	21.85%	18.45%	18.57%	16.82%	10.72%
SAP	31.53%	34.03%	27.80%	32.24%	8.81%	13.78%	21.75%	27.31%
Difference Ratio		67.4	28.6	34.9		277.5	228.8	83.4

Composition of the minimum variance and tangent portfolios for the American market under the short-sell allowed case. Showing also, the computed Difference Ratio.

## 5. Conclusion

Based on the methodology applied and the results obtained in the previous section we were able to answer the important question previously set in this research. Which return model performs better and under which circumstances?

To answer the above question, we compared for both opportunity sets, three main results: the difference between the expected and realized efficient frontier and portfolios, the difference between the in-sample MVT (control) and realized efficient frontier and portfolios, and finally, the difference in portfolio composition between in-sample MVT and each model analyzed throughout the present research.

Based on the [E] vs [R] analysis for the SSNA scenario, evidence suggest that, for both opportunity sets, Europe and America, the CCM shows a closer relationship between the expected efficient frontier and the realized one. Followed in order by the MFM and SIM. For the SSA scenario, evidence suggest the same pattern as before, for both opportunity sets, Europe and America, the CCM shows a closer relationship between the expected efficient frontier and the realized one. Followed in order by the MFM and SIM.

Regarding the in-sample MVT vs [R] analysis for the SSNA scenario, evidence suggest that, despite the fact that the chart shows several intersection and not a clear and straight path, the period of the efficient frontier demarked between the minimum variance portfolio and the tangent portfolio for both opportunity sets, Europe and America, the CCM shows a closer relationship between the control efficient frontier and the realized one. Followed in order for Europe by MFM and SIM and for America by SIM and MFM. For the SSA scenario, evidence suggest a divided scenario, for both opportunity sets, Europe and America, before the 20% of return in the graph, the SIM shows a closer relationship between the in-sample MVT efficient frontier and the realized one. Followed in order for Europe by CCM and MFM and for America by MFM and CCM. After the 20% of return, America continuous to show the pattern of SIM first but changing to second CCM and third MFM. While Europe changes to MFM, SIM, and CCM. Being MFM and SIM particular close to each other.

For the composition analysis and judging by the results of the difference ratio for the SSNA scenario, evidence suggests that, for both opportunity sets, Europe and America, the CCM shows a closer relationship between the control portfolio's composition and the

model's one. Followed in order for Europe by SIM and MFM and for America by MFM and SIM. For the SSA scenario, evidence suggests that, for both Europe, the CCM shows a closer relationship between the control portfolio's composition and the model's one for both the minimum variance and tangent portfolio. Followed in order for the minimum variance portfolio by SIM and MFM and for the tangent portfolio by MFM and SIM. For America, the minimum variance portfolio shows a priority of SIM, MFM and CCM while the tangent portfolio shows a priority of CCM, SIM and MFM.

Finally, based on all the evidence stated above, we can conclude that, if we only consider the model risk, for investors seeking a short-sell not allowed scenario, in both the Europe and America opportunity sets, the chosen model should be the simplest one, the CCM. Since it shows closest results to the in-sample MVT (Control) on risk-return basis and also in composition of portfolios. However, to investors seeking a short-sell allowed scenario, in both the Europe and America opportunity sets, the chosen model should be the SIM. Since it shows closest results to the in-sample MVT (Control) on risk-return basis even though CCM shows a closest result in composition of portfolios.

Future research studies could focus on prove that the pattern evidenced in this research, CCM for SSNA and SIM for SSA also is replicated under other opportunity sets, such as Japan, Canada and Singapore. Moreover, analyze a wider set of restrictions and the inclusion of different type of assets, such as bonds, in order to evidence if the pattern is replicated.



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## Appendix

*Table A. 1 European Investment Opportunity Set*

SuperSector Stoxx All Europe 800	Country	Name	Symbol
Industrial Goods & Services	ES	Abertis Infraestructuras S.A.	ABE.MC
Travel & Leisure	FR	Accor SA	AC.PA
Insurance	BE	Ageas SA/NV	AGS.BR
Retail	NL	Ahold Delhaize	AD.AS
Basic Resources	LU	Aperam	APAM.AS
Chemicals	DE	Bayer Aktiengesellschaft	BAYN.DE
Banks	FR	BNP Paribas	BNP.PA
Telecommunication	DE	Deutsche Telekom	DTE.F
Utility	PT	EDP	EDP.LS
Financial Services	IT	Exor N.V.	EXO.MI
Automobile & Parts	IT	Fiat Chrysler	FCA.MI
Real State	FR	Gecina SA	GFC.PA
Food & Beverage	NL	Heineken N.V.	HEIA.AS
Media	PT	NOS SGPS	NOS.LS
Oil & Gas	ES	Repsol S.A.	REP.MC
Technology	DE	SAP SE	SAP.F
Health Care	BE	UCB	UCB.BR
Personal & Household Goods	NL	Unilever N.V.	UNA.AS
Construction & Materials	AT	Wienerberger AG	WIE.VI

*Table A. 2 American Investment Opportunity Set*

SuperSector GICS SP 500	Country	Name	Symbol
Consumer Discretionary	US	Amazon.com, Inc.	AMZN
Consumer Discretionary	US	Yum! Brands, Inc.	YUM
Consumer Staples	US	The Hershey Company	HSY
Energy	US	Chesapeake Energy Corporation	CHK
Financial	US	Bank of America Corporation	BAC
Financial	US	BlackRock, Inc.	BLK
Health Care	US	Abbott Laboratories	ABT
Health Care	US	Pfizer Inc.	PFE
Industrials	US	FedEx Corporation	FDX
Industrials	US	General Electric Company	GE
Information Technologies	US	Apple Inc.	AAPL
Information Technologies	US	Electronic Arts Inc.	EA
Materials	US	The Dow Chemical Company	DOW
Real State	US	AvalonBay Communities, Inc.	AVB
Telecommunication	US	Verizon Communications Inc.	VZ
Utilities	US	Consolidated Edison, Inc.	ED

