

**MASTER OF SCIENCE IN
FINANCE**

**MASTER'S FINAL WORK
DISSERTATION**

THE BREXIT: A CASE OF FINANCIAL CONTAGION?

GUGLIELMO SAERRI

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SUPERVISOR:

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Abstract

Financial contagion is defined as an increase in the absolute value of cross-market correlations after a crisis (Forbes and Rigobon, 1999). The objective of this work is to analyze empirically if the Brexit shock produced a financial contagion into some of the most important Europe's financial markets. In order to do so, the log-returns of the stock market indexes of the United Kingdom, France and Germany (the two main representative countries of the Euro area and, at the same time, the main commercial partners of the United Kingdom) are used for a period between, approximately, the end of the Euro crisis in 2012 and February 2019. The idea is to use a sufficiently long period of time before and after the Brexit to ascertain if relevant changes in volatility and correlations were observed. The econometric methodology used was the following: after standard descriptive statistics and stationarity analysis, a trivariate VAR was estimated to account for the temporal dependence and dynamic interrelationships of the data. Then, a multivariate GARCH was estimated simultaneously with the identified VAR to model the conditional heteroskedasticity which is regularly observed in financial time series due to the volatility clustering phenomenon. Finally, the fitted conditional correlations and standard deviations were extracted from the VAR-MGARCH model. To test for the financial contagion possibly caused by the Brexit, a regression model for the conditional correlations and standard deviations was estimated by OLS with a dummy variable assuming unitary value on the occurrence of relevant Brexit-related information and a standard t-test was performed to evaluate the statistical significance of the coefficient. Moreover, to complement this statistical procedure, the averages of conditional correlations and standard deviations calculated sequentially along temporal sub-samples were compared. Both procedures resulted in sound statistical evidence that, in fact, the Brexit shock caused financial contagion on the analyzed financial stock markets.

JEL classification: C22, C32, F65

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I wrote it while I was working intensely full-time for a multinational company in Lisbon after coming back from an exchange program in Brazil. Because of the lack of time, a natural consequence of my high-pressure job, the only time that I could devote to writing my final work was at night after work: coming back home late and starting writing after dinner despite the desire of just resting was the main obstacle that my willpower had to overcome. Nevertheless, the desire of concluding my studies and, by consequent, returning to my country, from which I have been far for two years, was more determined and spurred me to find the energy to accomplish my duty.

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Table of contents

1. Introduction.....	1
2. Financial Contagion.....	5
2.1 Definition of financial contagion	5
2.2 Measurement of financial contagion.....	7
3. The Brexit	10
4. The data	14
5. The analysis	16
5.1 Analysis of the behavior of the series	16
5.2 Estimation of a VAR model.....	24
5.3 Estimation of a GARCH model.....	26
5.4 The conditional correlations and volatilities.....	31
5.5 The test	34
References	42
Annexes	45

List of Tables

TABLE I - <i>First ten UK's importing countries</i>	14
TABLE II - <i>ADF test for the UK's log returns series</i>	18
TABLE III - <i>ADF test for France's log returns series</i>	18
TABLE IV - <i>ADF test for Germany's log returns series</i>	18
TABLE V - <i>Lag order selection for VAR model</i>	25
TABLE VI - <i>Modulus of each root lies inside the unit circle</i>	25
TABLE VII - <i>Estimation of a VAR(1) model for the series of log returns of the United Kingdom, France and Germany</i>	30
TABLE VIII - <i>Information criteria coefficients for all the MGARCH models considered in the analysis</i>	30
TABLE IX - <i>Estimation of a MGARCH-DIAGVECH model for the series of log returns of the United Kingdom, France and Germany</i>	46
TABLE X - <i>Test for residual autocorrelations in the MGARCH estimation</i>	31
TABLE XI - <i>Estimation of OLS regression for the conditional correlation between the United Kingdom and France through the Brexit dummy</i>	34
TABLE XII - <i>Estimation of OLS regression for the conditional correlation between the United Kingdom and Germany through the Brexit dummy</i>	35
TABLE XIII - <i>Estimation of OLS regression for the conditional correlation between France and Germany through the Brexit dummy</i>	35
TABLE XIV - <i>Estimation of OLS regression for the conditional standard deviation for France through the Brexit dummy</i>	36
TABLE XV - <i>Estimation of OLS regression for the conditional standard deviation for Germany through the Brexit dummy</i>	36
TABLE XVI - <i>Estimation of OLS regression for the conditional standard deviation for the United Kingdom through the Brexit dummy</i>	37
TABLE XVII - <i>Average conditional standard deviations for time sub-samples for the United Kingdom, France and Germany</i>	38
TABLE XVIII - <i>Average conditional correlations for time sub-samples for the United Kingdom, France and Germany</i>	39

List of figures

FIGURE 1 - <i>Series of the FTSE100 log returns</i>	16
FIGURE 2 - <i>Series of the CAC40 log returns</i>	17
FIGURE 3 - <i>Series of the DAX30 log returns</i>	17
FIGURE 4 - <i>Comparison between theoretical and real quantiles for OLS residuals for the United Kingdom's series of log returns</i>	19
FIGURE 5 - <i>Comparison between theoretical and real quantiles for OLS residuals for France's series of log returns</i>	20
FIGURE 6 - <i>Comparison between theoretical and real quantiles for OLS residuals for Germany's series of log returns</i>	20
FIGURE 7 - <i>The null hypothesis of normality is rejected in the Jarque-Bera test for each series</i>	21
FIGURE 8 - <i>Comparison between t-student and real quantiles for OLS residuals for the United Kingdom's series of log returns</i>	22
FIGURE 9 - <i>Comparison between t-student and real quantiles for OLS residuals for France's series of log returns</i>	23
FIGURE 10 - <i>Comparison between t-student and real quantiles for OLS residuals for Germany's series of log returns</i>	23
FIGURE 11 - <i>Inverse roots of VAR polynomial characteristic equation inside the unit circle</i>	26
FIGURE 12 - <i>Conditional correlations through VAR-MGARCH model for the series of log returns of the United Kingdom, France and Germany</i>	32
FIGURE 13 - <i>Conditional standard deviations through VAR-MGARCH model for the series of log returns of the United Kingdom, France and Germany</i>	33

1. Introduction

The decision of the United Kingdom of leaving the European Union is one of the main events of our time: almost every day, since 2016, we have had news about how the process of separation from the European Union is developing. This happens because the Brexit has been and will be a pervasive shock capable of affecting our lives: it is about the exit of one of the most advanced economies from the richest and widest area of single market in the world. In summary, the Brexit is a historical event and, as such, deserves analysis.

The nature of the shock is twofold being both economic and political: on the one hand, flows of people, goods and financial instruments might be limited between the continent and the United Kingdom; on the other hand, the exit from the EU may trigger or strengthen centrifugal forces both in the United Kingdom and the European Union. One of the main immediate and current consequences of these scenarios is uncertainty around the future, a variable that in economy, and even more in finance, is crucial for economic agents to make decisions: the Brexit, despite not being a closed process, may have already had a dramatic impact on the financial markets of Europe.

Therefore, this work aims to understand whether the choice of the United Kingdom of leaving the European Union may have caused significant variations in the European financial stock markets. The focus is on the effects of the Brexit on the Eurozone stock markets given that, firstly, the Euro area is the most important monetary and economic sub-sample within the EU and, secondly, it is interesting to study the effects of a shock originated in a country that is not part of the monetary union. The basic objective of this dissertation is, in summary, to understand if the Brexit caused financial contagion in the Eurozone by using, as a sample, some of the most relevant Euro area's financial stock markets.

Answering the question as to whether the Brexit can be seen as a case of financial contagion for the Euro area is relevant for our understanding of international relationships. If the Brexit produces contagion, we can conclude that the level of interdependence of the European states is so deep that, independently of the political decision of staying out or in the Union, policy makers must understand that cooperation and integration are necessary and unavoidable to guarantee everyone's benefit. With regard to this point it is, furthermore, proper to make an observation: given that taking the Brexit as a shock that may have caused contagion means that the direction of causality flows clearly from the United Kingdom to the Eurozone, someone could argue that there would not be any necessity for the country from which the shock comes to seek more coordination with the countries affected. Even if this statement in principle can be true, especially in the case that the country

producing the shock is far bigger than the countries affected by the shock, nevertheless, evidence of contagion simply means an intrinsically strong interdependence which, reasonably, can let us suppose that the causality in the future, for some reason, may reverse its direction: therefore, it is reasonable to suppose that in the future the United Kingdom, because of its small size and interdependence with Europe, may be affected by shocks originated in the European Union.

As for the Brexit as a possible source of financial contagion, to my knowledge, no paper has been published yet and the choice of my topic is, thus, one of the main innovations introduced by my work. However, studies of financial contagion cases already exist. Here follow two works to which the idea of this work owes much and which inspired the methodology of this paper.