Table A. 3 Annualized expected returns of the European opportunity set for MVT in %

$$\bar{R}^{MVT} = \begin{vmatrix} 7.0 & 11.7 & 15.7 & 14.0 & 19.0 & 14.1 & 12.6 & 33.7 & 8.5 & 23.6 & 27.2 & 15.9 & 15.1 & 12.0 & 5.9 & 17.2 & 16.4 & 13.4 & 11.5 \end{vmatrix}$$

Table A. 4 Correlation Matrix of the European opportunity set for MVT

$$V^{MVT} = \begin{vmatrix} 0.065 & 0.039 & 0.051 & 0.019 & 0.055 & 0.034 & 0.055 & 0.025 & 0.031 & 0.048 & 0.052 & 0.028 & 0.020 & 0.033 & 0.046 & 0.023 & 0.022 & 0.021 & 0.026 \\ 0.039 & 0.103 & 0.062 & 0.023 & 0.074 & 0.047 & 0.074 & 0.032 & 0.034 & 0.068 & 0.078 & 0.043 & 0.029 & 0.040 & 0.051 & 0.033 & 0.028 & 0.027 & 0.039 \\ 0.051 & 0.062 & 0.145 & 0.022 & 0.080 & 0.045 & 0.098 & 0.036 & 0.041 & 0.072 & 0.085 & 0.043 & 0.023 & 0.050 & 0.066 & 0.033 & 0.033 & 0.025 & 0.038 \\ 0.019 & 0.023 & 0.022 & 0.042 & 0.030 & 0.024 & 0.028 & 0.021 & 0.018 & 0.026 & 0.029 & 0.017 & 0.017 & 0.017 & 0.023 & 0.016 & 0.016 & 0.020 & 0.016 \\ 0.055 & 0.074 & 0.080 & 0.030 & 0.212 & 0.059 & 0.098 & 0.042 & 0.044 & 0.085 & 0.098 & 0.050 & 0.025 & 0.049 & 0.076 & 0.037 & 0.030 & 0.026 & 0.057 \\ 0.034 & 0.047 & 0.045 & 0.024 & 0.059 & 0.072 & 0.059 & 0.033 & 0.029 & 0.053 & 0.058 & 0.032 & 0.027 & 0.032 & 0.045 & 0.032 & 0.030 & 0.030 & 0.031 \\ 0.055 & 0.074 & 0.098 & 0.028 & 0.098 & 0.059 & 0.155 & 0.046 & 0.049 & 0.089 & 0.099 & 0.050 & 0.028 & 0.052 & 0.080 & 0.038 & 0.031 & 0.032 & 0.051 \\ 0.025 & 0.032 & 0.036 & 0.021 & 0.042 & 0.033 & 0.046 & 0.243 & 0.027 & 0.039 & 0.039 & 0.024 & 0.018 & 0.025 & 0.036 & 0.025 & 0.017 & 0.021 & 0.023 \\ 0.031 & 0.034 & 0.041 & 0.018 & 0.044 & 0.029 & 0.049 & 0.027 & 0.064 & 0.039 & 0.042 & 0.025 & 0.017 & 0.034 & 0.040 & 0.021 & 0.019 & 0.020 & 0.022 \\ 0.048 & 0.068 & 0.072 & 0.026 & 0.085 & 0.053 & 0.089 & 0.039 & 0.039 & 0.121 & 0.115 & 0.044 & 0.028 & 0.044 & 0.062 & 0.037 & 0.031 & 0.031 & 0.043 \\ 0.052 & 0.078 & 0.085 & 0.029 & 0.098 & 0.058 & 0.099 & 0.039 & 0.042 & 0.115 & 0.193 & 0.047 & 0.030 & 0.049 & 0.071 & 0.039 & 0.034 & 0.032 & 0.049 \\ 0.028 & 0.043 & 0.043 & 0.017 & 0.050 & 0.032 & 0.050 & 0.024 & 0.025 & 0.044 & 0.047 & 0.062 & 0.021 & 0.028 & 0.034 & 0.022 & 0.019 & 0.020 & 0.028 \\ 0.020 & 0.029 & 0.023 & 0.017 & 0.025 & 0.027 & 0.028 & 0.018 & 0.017 & 0.028 & 0.030 & 0.021 & 0.044 & 0.018 & 0.022 & 0.020 & 0.017 & 0.026 & 0.016 \\ 0.033 & 0.040 & 0.050 & 0.017 & 0.049 & 0.032 & 0.052 & 0.025 & 0.034 & 0.044 & 0.049 & 0.028 & 0.018 & 0.089 & 0.040 & 0.022 & 0.021 & 0.019 & 0.031 \\ 0.046 & 0.051 & 0.066 & 0.023 & 0.076 & 0.045 & 0.080 & 0.036 & 0.040 & 0.062 & 0.071 & 0.034 & 0.022 & 0.040 & 0.107 & 0.030 & 0.024 & 0.027 & 0.035 \\ 0.023 & 0.033 & 0.033 & 0.016 & 0.037 & 0.032 & 0.038 & 0.025 & 0.021 & 0.037 & 0.039 & 0.022 & 0.020 & 0.022 & 0.030 & 0.048 & 0.019 & 0.021 & 0.020 \\ 0.022 & 0.028 & 0.033 & 0.016 & 0.030 & 0.030 & 0.031 & 0.017 & 0.019 & 0.031 & 0.034 & 0.019 & 0.017 & 0.021 & 0.024 & 0.019 & 0.061 & 0.019 & 0.017 \\ 0.021 & 0.027 & 0.025 & 0.020 & 0.026 & 0.030 & 0.032 & 0.021 & 0.020 & 0.031 & 0.032 & 0.020 & 0.026 & 0.019 & 0.027 & 0.021 & 0.019 & 0.038 & 0.015 \\ 0.026 & 0.039 & 0.038 & 0.016 & 0.057 & 0.031 & 0.051 & 0.023 & 0.022 & 0.043 & 0.049 & 0.028 & 0.016 & 0.031 & 0.035 & 0.020 & 0.017 & 0.015 & 0.132 \end{vmatrix}$$

Table A. 5 Annualized expected returns of the American opportunity set for MVT in %

$$\bar{R}^{MVT} = \begin{vmatrix} 30.7 & 18.9 & 19.9 & 0.8 & 13.0 & 13.8 & 10.3 & 14.2 & 15.6 & 16.7 & 24.8 & 28.3 & 18.8 & 17.1 & 14.4 & 12.8 \end{vmatrix}$$

Table A. 6 Correlation Matrix of the American opportunity set for MVT

$$V^{MVT} = \begin{pmatrix} 0.108 & 0.026 & 0.013 & 0.034 & 0.039 & 0.034 & 0.018 & 0.019 & 0.030 & 0.027 & 0.029 & 0.039 & 0.035 & 0.021 & 0.014 & 0.009 \\ 0.026 & 0.062 & 0.012 & 0.033 & 0.036 & 0.030 & 0.017 & 0.017 & 0.029 & 0.025 & 0.023 & 0.029 & 0.032 & 0.021 & 0.013 & 0.010 \\ 0.013 & 0.012 & 0.036 & 0.013 & 0.014 & 0.017 & 0.011 & 0.011 & 0.014 & 0.015 & 0.010 & 0.014 & 0.016 & 0.012 & 0.011 & 0.011 \\ 0.034 & 0.033 & 0.013 & 0.363 & 0.073 & 0.055 & 0.025 & 0.026 & 0.043 & 0.040 & 0.034 & 0.033 & 0.066 & 0.022 & 0.020 & 0.009 \\ 0.039 & 0.036 & 0.014 & 0.073 & 0.132 & 0.064 & 0.026 & 0.030 & 0.050 & 0.048 & 0.034 & 0.047 & 0.063 & 0.034 & 0.020 & 0.009 \\ 0.034 & 0.030 & 0.017 & 0.055 & 0.064 & 0.074 & 0.026 & 0.027 & 0.043 & 0.038 & 0.028 & 0.038 & 0.051 & 0.030 & 0.021 & 0.013 \\ 0.018 & 0.017 & 0.011 & 0.025 & 0.026 & 0.026 & 0.035 & 0.018 & 0.020 & 0.019 & 0.015 & 0.021 & 0.023 & 0.015 & 0.013 & 0.009 \\ 0.019 & 0.017 & 0.011 & 0.026 & 0.030 & 0.027 & 0.018 & 0.037 & 0.022 & 0.022 & 0.016 & 0.020 & 0.026 & 0.016 & 0.013 & 0.009 \\ 0.030 & 0.029 & 0.014 & 0.043 & 0.050 & 0.043 & 0.020 & 0.022 & 0.063 & 0.033 & 0.023 & 0.035 & 0.044 & 0.026 & 0.016 & 0.012 \\ 0.027 & 0.025 & 0.015 & 0.040 & 0.048 & 0.038 & 0.019 & 0.022 & 0.033 & 0.049 & 0.023 & 0.029 & 0.041 & 0.023 & 0.017 & 0.011 \\ 0.029 & 0.023 & 0.010 & 0.034 & 0.034 & 0.028 & 0.015 & 0.016 & 0.023 & 0.023 & 0.071 & 0.029 & 0.032 & 0.018 & 0.013 & 0.007 \\ 0.039 & 0.029 & 0.014 & 0.033 & 0.047 & 0.038 & 0.021 & 0.020 & 0.035 & 0.029 & 0.029 & 0.128 & 0.039 & 0.025 & 0.015 & 0.011 \\ 0.035 & 0.032 & 0.016 & 0.066 & 0.063 & 0.051 & 0.023 & 0.026 & 0.044 & 0.041 & 0.032 & 0.039 & 0.094 & 0.031 & 0.020 & 0.013 \\ 0.021 & 0.021 & 0.012 & 0.022 & 0.034 & 0.030 & 0.015 & 0.016 & 0.026 & 0.023 & 0.018 & 0.025 & 0.031 & 0.050 & 0.014 & 0.016 \\ 0.014 & 0.013 & 0.011 & 0.020 & 0.020 & 0.021 & 0.013 & 0.013 & 0.016 & 0.017 & 0.013 & 0.015 & 0.020 & 0.014 & 0.027 & 0.011 \\ 0.009 & 0.010 & 0.011 & 0.009 & 0.009 & 0.013 & 0.009 & 0.009 & 0.012 & 0.011 & 0.007 & 0.011 & 0.013 & 0.016 & 0.011 & 0.024 \end{pmatrix}$$