Firstly, Forbes and Rigobon (1999) defined the increase of cross-market correlation during the turmoil as financial contagion and analyzed the 1987 U.S. stock market crash, the 1994 Mexican peso collapse and the 1997 East Asian crises trying to find evidence of contagion. For each of the three crises, they divided, first, the respective sample period into several temporal intervals: some intervals were considered as “normal” periods, whereas others as periods of turmoil during which the hypothesis of financial contagion was possible. Then, they calculated the coefficients of correlations based on the selected model specification taking into account the heteroskedasticity phenomenon. In the end, they compared the adjusted correlation coefficients with those of the normal periods getting no evidence of contagion.

Secondly, Mitra, Iyer, and Joseph (2015), focused, instead, more on volatility than cross-market correlation to explain contagion between financial markets. In their work, they attempted to track the transmission of volatility across the ten main stock markets in the world during a period of twenty years. To capture the volatility spillover effect from one market to the other, they resorted to a bivariate GARCH model (more specifically a VAR-EGARCH specification). They found that spillover between the international stock markets is persistent at all times and is not random in 95% of cases corroborating the so-called *meteor shower hypothesis*: present volatility of a stock market is function of past volatility from other market(s).

The present work, following the example of Forbes and Rigobon (1999), focuses on the estimate of the cross-market correlations as well as tests the variation of average estimated correlations for sequential temporal sub-samples. Nevertheless, adopting the approach of Mitra, Yver and Joseph (2015), it also uses the volatility estimate as a key and primary variable to test directly contagion: consequently, according to the same approach used for sequential average correlations, sequential averages of volatility estimates are calculated. Moreover, following the example of Mitra, Yver and Joseph (2015) a VAR-MGARCH model has been used to allow for interdependence and

heteroskedasticity. Finally, given that the results provided by the calculation of the sequential averages of the correlation and volatility estimates depend on the size of the temporal sub-samples, a dummy variable, which assumes unitary value on the occurrence of Brexit events, has been introduced in order to test the estimates of correlations and volatilities in function of the Brexit shocks.

In summary, this work introduces an innovation with regard to the topic, in that the Brexit as a case of financial contagion has not been studied yet, and an innovation with regard to the methodology attempting to unify the main approaches to the financial contagion measurement into one together with the addition of a dummy variable analysis.

On the basis of the results just presented above, addressing the question as to whether the Brexit can be regarded as a cause of financial contagion turns into verifying if stock markets correlations and volatilities varied in a significant way due to some important events related to the process of separation from the European Union. In order to do so, first it is necessary to get these two variables from a reasonable sample in such a way that they can be used to test the hypothesis of contagion. That is the reason why I resorted to a VAR-MGARCH model: on the one side, the VAR part allows to model the interdependence between stock markets in a relatively simple way without the risk of specifying too complicated or too simple economic models; on the other side, the GARCH part allows to take into account the time-varying nature of volatility in stock markets. The result is, therefore, two robust estimates of the variables that are necessary to test the hypothesis of contagion. Testing the hypothesis of financial contagion due to the Brexit, at this stage, simply means to check if information related to the Brexit can statistically explain shifts in the correlations and volatilities as well as determine if such a change is, in fact, significant in statistical and practical terms.

The analysis presented in this work shows empirically that the Brexit behaved as a shock capable of producing contagion in the Eurozone. More specifically, utilizing a constant and a dummy with unitary value on the occurrence of crucial Brexit-related information as independent variables, the OLS regression for the conditional correlations and standard deviations drawn from the VAR-MGARCH model displays statistically significant coefficients for the dummy variable: therefore, the changes in conditional correlations and standard deviations during the sample period can be explained, causally, by the Brexit informational innovations. Moreover, the sequential sub-sample averages for the conditional correlations and standard deviations show, in absolute terms, abrupt and intense variations during the temporal intervals with more flow of significant Brexit-related events. In summary, volatilities and correlations between stock markets, the two key

variables to explain financial contagion, shift in a statistically significant way because of the Brexit events.

The dissertation is structured as follows. In the first section, the idea of financial contagion as provided by the literature is reviewed. More in detail, definitions and ways to measure financial contagion are listed. The second section summarizes the chronological path that lead to the Brexit: dates of remarkable importance are presented on which significant variations in correlations and volatilities may have occurred. In the third section, the database is described.

The fourth section explains the econometric analysis that was carried out in order to get evidence of financial contagion: first of all, I created the series of log returns from the returns of the stock market indexes of the United Kingdom, France and Germany and realized a graphical analysis. Then, to complete the first step of my analysis, I created a dummy variable linked to the Brexit that took unitary value on the occurrence of remarkable Brexit-related events. Secondly, I checked the stationarity of the series created and the most suitable error distribution model. Thirdly, I ran a Vector Autoregressive (VAR) model after carrying out an optimal number of lags selection in order to take into account the interdependence amongst all the stock markets involved in the analysis. In the following step, I realized a GARCH model upon the VAR to account for the heteroskedasticity exhibited by the financial time-series, the so-called *volatility clustering effect*. Several kinds of GARCH models were tested: the model chosen was the one which minimized the information criteria. Finally, I carried out a test on the MGARCH model to prevent residual ARCH effects from being still active. Using the VAR-MGARCH model above-mentioned, therefore, I drew the conditional correlations and standard deviations for the sample period. The final step of the analysis was to create an OLS model for the conditional correlations and volatilities in function of a constant and the dummy linked to the Brexit events. Using this model, the significance of the dummy variable was tested. Moreover, a sequential analysis of the value of the average of the VAR-MGARCH conditional correlations and standard deviations was carried out. The fifth and last section of my work, in the end, on the basis of the results from the above-mentioned tests, presents the conclusions of the analysis.

2. Financial Contagion

2.1 Definition of financial contagion

On the 23rd of June 2016, the United Kingdom expressed in a referendum its will of leaving the European Union. This event caused, immediately, sensible fluctuations in the main stock markets. In fact, this example showed how an important shock in a specific country with international impact may have noticeable effects on several financial markets independently of its size, structure or location.

Nevertheless, temporary fluctuations in the volatility and correlations of financial stock markets after specific news events do not necessarily mean that there was a case of financial contagion. The existing literature provides several definitions of *financial contagion* and, consequently to the best of my knowledge, there does not exist consensus upon what financial contagion is.

As a general idea, the contagion literature chose the criterion of correlation shift as the rule to set apart normal from contagious periods (Forbes and Rigobon, 1999). A relevant limitation of this criterion is that changes in correlation are quite natural during periods of innovation in regulation and financial technology, and increase in the number of market participants. Thus, correlation shifts, spillover effects and amplification processes can be at the same time either effects of financial contagion or processes endogenously generated inside the market.

Pericoli and Sbracia (2001) listed five possible definitions of financial contagion:

Definition 1 Contagion is a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country.

This definition is usually linked to empirical research upon the international consequences due to plunge of exchange rate. It accounts for the fact that currency exchange rate crises involve an extended set of countries amongst which some can avoid contagion despite being affected by speculative attacks. Generally speaking, this is a really broad definition that can include all of the following.

Definition 2 Contagion takes place when volatility spills over from the crisis-originating country to the financial markets of other countries.

Asset price volatility occurs during periods of financial turmoil: crises are identified with peaks in volatility and contagion is measured as volatility spillovers from one country to another. Hence,

contagion is a significant simultaneous rise in volatility in different markets not due to just simple interdependence. Since volatility is regarded as a good approximation of uncertainty in finance, contagion is basically related to the spread of uncertainty through the financial markets. Such definition corroborates the following.

Definition 3 Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets.

This definition stresses the quantitative dimension relating contagion to excessive co-movements in comparison with standard interdependence effects. The virtue of this definition is that it fits with what is commonly perceived as contagion in the context of the 2008 crisis following the Lehman Brothers bankruptcy. What is crucial is how to draw a distinction between ordinary and extra-ordinary co-movements.

Definition 4 Contagion occurs when the transmission channel is different after a shock in one market.

Shift contagion occurs if the transmission channel is intensified in response to a crisis in one country. For instance, it may be the case that some channels of crisis transmission are activated only during a shock. In general, it also means that contagion is not necessarily dependent on the idea of a tighter interdependence between countries, but it can just be explained by a different way of crisis transmission. This definition, as in the third definition, can be translated into excessive co-movements in prices and quantity or into jumps between multiple equilibria (see Definition 5), but it also enables the inclusion of economic variables behavior discontinuities produced by learning processes, sudden and excessive reception of information or imitative behavior by market agents.

Definition 5 Contagion occurs when co-movements cannot be explained by fundamentals.