Table A. 7 Correlation Matrix of the American opportunity set for CCM

$$V^{CCM} = \begin{pmatrix} 0.108 & 0.029 & 0.022 & 0.071 & 0.043 & 0.032 & 0.022 & 0.022 & 0.029 & 0.026 & 0.031 & 0.042 & 0.036 & 0.026 & 0.019 & 0.018 \\ 0.029 & 0.062 & 0.017 & 0.053 & 0.032 & 0.024 & 0.017 & 0.017 & 0.022 & 0.020 & 0.024 & 0.032 & 0.027 & 0.020 & 0.015 & 0.014 \\ 0.022 & 0.017 & 0.036 & 0.041 & 0.025 & 0.019 & 0.013 & 0.013 & 0.017 & 0.015 & 0.018 & 0.024 & 0.021 & 0.015 & 0.011 & 0.011 \\ 0.071 & 0.053 & 0.041 & 0.363 & 0.078 & 0.059 & 0.040 & 0.041 & 0.054 & 0.048 & 0.057 & 0.077 & 0.066 & 0.048 & 0.035 & 0.033 \\ 0.043 & 0.032 & 0.025 & 0.078 & 0.132 & 0.035 & 0.024 & 0.025 & 0.032 & 0.029 & 0.035 & 0.046 & 0.040 & 0.029 & 0.021 & 0.020 \\ 0.032 & 0.024 & 0.019 & 0.059 & 0.035 & 0.074 & 0.018 & 0.019 & 0.024 & 0.022 & 0.026 & 0.035 & 0.030 & 0.022 & 0.016 & 0.015 \\ 0.022 & 0.017 & 0.013 & 0.040 & 0.024 & 0.018 & 0.035 & 0.013 & 0.017 & 0.015 & 0.018 & 0.024 & 0.021 & 0.015 & 0.011 & 0.010 \\ 0.022 & 0.017 & 0.013 & 0.041 & 0.025 & 0.019 & 0.013 & 0.037 & 0.017 & 0.015 & 0.018 & 0.024 & 0.021 & 0.015 & 0.011 & 0.009 \\ 0.029 & 0.022 & 0.017 & 0.054 & 0.032 & 0.024 & 0.017 & 0.017 & 0.063 & 0.020 & 0.024 & 0.032 & 0.027 & 0.020 & 0.015 & 0.014 \\ 0.026 & 0.020 & 0.015 & 0.048 & 0.029 & 0.022 & 0.015 & 0.015 & 0.020 & 0.049 & 0.021 & 0.028 & 0.024 & 0.018 & 0.013 & 0.012 \\ 0.031 & 0.024 & 0.018 & 0.057 & 0.035 & 0.026 & 0.018 & 0.018 & 0.024 & 0.021 & 0.071 & 0.034 & 0.029 & 0.021 & 0.016 & 0.015 \\ 0.042 & 0.032 & 0.024 & 0.077 & 0.046 & 0.035 & 0.024 & 0.024 & 0.032 & 0.028 & 0.034 & 0.128 & 0.039 & 0.029 & 0.021 & 0.020 \\ 0.036 & 0.027 & 0.021 & 0.066 & 0.040 & 0.030 & 0.021 & 0.021 & 0.027 & 0.024 & 0.029 & 0.039 & 0.094 & 0.025 & 0.018 & 0.017 \\ 0.026 & 0.020 & 0.015 & 0.048 & 0.029 & 0.022 & 0.015 & 0.015 & 0.020 & 0.018 & 0.021 & 0.029 & 0.025 & 0.050 & 0.013 & 0.012 \\ 0.019 & 0.015 & 0.011 & 0.035 & 0.021 & 0.016 & 0.011 & 0.011 & 0.015 & 0.013 & 0.016 & 0.021 & 0.018 & 0.013 & 0.027 & 0.009 \\ 0.018 & 0.014 & 0.011 & 0.033 & 0.020 & 0.015 & 0.010 & 0.009 & 0.014 & 0.012 & 0.015 & 0.020 & 0.017 & 0.012 & 0.009 & 0.024 \end{pmatrix}$$