The spread of a crisis arises in the presence of a coordination problem involving an arbitrary switch from one equilibrium to another one without fundamentals being able to explain the deviation. More precisely, this definition refers to models allowing multiple instantaneous equilibria in the presence of a coordination game: if the spread of a crisis reflects the arbitrary change from one equilibrium to another, fundamentals alone cannot explain the process; yet, they can explain why some countries are more vulnerable to crises than others. For example, in case of financial contagion via a liquidity crisis, countries whose relative level of international reserves in foreign currency is low are more likely to be hit.

Forbes and Rigobon (1999), similarly to the fourth definition above-mentioned, focused on the concept of shift contagion as defined as a change in the intensity of correlations between financial asset prices during a period of crisis.

The third and the fifth definition exposed by Pericoli and Sbracia (2001) explained contagion as an excess of co-movements. Masson (1999) and Claessens (2001) used the same approach, investigating the cause of the excess of co-movements in the irrationality of operators, herd behavior and financial panic.

Moreover, four stylized facts can be listed to corroborate the analysis of financial contagion through volatility and correlation shifts (Corsetti et al., 2010):

- (i) Financial crises are often related with an increase in covariances of returns between countries;
- (ii) Correlations of returns during crises often rise, but there are cases of crisis in which correlations decrease or remain at the same level: what matters is the variation in absolute terms;
- (iii) The occurrence of an unexpected downturn in stock market prices within a group of country;
- (iv) The evidence of the increase of volatility in concurrence with financial crises.

2.2 Measurement of financial contagion

In agreement with the definitions above-mentioned and literature review, to test and measure financial contagion after the Brexit shock, I decided to proceed as follows: estimate cross-market correlations and, then, verify whether or not a significant increase of correlation occurred after the shock. A significant increase in correlation is considered as statistical evidence favoring the hypothesis of financial contagion. The same idea, moreover, can be applied to another crucial variable, the volatility, namely to its approximation represented by the standard deviation.

King and Wadhani (1990) resorted to such approach to test contagion amongst the United States, the United Kingdom and Japan after the 1987 downturn in the American stock market: rise in correlations level after the shock was taken as evidence of financial contagion. For the same crisis Forbes and Rigobon (1999), instead, used an estimate of correlation accounting for heteroskedasticity effects and found no evidence of contagion.

Likewise, Lee and Kim (1993) applied the same methodology to a wider extent by analyzing twelve markets affected by the American 1987 downturn obtaining evidence of contagion, namely a 70% increase in the after-shock correlation level. Calvo and Reinhart (1996) analyzed the effects of 1994 Mexican crisis on emerging markets in terms of stock prices and sovereign bonds finding a relevant increase in cross-market correlations interpretable as financial contagion.

A second common approach is based on ARCH and GARCH models for the estimation of volatility-driven mechanisms of transmission from one country to another: through these models it is possible to include the problem of change in volatility through time and consider the time-varying dimension of standard deviations and correlations. For example, through this approach, Edwards (1998) studied the possibility of contagion within the debentures markets after the Mexican peso crisis and observed significant spillovers between Argentina and Mexico, but not between Mexico and Chile, whereas Hamao, Masulis and Ng (1990) noticed the presence of statistically significant volatility spillovers from New York to London and Tokyo, and from London to Tokyo after the 1987 American crisis.

Other researchers resorted to direct measurement to explain financial contagion, namely they examined how several variables directly affected the vulnerability of one country to crisis. Eichengreen, Rose and Wyploz (1996) used a probit model to estimate the probability of a crisis occurring for a set of industrialized countries from 1959 to 1993 finding that such probability is related to speculative attacks taking place in other countries at the same time. Forbes (2000), using a different strategy, estimated the consequences of the Russian and Asian crises upon approximately ten thousand companies all over the world, concluding that commercial ties are key variables to explain vulnerability to financial shocks.

Nevertheless, as many other papers do, such findings test the channels through which financial contagion may spread, not directly the existence of financial contagion itself. Moreover, specifying the whole set of explicative variables can be difficult: that is one of the reasons why researchers often resort to VAR models which avoid the need of model specification, but at the same time accounts for interdependence between financial markets.

All the models presented so far are suitable to adjust the change in variance along time. Yet, they still have a problem in that the estimates of correlations for different sub-samples depend on the choice of the sub-samples themselves. One way to mitigate this drawback is to ascertain the statistical significance of the extremes of the temporal sub-samples by using a dummy variable dependent on key information related to the event analyzed.

The key points of financial contagion, therefore, can be summarized as follows:

- a) Volatility and correlations are the key variables to define and measure financial contagion;
- b) A reasonable way of measuring financial contagion is to calculate how correlations and volatility change sequentially along time from one period to another. Furthermore, dummy variables can help testing statistical significance of the impact of information on the occurrence of events reasonably capable of causing contagion;
- c) Interdependence between pairs of countries can be studied by a VAR model, which, besides, has the noticeable advantage of not needing model specification;
- d) Heteroskedasticity must be taken into consideration in analyzing financial contagion: GARCH models suit this need.

For all these reasons, thus, the approach used in this analysis was to implement a VAR-MGARCH model and test the change in volatility and correlation through time by using auxiliary dummies and sequential change in the averages.

3. The Brexit

In order to better understand the temporal sample selection and the Brexit process itself, it is useful to briefly recapitulate the main steps that lead to the United Kingdom's exit from the European Union:

- **23th January 2013** – David Cameron, the British Prime Minister, states that he is in favor of a referendum on the UK's membership of the EU;
- **22nd May 2014** – The United Kingdom Independence Party (UKIP) led by Nigel Farage wins 26% of the votes in European elections and becomes the UK's most important representation in the European Parliament. The party, whose main policy is to leave the EU, also gets large gains in local elections becoming one of the most relevant British parties;
- **7th May 2015** – David Cameron wins the general British elections with a political program that includes the commitment to hold an in/out referendum;
- **22nd February 2016** – David Cameron announces that the referendum on the UK remaining in or withdrawing from the EU will be held on the 23rd of June of the same year;
- **23rd June 2016** – With 51.9% of the votes against and 48.1% for remaining in the EU, the referendum sees Leave campaigners win. David Cameron, because of that, resigns immediately as Prime Minister;
- **13th July 2016** – Theresa May becomes the new British prime minister;
- **27th July 2016** – Michel Barnier is nominated chief negotiator on behalf of the EU in the Brexit negotiation;
- **8th September 2016** – Guy Verhostadt is nominated by the European Parliament to lead Parliament-related negotiations regarding Brexit;
- **17th January 2017** – May gives a speech specifying the government's Brexit plans;
- **24th January 2017** – the Supreme Court states that the House of Commons must be consulted before the activation of the Article 50 which can kickstart the process of separation from the EU;
- **2nd February 2017** – The UK government publishes for the first time since the referendum its white paper on the Brexit, officially declaring what direction it will take during negotiations with the European Union;
- **29th March 2017** – The Article 50 is triggered, which starts the clock on the process of the UK leaving the EU;
- **8th April 2017** –The Prime Minister May calls a General Election for later in 2017;

- **8th June 2017** – Because of the unexpected call for general elections, Theresa May loses her majority in Parliament. Northern Ireland’s Democratic Unionist Party (DUP) makes a deal with the Conservatives and its votes allow Theresa May to stay in power;
- **9th June 2017**- Theresa May informs the Queen that she intends to form a government with the DUP;
- **19th June 2017**- The first round of UK-EU exit negotiations begins;
- **26th June 2017** – Formal negotiations on withdrawal begin between the UK and the EU;
- **2nd July 2017** – the UK announces abandonment of 1964 fishing convention;
- **22nd September 2017** – In an attempt to break the political deadlock caused by the inexistence of a shared vision over the Brexit negotiations, Theresa May talks about several key points during a speech in Florence, Italy. She declares that important issues for the Brexit to succeed are a transition period, fishing grounds, the border between Ireland and Northern Ireland and leaving the single European market;
- **23rd September 2017** – The American rating agency Moody’s downgrades UK’s rating from AA1 to AA2;
- **10th November 2017** – Theresa May proposes UK’s exit date on the 29th March 2019;
- **13th December 2017** – Rebel Tory MPs, because of contrasts with May’s government about the Brexit negotiations, side with the Opposition, forcing the government to guarantee a vote on the final Brexit deal;
- **15th December 2017** – The EU agrees to move on to the second phase of negotiations after an agreement is reached on the Brexit *divorce bill*, Irish border and EU citizens’ rights;
- **17th December 2017** – the European Union Referendum Act 2015, which decides upon the United Kingdom staying in the European Union, receives the royal assent and comes into force;
- **28th February 2018** – The European Commission publishes the draft of the Withdrawal Agreement between the European Union and the United Kingdom;
- **19 March 2018** – The UK and EU make decisive steps in negotiations. Agreements include dates for a transitional period after the Brexit day, the status of EU citizens in the UK before and after that time and fishing policy. Nevertheless, thorny issues remain, among which the main is about the Northern Ireland’s border;
- **26th June 2018** – The European Union (Withdrawal) Bill receives Royal Assent: the European Union Withdrawal Act becomes Act of Parliament;
- **9th July 2018** – Boris Johnson resigns as Foreign Secretary, stating that the Prime Minister was leading the UK into a *semi-Brexit* with the *status of a colony*. Then David Davis, the