Table A. 8 Correlation Matrix of the European opportunity set for CCM

$$V^{CCM}_1 = \begin{vmatrix} 0.065 & 0.032 & 0.038 & 0.020 & 0.046 & 0.027 & 0.039 & 0.049 & 0.025 & 0.035 & 0.044 & 0.025 & 0.021 & 0.030 & 0.033 & 0.022 & 0.025 & 0.019 & 0.036 \\ 0.032 & 0.103 & 0.048 & 0.026 & 0.058 & 0.034 & 0.050 & 0.062 & 0.032 & 0.044 & 0.055 & 0.031 & 0.026 & 0.038 & 0.041 & 0.027 & 0.031 & 0.025 & 0.046 \\ 0.038 & 0.048 & 0.145 & 0.031 & 0.069 & 0.040 & 0.059 & 0.074 & 0.038 & 0.052 & 0.066 & 0.037 & 0.031 & 0.045 & 0.049 & 0.033 & 0.037 & 0.029 & 0.054 \\ 0.020 & 0.026 & 0.031 & 0.042 & 0.037 & 0.022 & 0.032 & 0.040 & 0.020 & 0.028 & 0.035 & 0.020 & 0.017 & 0.024 & 0.026 & 0.018 & 0.020 & 0.016 & 0.029 \\ 0.046 & 0.058 & 0.069 & 0.037 & 0.212 & 0.048 & 0.071 & 0.089 & 0.046 & 0.063 & 0.079 & 0.045 & 0.038 & 0.054 & 0.059 & 0.040 & 0.045 & 0.035 & 0.066 \\ 0.027 & 0.034 & 0.040 & 0.022 & 0.048 & 0.072 & 0.041 & 0.052 & 0.027 & 0.037 & 0.046 & 0.026 & 0.022 & 0.031 & 0.034 & 0.023 & 0.026 & 0.020 & 0.038 \\ 0.039 & 0.050 & 0.059 & 0.032 & 0.071 & 0.041 & 0.155 & 0.076 & 0.039 & 0.054 & 0.068 & 0.039 & 0.032 & 0.046 & 0.051 & 0.034 & 0.038 & 0.030 & 0.056 \\ 0.049 & 0.062 & 0.074 & 0.040 & 0.089 & 0.052 & 0.076 & 0.243 & 0.049 & 0.067 & 0.085 & 0.048 & 0.040 & 0.058 & 0.063 & 0.042 & 0.048 & 0.038 & 0.070 \\ 0.025 & 0.032 & 0.038 & 0.020 & 0.046 & 0.027 & 0.039 & 0.049 & 0.064 & 0.035 & 0.044 & 0.025 & 0.021 & 0.030 & 0.033 & 0.022 & 0.024 & 0.019 & 0.036 \\ 0.035 & 0.044 & 0.052 & 0.028 & 0.063 & 0.037 & 0.054 & 0.067 & 0.035 & 0.121 & 0.060 & 0.034 & 0.029 & 0.041 & 0.045 & 0.030 & 0.034 & 0.027 & 0.050 \\ 0.044 & 0.055 & 0.066 & 0.035 & 0.079 & 0.046 & 0.068 & 0.085 & 0.044 & 0.060 & 0.193 & 0.043 & 0.036 & 0.052 & 0.056 & 0.038 & 0.042 & 0.034 & 0.063 \\ 0.025 & 0.031 & 0.037 & 0.020 & 0.045 & 0.026 & 0.039 & 0.048 & 0.025 & 0.034 & 0.043 & 0.062 & 0.020 & 0.029 & 0.032 & 0.021 & 0.024 & 0.019 & 0.036 \\ 0.021 & 0.026 & 0.031 & 0.017 & 0.038 & 0.022 & 0.032 & 0.040 & 0.021 & 0.029 & 0.036 & 0.020 & 0.044 & 0.025 & 0.027 & 0.018 & 0.020 & 0.016 & 0.030 \\ 0.030 & 0.038 & 0.045 & 0.024 & 0.054 & 0.031 & 0.046 & 0.058 & 0.030 & 0.041 & 0.052 & 0.029 & 0.025 & 0.089 & 0.038 & 0.026 & 0.029 & 0.023 & 0.043 \\ 0.033 & 0.041 & 0.049 & 0.026 & 0.059 & 0.034 & 0.051 & 0.063 & 0.033 & 0.045 & 0.056 & 0.032 & 0.027 & 0.038 & 0.107 & 0.028 & 0.032 & 0.025 & 0.047 \\ 0.022 & 0.027 & 0.033 & 0.018 & 0.040 & 0.023 & 0.034 & 0.042 & 0.022 & 0.030 & 0.038 & 0.021 & 0.018 & 0.026 & 0.028 & 0.048 & 0.021 & 0.017 & 0.031 \\ 0.025 & 0.031 & 0.037 & 0.020 & 0.045 & 0.026 & 0.038 & 0.048 & 0.024 & 0.034 & 0.042 & 0.024 & 0.020 & 0.029 & 0.032 & 0.021 & 0.061 & 0.019 & 0.035 \\ 0.019 & 0.025 & 0.029 & 0.016 & 0.035 & 0.020 & 0.030 & 0.038 & 0.019 & 0.027 & 0.034 & 0.019 & 0.016 & 0.023 & 0.025 & 0.017 & 0.019 & 0.038 & 0.028 \\ 0.036 & 0.046 & 0.054 & 0.029 & 0.066 & 0.038 & 0.056 & 0.070 & 0.036 & 0.050 & 0.063 & 0.036 & 0.030 & 0.043 & 0.047 & 0.031 & 0.035 & 0.028 & 0.132 \end{vmatrix}$$

Table A. 9 Annualized expected returns of the European opportunity set for SIM in %

$$\bar{R}^{SIM} = \begin{vmatrix} 6.6 & 10.9 & 13.7 & 13.4 & 21.4 & 14.3 & 10.3 & 34.7 & 8.4 & 22.9 & 28.4 & 15.8 & 13.2 & 12.9 & 5.3 & 16.4 & 13.9 & 12.5 & 9.1 \end{vmatrix}$$

Table A. 10 Annualized expected returns of the American opportunity set for SIM in %

$$\bar{R}^{SIM} = \begin{vmatrix} 30.6 & 18.4 & 19.7 & -1.8 & 12.9 & 14.6 & 10.8 & 13.7 & 16.5 & 16.1 & 26.0 & 28.9 & 18.5 & 17.8 & 16.3 & 13.7 \end{vmatrix}$$

Table A. 11 Correlation Matrix of the European opportunity set for SIM

$$V^{SIM} = \begin{vmatrix} 0.038 & 0.019 & 0.025 & 0.011 & 0.039 & 0.016 & 0.030 & 0.016 & 0.012 & 0.022 & 0.035 & 0.016 & 0.012 & 0.020 & 0.023 & 0.016 & 0.006 & 0.007 & 0.022 \\ 0.019 & 0.076 & 0.030 & 0.011 & 0.046 & 0.019 & 0.036 & 0.018 & 0.015 & 0.026 & 0.041 & 0.019 & 0.015 & 0.024 & 0.028 & 0.019 & 0.008 & 0.008 & 0.026 \\ 0.025 & 0.030 & 0.094 & 0.017 & 0.061 & 0.026 & 0.048 & 0.025 & 0.020 & 0.034 & 0.055 & 0.026 & 0.020 & 0.032 & 0.037 & 0.025 & 0.010 & 0.010 & 0.034 \\ 0.011 & 0.011 & 0.017 & 0.032 & 0.027 & 0.011 & 0.021 & 0.011 & 0.009 & 0.015 & 0.024 & 0.011 & 0.009 & 0.014 & 0.016 & 0.011 & 0.004 & 0.005 & 0.015 \\ 0.039 & 0.046 & 0.061 & 0.027 & 0.215 & 0.039 & 0.073 & 0.037 & 0.030 & 0.052 & 0.084 & 0.039 & 0.030 & 0.049 & 0.056 & 0.038 & 0.015 & 0.016 & 0.052 \\ 0.016 & 0.019 & 0.026 & 0.011 & 0.039 & 0.044 & 0.030 & 0.016 & 0.013 & 0.022 & 0.035 & 0.016 & 0.013 & 0.021 & 0.023 & 0.016 & 0.006 & 0.007 & 0.022 \\ 0.030 & 0.036 & 0.048 & 0.021 & 0.073 & 0.030 & 0.086 & 0.029 & 0.023 & 0.040 & 0.065 & 0.031 & 0.023 & 0.038 & 0.043 & 0.029 & 0.012 & 0.012 & 0.041 \\ 0.016 & 0.018 & 0.025 & 0.011 & 0.037 & 0.016 & 0.029 & 0.269 & 0.012 & 0.021 & 0.034 & 0.016 & 0.012 & 0.020 & 0.022 & 0.015 & 0.006 & 0.006 & 0.021 \\ 0.012 & 0.015 & 0.020 & 0.009 & 0.030 & 0.013 & 0.023 & 0.012 & 0.043 & 0.017 & 0.027 & 0.013 & 0.010 & 0.016 & 0.018 & 0.012 & 0.005 & 0.005 & 0.017 \\ 0.022 & 0.026 & 0.034 & 0.015 & 0.052 & 0.022 & 0.040 & 0.021 & 0.017 & 0.074 & 0.047 & 0.022 & 0.017 & 0.027 & 0.031 & 0.021 & 0.009 & 0.009 & 0.029 \\ 0.035 & 0.041 & 0.055 & 0.024 & 0.084 & 0.035 & 0.065 & 0.034 & 0.027 & 0.047 & 0.185 & 0.035 & 0.027 & 0.044 & 0.050 & 0.034 & 0.014 & 0.014 & 0.047 \\ 0.016 & 0.019 & 0.026 & 0.011 & 0.039 & 0.016 & 0.031 & 0.016 & 0.013 & 0.022 & 0.035 & 0.045 & 0.013 & 0.021 & 0.024 & 0.016 & 0.007 & 0.007 & 0.022 \\ 0.012 & 0.015 & 0.020 & 0.009 & 0.030 & 0.013 & 0.023 & 0.012 & 0.010 & 0.017 & 0.027 & 0.013 & 0.033 & 0.016 & 0.018 & 0.012 & 0.005 & 0.005 & 0.017 \\ 0.020 & 0.024 & 0.032 & 0.014 & 0.049 & 0.021 & 0.038 & 0.020 & 0.016 & 0.027 & 0.044 & 0.021 & 0.016 & 0.092 & 0.029 & 0.020 & 0.008 & 0.008 & 0.027 \\ 0.023 & 0.028 & 0.037 & 0.016 & 0.056 & 0.023 & 0.043 & 0.022 & 0.018 & 0.031 & 0.050 & 0.024 & 0.018 & 0.029 & 0.067 & 0.023 & 0.009 & 0.009 & 0.031 \\ 0.016 & 0.019 & 0.025 & 0.011 & 0.038 & 0.016 & 0.029 & 0.015 & 0.012 & 0.021 & 0.034 & 0.016 & 0.012 & 0.020 & 0.023 & 0.040 & 0.006 & 0.006 & 0.021 \\ 0.006 & 0.008 & 0.010 & 0.004 & 0.015 & 0.006 & 0.012 & 0.006 & 0.005 & 0.009 & 0.014 & 0.007 & 0.005 & 0.008 & 0.009 & 0.006 & 0.048 & 0.003 & 0.009 \\ 0.007 & 0.008 & 0.010 & 0.005 & 0.016 & 0.007 & 0.012 & 0.006 & 0.005 & 0.009 & 0.014 & 0.007 & 0.005 & 0.008 & 0.009 & 0.006 & 0.003 & 0.023 & 0.009 \\ 0.022 & 0.026 & 0.034 & 0.015 & 0.052 & 0.022 & 0.041 & 0.021 & 0.017 & 0.029 & 0.047 & 0.022 & 0.017 & 0.027 & 0.031 & 0.021 & 0.009 & 0.009 & 0.107 \end{vmatrix}$$

Table A. 12 Annualized expected returns of the European opportunity set for MFM in %

$$\bar{R}^{MFM} = \begin{vmatrix} 7.9 & 12.6 & 15.3 & 15.3 & 23.0 & 14.6 & 11.8 & 36.3 & 9.7 & 24.7 & 27.7 & 17.7 & 15.1 & 14.6 & 6.9 & 18.2 & 15.9 & 14.5 & 10.7 \end{vmatrix}$$

Table A. 13 Annualized expected returns of the American opportunity set for MFM in %

$$\bar{R}^{MFM} = \begin{vmatrix} 10.8 & 30.6 & 26.0 & 17.8 & 12.9 & 14.6 & -1.8 & 13.7 & 18.5 & 28.9 & 16.5 & 16.1 & 19.7 & 13.7 & 13.3 & 15.4 \end{vmatrix}$$