Secretary of State for Exiting the European Union, also resigns and is replaced by Dominic Raab;

- **31st October 2018** – The EU’s chief negotiator says negotiations must be complete before the end of October to give the 27 EU countries time to sign off the deal. British MPs, moreover, agree to vote on the final deal in the UK Parliament before the 29th of March 2019;
- **14th November 2018** – Following months of tricky negotiations, the official withdrawal agreement is released. However, the deal faces fierce criticism from the opposition as well as from within May’s own party;
- **15th November 2018** – Dominic Raab resigns as Secretary of State for Exiting the European Union. Raab, who succeeds David Davis to that position in July, is replaced by Stephen Barclay;
- **25th November 2018** – All 27 Member States remaining in the European Union approve Theresa May’s Brexit plan, allowing the plan to be submitted to the House of Commons;
- **7th December 2018** – A journalistic inquiry reveals that Northern Ireland might remain in the EU permanently if the EU and UK should not find agreement upon trade relationships;
- **10th December 2018** – The Court of Justice of the European Union states that *The United Kingdom is free to revoke unilaterally the notification of its intention to withdraw from the EU*. This statement would mean that, under Article 50 of the EU Treaty, the UK could legally remain a member of the EU under its current, unchanged terms if it chose to do so;
- **10th December 2018** – The British Parliamentary vote on the Prime Minister May’s EU-approved Brexit deal is delayed, after initially being scheduled for the 11th of December. Theresa May acknowledges that the vote was postponed because it *would be rejected by a significant margin* if put to Parliament;
- **12th December 2018** – The Prime Minister faces– and survives– a vote about the leadership of the Conservative Party. Theresa May, furthermore, faces a vote of no-confidence, but wins the vote with 200 votes in favor and 117 against;
- **17th December 2018** – Theresa May announces crucial Brexit parliamentary vote for mid-January 2019. The vote, however, had originally been scheduled for December;
- **7th January 2019** – The vote on Theresa May’s Brexit proposal is finally publically announced for Tuesday, January 15th 2019;
- **9th January 2019** – A motion is passed in the House of Commons according to which the Prime Minister Theresa May will have to provide a ‘Plan B’ within 3 days, if her Brexit Plan is not supported;

- **15th January 2019** – Theresa May’s Brexit Withdrawal Agreement is resoundingly defeated in the House of Commons: with 432 contrary votes against just 202 in favor of the agreement, the vote is one of the largest government defeats in British political history. The Prime Minister, subsequently, agrees to allow discussion on any parliamentary vote of no-confidence against the government: one is immediately tabled by the Opposition Leader Jeremy Corbyn;
- **16th January 2019** – The Prime Minister survives a parliamentary no confidence vote with 325 votes in its favor against 306 in opposition;
- **29th January 2019** – British MPs support Conservative minister Sir Graham Brady’s Brexit amendment with 317 votes in favor against 301 in opposition: the non-binding amendment aims at *alternative arrangements* for the Irish border arrangement *backstop* proposed in the current EU-approved Brexit negotiations. Anyway, the European Union quickly rules out re-negotiating the current agreement.

MPs, furthermore, also vote to support a non-binding amendment proposed by the Conservative minister Dame Caroline Spelman and the Labor minister Jack Dromey that rejects the scenario a no-deal Brexit. The proposal is supported by 318 votes in favor versus 310 against.

Sources: www.bbc.com , www.wikitribune.com , www.aljazeera.com , www.parliament.uk and <https://about-britain.com> (for more details see the References Section).

4. The data

The data used in this analysis were the series of three European stock market indexes: the French CAC40, the German DAX30 and the British FTSE100. These indexes are regarded as the most comprehensive stock markets indexes for the respective countries and, consequently, the best *proxies* to verify the impact of Brexit on the corresponding stock markets. As for the selected countries, France and Germany were chosen as subjects of analysis together with the United Kingdom for three main reasons:

-Implementing a statistical model involves naturally choosing an appropriate sample: in order not to have too complicated time-series models with too many coefficients to estimate, I narrowed the number of stock markets potentially affected by the Brexit to two;

-France and Germany may be considered as the main European and Euro economies;

-France and Germany are some of the most important commercial partners of the United Kingdom. This fact is evident from **Table I** which lists the countries with highest weight in the total UK exports. Here it can be seen that France and Germany occupy the first places in the percentage of total UK exports. Therefore, it may be conjectured that these countries will be the most affected by the Brexit shock and, hence, are in the pole position in a study about financial contagion caused by Brexit.

TABLE I-*First ten UK's importing countries*

Countries	Percentage of total UK exports
United States	13,30%
Germany	9,70%
Netherlands	6,90%
France	5,90%
Ireland	5,70%
China	5,20%
Switzerland	4,00%
Belgium	2,90%
Italy	2,90%

Source: www.worldstopexports.com

I began this study by collecting the daily prices of CAC40, DAX30 and FTSE100 from *Yahoo Finance* for the period between the 2nd of January 2012 until the 15th of February 2019. The first date was chosen as an approximate stillness date after the sovereign debt crisis and the final date as the last information available at the moment of the draft of this work. For each

series, the log returns were calculated. Since the stock markets in France, Germany and the United Kingdom sometimes open and close irregularly due to national holidays, the data were homogenized deleting the days that did not match amongst the series of the log returns. The final dataset corresponded to three time-series of log returns from the 4th of January 2012 to the 15th of February 2019 for a total of 1770 observations for each series.

For the remaining sections, in order not to weigh the text too much with exhaustive algebraic explanations about the mathematical structure of each statistical test and econometric model used in the analysis, I preferred giving, as it was considered of benefit for the reader, the general intuition to understand the techniques implemented and follow the logical steps in a straightforward way.

The software used to manage the data and obtain the necessary output was EViews 10.

5. The analysis

5.1 Analysis of the behavior of the series

The plots of the FTSE100, CAC40 and DAX 30 log returns are illustrated in **Figures 1, 2 and 3**, respectively: the horizontal axis represents the years and months included in the sample, whereas the vertical axis represents the log-returns in decimals. The graphs of the three series show remarkable similarity suggesting intrinsic interdependence among them. All of them, generally speaking, show the so-called *volatility clustering* effect, namely several consecutive observations which can be categorized in a high volatility regime followed by many consecutive periods of lower volatility and vice-versa. More in detail, the UK returns seems to be more unstable with several observations deviating substantially out of the sub-period average after 2015. Germany's and France's returns, instead, exhibit a more regular pattern but with a noticeable increase in volatility during the 2015 and 2016 and, above all, extreme observations around the 24th of June 2016, the day after the Brexit referendum.