Table A. 14 Correlation Matrix of the American opportunity set for SIM

$$V^{SIM} = \begin{pmatrix} 0.082 & 0.013 & 0.003 & 0.028 & 0.029 & 0.027 & 0.016 & 0.014 & 0.024 & 0.024 & 0.017 & 0.016 & 0.029 & 0.012 & 0.008 & 0.001 \\ 0.013 & 0.041 & 0.002 & 0.019 & 0.020 & 0.018 & 0.011 & 0.009 & 0.016 & 0.016 & 0.012 & 0.011 & 0.020 & 0.008 & 0.005 & 0.001 \\ 0.003 & 0.002 & 0.031 & 0.005 & 0.005 & 0.004 & 0.003 & 0.002 & 0.004 & 0.004 & 0.003 & 0.003 & 0.005 & 0.002 & 0.001 & 0.000 \\ 0.028 & 0.019 & 0.005 & 0.302 & 0.041 & 0.038 & 0.022 & 0.020 & 0.034 & 0.035 & 0.025 & 0.023 & 0.042 & 0.017 & 0.012 & 0.001 \\ 0.029 & 0.020 & 0.005 & 0.041 & 0.124 & 0.039 & 0.023 & 0.020 & 0.035 & 0.035 & 0.025 & 0.024 & 0.042 & 0.017 & 0.012 & 0.001 \\ 0.027 & 0.018 & 0.004 & 0.038 & 0.039 & 0.059 & 0.021 & 0.019 & 0.032 & 0.033 & 0.024 & 0.022 & 0.039 & 0.016 & 0.011 & 0.001 \\ 0.016 & 0.011 & 0.003 & 0.022 & 0.023 & 0.021 & 0.032 & 0.011 & 0.019 & 0.019 & 0.014 & 0.013 & 0.023 & 0.009 & 0.006 & 0.001 \\ 0.014 & 0.009 & 0.002 & 0.020 & 0.020 & 0.019 & 0.011 & 0.027 & 0.017 & 0.017 & 0.012 & 0.012 & 0.021 & 0.008 & 0.006 & 0.001 \\ 0.024 & 0.016 & 0.004 & 0.034 & 0.035 & 0.032 & 0.019 & 0.017 & 0.047 & 0.029 & 0.021 & 0.020 & 0.035 & 0.014 & 0.010 & 0.001 \\ 0.024 & 0.016 & 0.004 & 0.035 & 0.035 & 0.033 & 0.019 & 0.017 & 0.029 & 0.056 & 0.021 & 0.020 & 0.035 & 0.014 & 0.010 & 0.001 \\ 0.017 & 0.012 & 0.003 & 0.025 & 0.025 & 0.024 & 0.014 & 0.012 & 0.021 & 0.021 & 0.062 & 0.014 & 0.026 & 0.010 & 0.007 & 0.001 \\ 0.016 & 0.011 & 0.003 & 0.023 & 0.024 & 0.022 & 0.013 & 0.012 & 0.020 & 0.020 & 0.014 & 0.114 & 0.024 & 0.010 & 0.007 & 0.001 \\ 0.029 & 0.020 & 0.005 & 0.042 & 0.042 & 0.039 & 0.023 & 0.021 & 0.035 & 0.035 & 0.026 & 0.024 & 0.076 & 0.017 & 0.012 & 0.001 \\ 0.012 & 0.008 & 0.002 & 0.017 & 0.017 & 0.016 & 0.009 & 0.008 & 0.014 & 0.014 & 0.010 & 0.010 & 0.017 & 0.039 & 0.005 & 0.000 \\ 0.008 & 0.005 & 0.001 & 0.012 & 0.012 & 0.011 & 0.006 & 0.006 & 0.010 & 0.010 & 0.007 & 0.007 & 0.012 & 0.005 & 0.028 & 0.000 \\ 0.001 & 0.001 & 0.000 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.000 & 0.000 & 0.023 \end{pmatrix}$$

Table A. 15 Correlation Matrix of the American opportunity set for MFM

$$V^{MFM} = \begin{pmatrix} 0.032 & 0.020 & 0.016 & 0.012 & 0.014 & 0.018 & 0.013 & 0.003 & 0.022 & 0.016 & 0.018 & 0.019 & 0.007 & 0.012 & 0.008 & 0.014 \\ 0.020 & 0.082 & 0.030 & 0.011 & 0.010 & 0.015 & -0.017 & 0.000 & 0.028 & 0.006 & 0.018 & 0.026 & 0.012 & 0.016 & 0.016 & 0.021 \\ 0.016 & 0.030 & 0.062 & 0.011 & 0.013 & 0.017 & 0.002 & 0.002 & 0.025 & 0.012 & 0.019 & 0.022 & 0.009 & 0.014 & 0.011 & 0.017 \\ 0.012 & 0.011 & 0.011 & 0.039 & 0.005 & 0.017 & 0.020 & 0.007 & 0.020 & 0.027 & 0.019 & 0.016 & 0.010 & 0.011 & 0.005 & 0.015 \\ 0.014 & 0.010 & 0.013 & 0.005 & 0.124 & 0.036 & 0.057 & -0.006 & 0.033 & 0.009 & 0.027 & 0.026 & -0.008 & 0.013 & 0.003 & 0.005 \\ 0.018 & 0.015 & 0.017 & 0.017 & 0.036 & 0.059 & 0.048 & 0.003 & 0.035 & 0.028 & 0.031 & 0.028 & 0.005 & 0.016 & 0.006 & 0.016 \\ 0.013 & -0.017 & 0.002 & 0.020 & 0.057 & 0.048 & 0.302 & 0.005 & 0.037 & 0.044 & 0.039 & 0.025 & -0.003 & 0.014 & -0.004 & 0.008 \\ 0.003 & 0.000 & 0.002 & 0.007 & -0.006 & 0.003 & 0.005 & 0.023 & 0.004 & 0.014 & 0.005 & 0.003 & 0.006 & 0.003 & 0.001 & 0.006 \\ 0.022 & 0.028 & 0.025 & 0.020 & 0.033 & 0.035 & 0.037 & 0.004 & 0.076 & 0.030 & 0.034 & 0.033 & 0.010 & 0.020 & 0.011 & 0.022 \\ 0.016 & 0.006 & 0.012 & 0.027 & 0.009 & 0.028 & 0.044 & 0.014 & 0.030 & 0.114 & 0.031 & 0.022 & 0.015 & 0.015 & 0.004 & 0.021 \\ 0.018 & 0.018 & 0.019 & 0.019 & 0.027 & 0.031 & 0.039 & 0.005 & 0.034 & 0.031 & 0.056 & 0.027 & 0.009 & 0.017 & 0.008 & 0.018 \\ 0.019 & 0.026 & 0.022 & 0.016 & 0.026 & 0.028 & 0.025 & 0.003 & 0.033 & 0.022 & 0.027 & 0.047 & 0.008 & 0.017 & 0.010 & 0.018 \\ 0.007 & 0.012 & 0.009 & 0.010 & -0.008 & 0.005 & -0.003 & 0.006 & 0.010 & 0.015 & 0.009 & 0.008 & 0.031 & 0.006 & 0.005 & 0.011 \\ 0.012 & 0.016 & 0.014 & 0.011 & 0.013 & 0.016 & 0.014 & 0.003 & 0.020 & 0.015 & 0.017 & 0.017 & 0.006 & 0.027 & 0.006 & 0.012 \\ 0.008 & 0.016 & 0.011 & 0.005 & 0.003 & 0.006 & -0.004 & 0.001 & 0.011 & 0.004 & 0.008 & 0.010 & 0.005 & 0.006 & 0.028 & 0.008 \\ 0.014 & 0.021 & 0.017 & 0.015 & 0.005 & 0.016 & 0.008 & 0.006 & 0.022 & 0.021 & 0.018 & 0.018 & 0.011 & 0.012 & 0.008 & 0.041 \end{pmatrix}$$