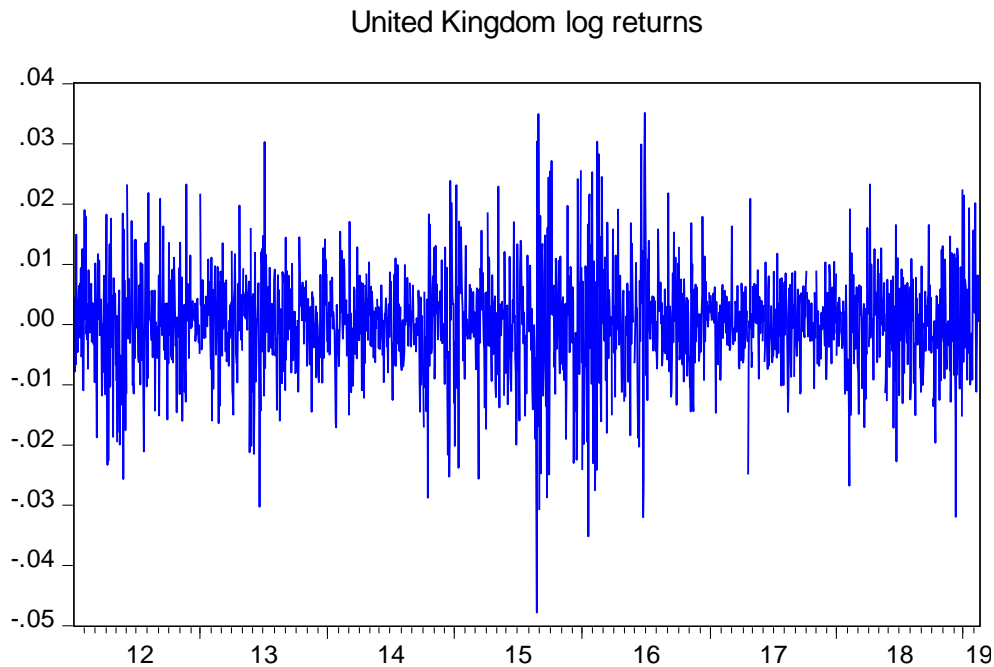


FIGURE 1 – *Series of the FTSE100 log returns*

Source: EViews 10 output

France log returns

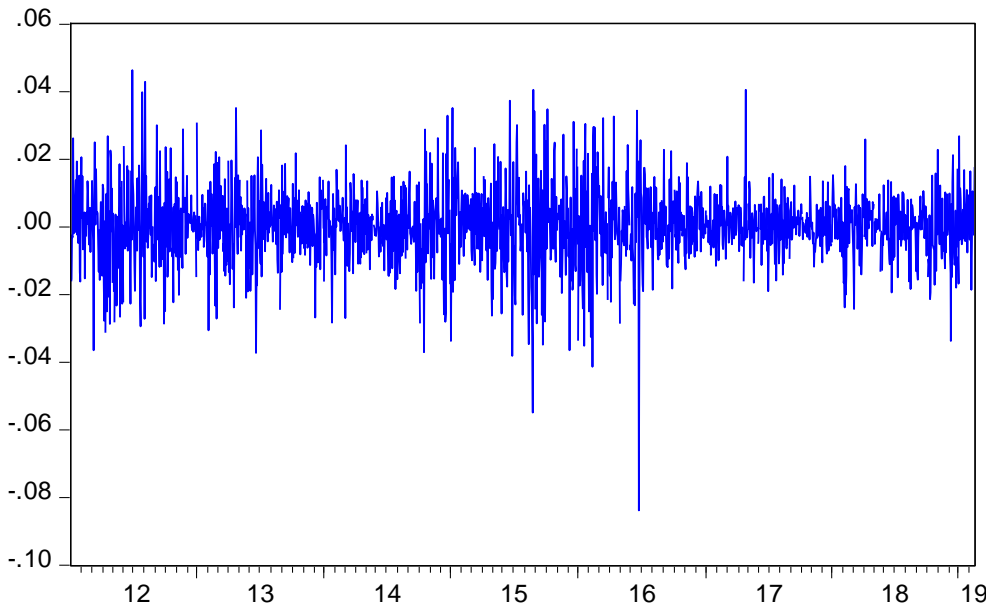


FIGURE 2 - Series of the CAC40 log returns

Source: EViews 10 output

Germany log returns

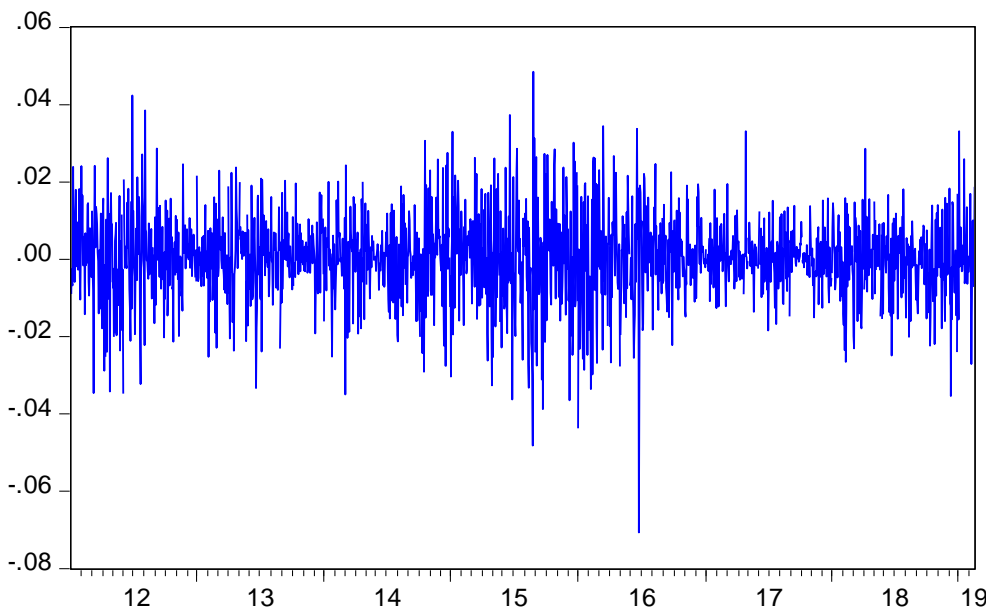


FIGURE 3 – Series of the DAX30 log returns

Source: Eviews 10 output

An important first step to carry out in time-series analysis is to check if the series are stationary. A series is (second order) stationary if it has the property that the mean, variance and autocorrelation function of its underlying Data Generating Process (DGP) do not change over time.

The stationarity was verified through the Augmented Dickey-Fuller (ADF) test: the null hypothesis of the ADF test is that the series has a unique unit root, i.e., has, exactly, one root equal to unity in the characteristic equation of the DGP of the series (with the other roots being higher than one, in modulus). The alternative hypothesis of this test is that the series has no unit root which, for the

analyzed series, corresponds to stationarity. The results of the application of the ADF test to each return series in EViews are shown in **Tables II-IV**: several lag lengths were tested getting the same result (rejection of null hypothesis) and a lag length equal to 24 was finally chosen to test the hypothesis of stationarity. Given that the null hypothesis was rejected for each series (p-value lower than any reasonable significance level) the statistical evidence suggests that all these series are stationary. Given stationarity, this dataset is suitable for the application of the VAR and GARCH econometric methodology.

Null Hypothesis: UK_LNRET has a unit root
 Exogenous: Constant
 Lag Length: 24 (Automatic - based on t-statistic, lagpval=0.1, maxlag=24)

TABLE II - ADF test for the UK's log returns series

	t-Statistic	Prob.*
<u>Augmented Dickey-Fuller test statistic</u>	-8.969741	0.0000
Test critical values: 1% level	-3.433895	
5% level	-2.862992	
10% level	-2.567590	

Source: EViews 10 output

Null Hypothesis: FR_LNRET has a unit root
 Exogenous: Constant
 Lag Length: 23 (Automatic - based on t-statistic, lagpval=0.1, maxlag=24)

TABLE III - ADF test for France's log returns series

	t-Statistic	Prob.*
<u>Augmented Dickey-Fuller test statistic</u>	-9.495890	0.0000
Test critical values: 1% level	-3.433892	
5% level	-2.862991	
10% level	-2.567590	

Source: Eview 10 output

Null Hypothesis: GER_LNRET has a unit root
 Exogenous: Constant
 Lag Length: 24 (Automatic - based on t-statistic, lagpval=0.1, maxlag=24)

TABLE IV - ADF test for Germany's log returns series

	t-Statistic	Prob.*
<u>Augmented Dickey-Fuller test statistic</u>	-8.375178	0.0000
Test critical values: 1% level	-3.433895	
5% level	-2.862992	
10% level	-2.567590	

Source: EViews 10 output

The second basic step, then, was to study the empirical distribution of the returns in order to understand which theoretical distribution makes the best fit to these series as this is an important input for the class of econometric models I am going to apply. With this purpose, I realized two different checks on the series of the mean-adjusted returns (returns minus the sample average). Both checks found evidence against the application of the normal distribution to the return series.

The first check was based on the empirical distribution of the mean-adjusted returns: the empirical quantiles of the mean-adjusted returns are plotted against the quantiles of a normal distribution, which means that if the observations come from a normal population, then the resulting chart should be a perfectly 45-degree line. The resulting charts, which are known as Q-Q plots, are illustrated in **Figures 4, 5** and **6** for the UK, France and Germany series, respectively.

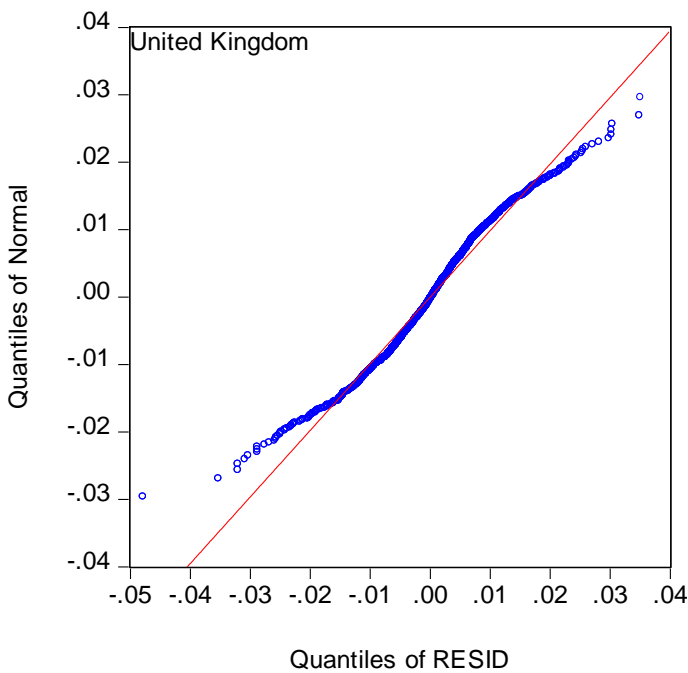


FIGURE 4 - *Q-Q plot of the UK's series of mean-adjusted log returns against the Normal distribution*

Source: EViews 10 output

The empirical quantiles of the UK's mean-adjusted returns, as can be seen in **Figure 4**, deviate significantly from the theoretical quantiles of the Normal exhibiting fatter tails suggesting that a different distribution allowing for thicker tails could be more suitable to develop the analysis.

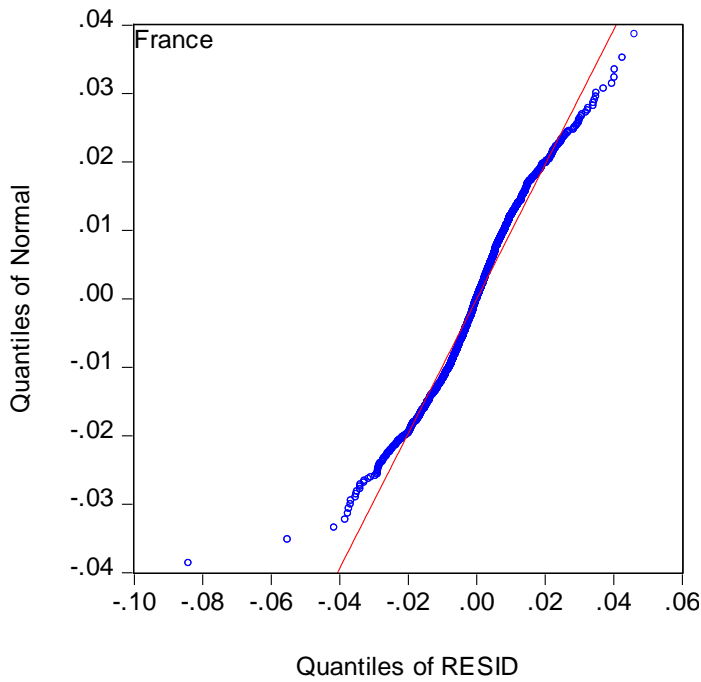


FIGURE 5 - *Q-Q plot of the France's series of mean-adjusted log returns against the Normal distribution*

Source: EViews 10 output

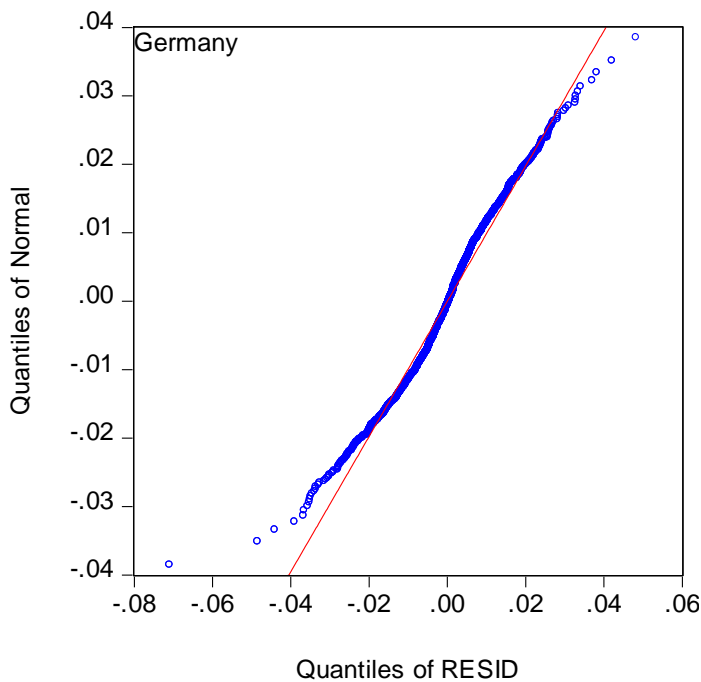


FIGURE 6 - *Q-Q plot of the Germany's series of mean-adjusted log returns against the Normal distribution*

Source: EViews 10 output

Equally for the case of the United Kingdom, the mean-adjusted return series of France and Germany (**Figure 5** and **Figure 6**), show fatter tails than the Normal distribution, which suggests that a different distribution allowing for this kind of pattern should be applied.

Using the same series, I also applied the Jarque-Bera test, whose null hypothesis is that the mean-adjusted return series is normally distributed.

The Jarque-Bera test verifies whether sample data fit the values of skewness and kurtosis of a Normal distribution which are equal, respectively, to 0 and 3. The value of the Jarque-Bera statistic must be compared with the critical value of a Chi-squared distribution with two degrees of freedom: the null hypothesis is rejected if the observed value of the test statistic is higher than the critical value.

The EViews outputs of this test are shown in **Figure 7**. Given that the p-values are always very low and, approximately, equal to 0, the null hypothesis in all cases is rejected and evidence is found against the Normal distribution for the mean-adjusted returns.

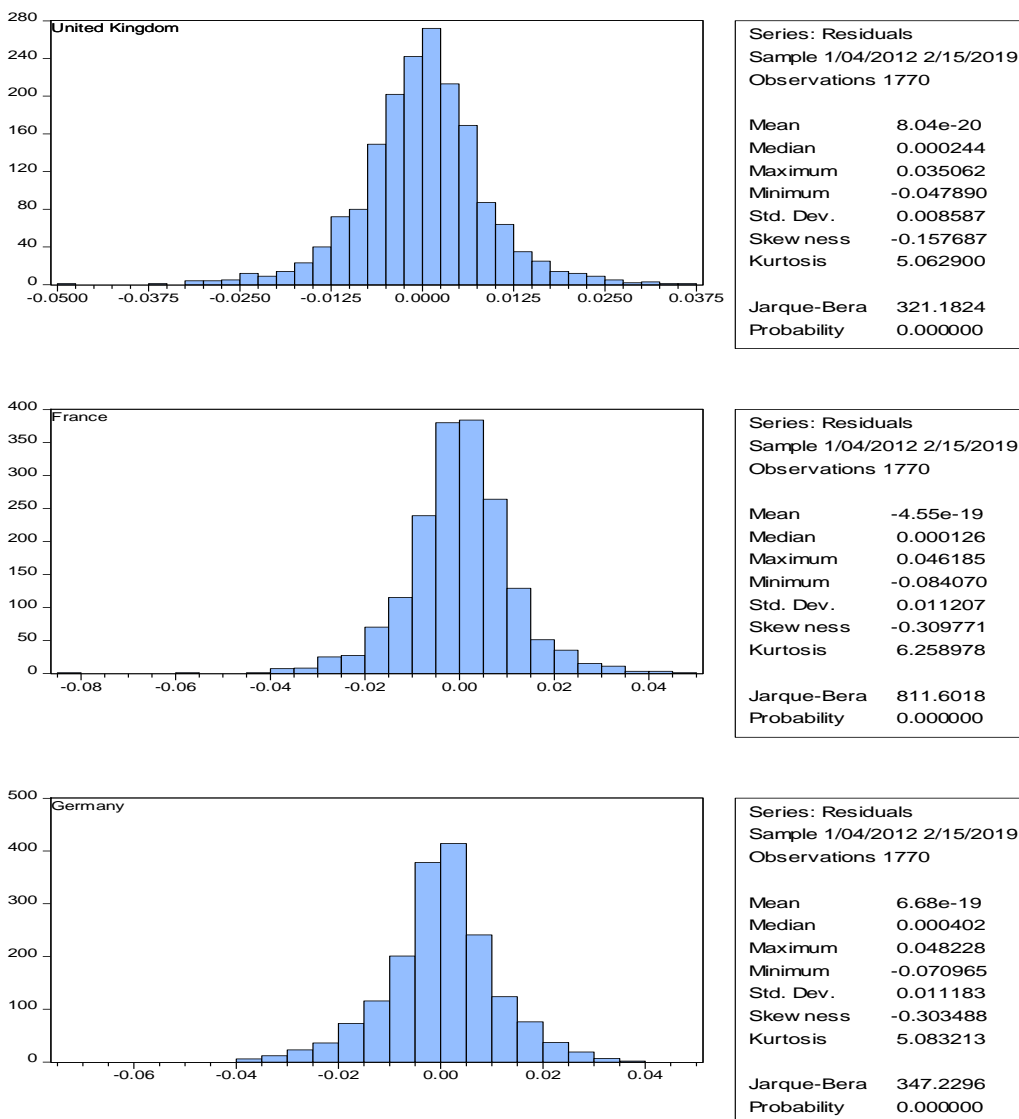


FIGURE 7 - *The null hypothesis of normality is rejected in the Jarque-Bera test for each series*

Source: EViews 10 output

Given that the hypothesis of the Normal distribution is rejected, it is recommended to choose another distribution in the model specification process whose fitting could perform better. Given

that both checks gave evidence of fat tails, a reasonable specification can be a Student's t -distribution. Q-Q plots of the mean-adjusted returns against the Student's t -distribution are shown in **Figures 8, 9** and **10**. Using the same approach as in the first test afore-mentioned, it can be noticed that the quantiles of the mean-adjusted returns against the theoretical quantiles of a Student's t -distribution display a better fit. In the case of the United Kingdom (**Figure 8**) the fitting is almost perfect with the exception of a slight deviation in the extreme quantiles.