Table A. 16 Correlation Matrix of the European opportunity set for MFM

0.038	0.020	0.022	0.015	0.039	0.020	0.025	0.013	0.018	0.021	0.032	0.018	0.016	0.033	0.023	0.021	0.013	0.012	0.020
0.020	0.077	0.024	0.014	0.041	0.020	0.027	0.014	0.017	0.022	0.034	0.018	0.015	0.030	0.024	0.019	0.011	0.010	0.021
0.022	0.024	0.095	0.008	0.058	0.020	0.036	0.004	0.019	0.025	0.042	0.018	0.010	0.026	0.023	0.017	0.006	0.002	0.037
0.015	0.014	0.008	0.033	0.018	0.017	0.017	0.026	0.012	0.016	0.022	0.015	0.017	0.025	0.021	0.018	0.013	0.015	0.003
0.020	0.020	0.020	0.017	0.216	0.035	0.056	0.008	0.034	0.043	0.068	0.031	0.021	0.052	0.041	0.032	0.016	0.009	0.057
0.020	0.020	0.020	0.017	0.035	0.045	0.027	0.023	0.016	0.022	0.033	0.018	0.017	0.029	0.026	0.020	0.012	0.013	0.015
0.025	0.027	0.036	0.017	0.056	0.027	0.088	0.036	0.019	0.032	0.053	0.024	0.019	0.024	0.036	0.022	0.007	0.010	0.028
0.013	0.014	0.004	0.026	0.023	0.023	0.036	0.272	0.005	0.023	0.038	0.021	0.025	0.003	0.038	0.017	0.005	0.021	-0.015
0.018	0.017	0.019	0.012	0.016	0.016	0.019	0.005	0.044	0.017	0.026	0.014	0.012	0.029	0.018	0.017	0.011	0.009	0.019
0.021	0.022	0.025	0.016	0.022	0.022	0.032	0.023	0.017	0.075	0.038	0.020	0.017	0.028	0.028	0.020	0.011	0.011	0.021
0.032	0.034	0.042	0.022	0.033	0.033	0.053	0.038	0.026	0.038	0.187	0.030	0.024	0.037	0.043	0.029	0.013	0.014	0.034
0.018	0.018	0.018	0.015	0.018	0.018	0.024	0.021	0.014	0.020	0.030	0.046	0.016	0.026	0.023	0.018	0.011	0.012	0.013
0.016	0.015	0.010	0.017	0.017	0.017	0.019	0.025	0.012	0.017	0.024	0.016	0.034	0.025	0.022	0.018	0.012	0.015	0.005
0.033	0.030	0.026	0.025	0.029	0.029	0.024	0.003	0.029	0.028	0.037	0.026	0.025	0.093	0.030	0.035	0.028	0.022	0.027
0.023	0.024	0.023	0.021	0.026	0.026	0.036	0.038	0.018	0.028	0.043	0.023	0.022	0.030	0.068	0.024	0.013	0.017	0.015
0.021	0.019	0.017	0.018	0.020	0.020	0.022	0.017	0.017	0.020	0.029	0.018	0.018	0.035	0.024	0.041	0.016	0.015	0.014
0.013	0.011	0.006	0.013	0.012	0.012	0.007	0.005	0.011	0.011	0.013	0.011	0.012	0.028	0.013	0.016	0.048	0.012	0.006
0.012	0.010	0.002	0.015	0.013	0.013	0.010	0.021	0.009	0.011	0.014	0.012	0.015	0.022	0.017	0.015	0.012	0.023	-0.002
0.020	0.021	0.037	0.003	0.015	0.015	0.028	-0.015	0.019	0.021	0.034	0.013	0.005	0.027	0.015	0.014	0.006	-0.002	0.109