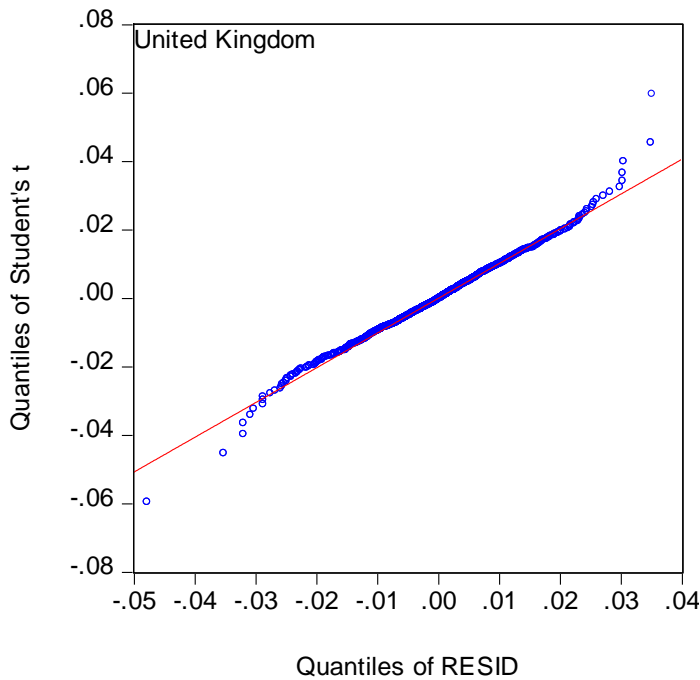


FIGURE 8 - *Q-Q plot of the UK's series of mean-adjusted log returns against the Student's t -distribution*

Source: EViews 10 output

In the case of France (**Figure 9**), the fitting improves too, but not as much as in the case of the United Kingdom and some deviations can be observed on the lower and upper quantiles.

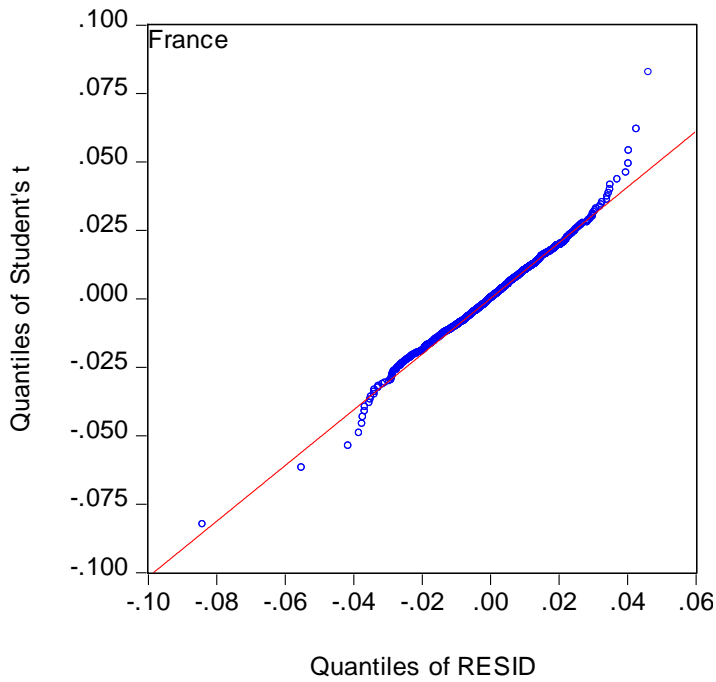


FIGURE 9 - *Q-Q plot of the France's series of mean-adjusted log returns against the Student's t-distribution*

Source: EViews 10 output

Finally, in the case of Germany (**Figure 10**), we also observe deviations, especially in the upper extreme, but the fitting improves slightly. Hence, in summary, I consider this as evidence that Student's t-distribution is more adequate for modelling the return series under analysis.

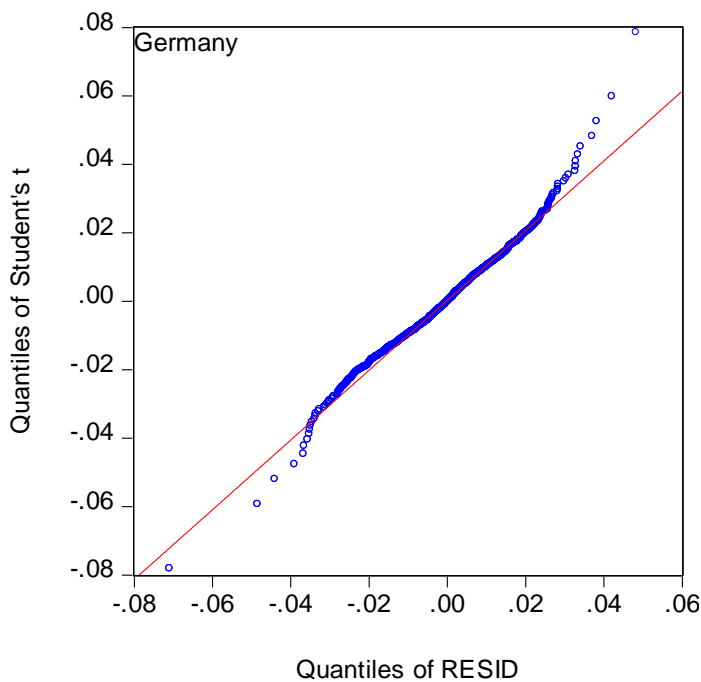


FIGURE 10 - *Q-Q plot of the Germany's series of mean-adjusted log returns against the Student's t-distribution*

Source: EViews 10 output

5.2 Estimation of a VAR model

As previously seen on the graphs of the log-returns (**Figures 1, 2 and 3**), it is possible to notice a common pattern amongst the three series due to the tight relationships between the financial markets of these countries. In order to allow for the existing interdependence without the need of specifying a detailed economic model, I applied the VAR methodology to the three series under analysis, the log-returns of FTSE100, CAC40 and DAX30.

A VAR model is a system of simultaneous equations which represents the multivariate generalization of an univariate AR model : a k -variable vector autoregression of order p , VAR(p), is a system of k linear equations, with each equation describing the dynamics of one variable as a linear function of the previous p lags of every variable in the system, including its own p lags.

The general equation of a VAR(p) over the sample period is, therefore,

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + e_t \quad \text{for } t = 1, \dots, T \quad (1)$$

where y_t is a k -vector (or a $k \times 1$ matrix) of endogenous variables, i.e., the i^{th} element at time t is $y_{i,t}$; c is a k -vector of constants (i.e. the intercepts); A_i is a time-invariant ($k \times k$)-matrix; y_{t-i} is the (lagged) observation at i periods back; and e_t is a k -vector (or a $k \times 1$ matrix) of unobservable i.i.d. zero-mean error terms, i.e., $\mathbf{E}(e_t) = 0$ (white noise).

In the context of our analysis, the estimation of a VAR model to these three returns series demands the choice of the order p , which represents the number of lags, and that the stability condition of VAR models is satisfied.

Table V shows the values of different information criteria according the VAR lag order. The selection of the lag order was driven by the minimization of the information criteria, as it is standard in the literature. Hence, I selected a VAR model with one lag as this is the lag that minimizes most of the information criteria as seen in **Table V**.

VAR Lag Order Selection Criteria
 Endogenous variables: FR_LNRET UK_LNRET GER_LNRET
 Exogenous variables: C
 Date: 03/02/19 Time: 11:49
 Sample: 1/04/2012 2/15/2019
 Included observations: 1762

TABLE V - Lag order selection for VAR model

Source: EViews 10 output

Lag	LogL	LR	FPE	AIC	SC	HQ
0	19525.51	NA	4.77e-14	-22.15949	-22.15017	-22.15605*
1	19538.89	26.70097	4.75e-14*	-22.16447*	-22.15022*	-22.15069
2	19540.75	3.689637	4.79e-14	-22.15635	-22.09111	-22.13224
3	19546.61	11.65373	4.81e-14	-22.15279	-22.05958	-22.11835
4	19557.51	21.65338*	4.80e-14	-22.15495	-22.03379	-22.11018
5	19565.40	15.63004	4.80e-14	-22.15369	-22.00456	-22.09858
6	19570.89	10.85111	4.82e-14	-22.14970	-21.97261	-22.08426
7	19572.51	3.214874	4.86e-14	-22.14133	-21.93628	-22.06556
8	19577.19	9.227260	4.88e-14	-22.13643	-21.90342	-22.05032

Thus, in summary, the equation (1) can be expressed explicitly for the particular case of this analysis as a system of $k = 3$ equations, $p = 1$ order of lag, and $T = 1769$ observations (i.e., the total number of observations minus the lag order), namely, a trivariate VAR (1) as follows:

$$\begin{cases} y_{UK,t} = c_1 + a_{1,1}y_{UK,t-1} + a_{1,2}y_{FR,t-1} + a_{1,3}y_{GER,t-1} + e_{1,t} \\ y_{FR,t} = c_2 + a_{2,1}y_{UK,t-1} + a_{2,2}y_{FR,t-1} + a_{2,3}y_{GER,t-1} + e_{2,t} \\ y_{GER,t} = c_3 + a_{3,1}y_{UK,t-1} + a_{3,2}y_{FR,t-1} + a_{3,3}y_{GER,t-1} + e_{3,t} \end{cases} \quad (2)$$

where, respectively, the left-hand side of each equations represents the observation at time t of the log-returns of the FTSE100, CAC40 and DAX30 analyzed in Section 5.1.

As for the second step, with a reasoning similar to the one that supported the check for stationarity in the time-series samples of log returns, it was necessary to verify if the roots of the VAR (1) characteristic polynomial equation were inside the unit circle: in case they were not, the model would be considered as unstable and not suitable to carry out a statistical inference analysis. The calculation of the roots (**Table VI**) proved the model to be stable and, therefore, suitable to continue the analysis, as shown here below graphically (**Figure 11**).

Roots of Characteristic Polynomial
 Endogenous variables: FR_LNRET
 UK_LNRET GER_LNRET
 Exogenous variables: C
 Lag specification: 1 1
 Date: 03/02/19 Time: 11:53

TABLE VI - Modulus of each root lies inside the unit circle

Source: EViews 10 output

Root	Modulus
-0.032044 - 0.050350i	0.059682
-0.032044 + 0.050350i	0.059682
0.052758	0.052758

No root lies outside the unit circle.
 VAR satisfies the stability condition.

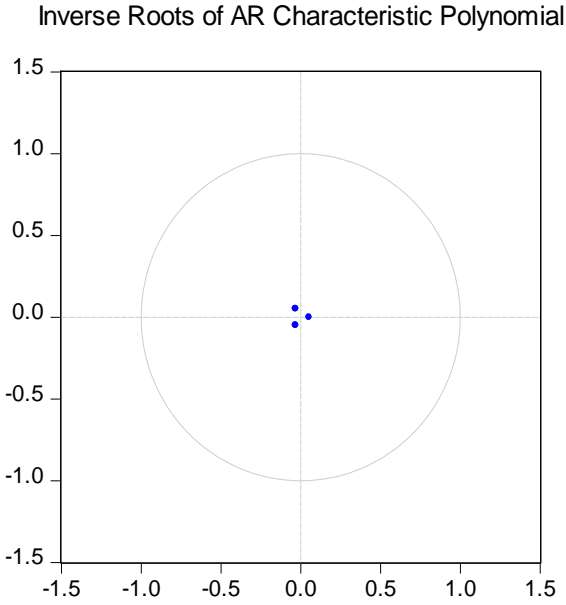


FIGURE 11 - *Inverse roots of VAR polynomial characteristic equation inside the unit circle*

Source: EViews 10 output

For completeness, it is possible to check the whole output of the VAR (1) estimation in **Table VII** available in the Annexes section.

5.3 Estimation of a GARCH model

After taking into consideration the interdependence through a VAR model, it is necessary to model the conditional autoregressive heteroskedasticity (ARCH effects) of the DGP as this is a pattern regularly observed in financial time series. In fact, as argued in Section 5.1, the time-series exhibit volatility clustering which is likely to generate these ARCH effects. To do so, I estimated a MGARCH (1,1) model simultaneously with the VAR (1) model.

The GARCH model, very briefly, acts as an ARMA¹ (*Autoregressive Moving Average*) process for the error variance.

The general GARCH (p, q), by consequent, can be expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_q \sigma_{t-q}^2 \quad (3)$$

¹ The notation ARMA (p, q) refers to the model with p autoregressive terms and q moving-average terms. Thus, this model contains the AR (p) model and the MA (q) model. Generally speaking, an ARMA (p, q) model has the following equation: $X_t = c + \varepsilon_t + \sum_{i=1}^p \varphi_i X_{t-i} + \sum_{i=1}^q \vartheta_i \varepsilon_{t-i}$.

where, p is the lag order of the squared error ε^2 and q is the lag order of the variance σ^2 : the first part of the equation (3) up to $\alpha_p \varepsilon_{t-p}^2$ is the ARCH (p) part, whereas the remaining part up to $\beta_q \sigma_{t-q}^2$ is the GARCH (q) part. Typically, given a process $\{\varepsilon_t\}_t$ for the stock returns, the hypothesis is that $\varepsilon_t = \sigma_t z_t$, where $z_t \sim N(0,1)$ for every t and σ_t follows the equation, or the ARCH part of it, as expressed in (3).

The MGARCH model is the multivariate extension of the univariate GARCH model which allows the conditional-on-past-history covariance matrix of the endogenous variables to follow a flexible dynamic structure.

The general equation of a MGARCH model (*vech representation*) as presented by Engle and Kroner (1995) can be written as follows:

$$\begin{cases} y_t = \mathbf{C}x_t + \varepsilon_t \\ \varepsilon_t = \mathbf{H}_t^{\frac{1}{2}}v_t \end{cases} \quad (4)$$

where, y_t is a k -vector of dependent variables, \mathbf{C} is a $k \times m$ parameter matrix, x_t is a k -vector of explanatory variables, possibly including lags of y_t , $\mathbf{H}_t^{\frac{1}{2}}$ is the Cholesky² factor of time-varying conditional matrix \mathbf{H}_t , and v_t is a k -vector of zero-mean, unit-variance i.i.d. innovations. For general purposes, \mathbf{H}_t is a matrix generalization of univariate GARCH models, i.e., it is the symmetric conditional covariance matrix and it satisfies the following properties:

1. Diagonal elements of \mathbf{H}_t must be strictly positive;
2. The matrix \mathbf{H}_t is positive-definite³;
3. Stationarity: $\mathbf{E}[\mathbf{H}_t]$ exists, finite and constant with regard to t .

For instance, a MGARCH (1,1) model can be expressed as

$$\text{vech}(\mathbf{H}_t) = s + \mathbf{A} \text{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}) + \mathbf{B} \text{vech}(\mathbf{H}_{t-1}) \quad (5)$$

where, *vech* denotes the vector-half operator, which stacks the lower triangular elements of a $N \times N$ matrix as an a $[N(N+1)/2] \times 1$ vector. For example, let \mathbf{M} a (2×2) matrix, then *vech* (\mathbf{M}) is

² Every positive-definite matrix $\mathbf{A} \in \mathbf{R}^{n \times n}$ can be factored as $\mathbf{A} = \mathbf{R}^T \mathbf{R}$, where \mathbf{R} is upper triangular with positive diagonal elements. \mathbf{R} is called the *Cholesky factor* of \mathbf{A} and can be interpreted as the “square root” of a positive definite matrix.

³ A symmetric matrix $\mathbf{A} \in \mathbf{R}^{n \times n}$ is *positive definite* if $x^T \mathbf{A} x > 0$ for all $x \neq 0$.

$$vech(\mathbf{M}) = \begin{bmatrix} m_{11} \\ m_{21} \\ m_{22} \end{bmatrix} \quad (6)$$

Given that the conditional covariance matrix \mathbf{H}_t is symmetric, $vech(\mathbf{H}_t)$ contains all the unique elements in \mathbf{H}_t .

Thus, the equation (5) for the example case $N = 2$, becomes:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \quad (7)$$

As the example of equation (7) shows that the even simpler MGARCH model suffers the so-called *curse of dimensionality*, i.e., even for low dimensions of N and p and q , the number of parameters to estimate is very large. Precisely, the number is: $1 + (p + q)[N(N + 1)/2]^2$.

The lag orders (MGARCH (1,1)) was selected based on the concept of parsimony: the GARCH (1,1) is parsimonious in terms of coefficients to estimate but empirical evidence also systematically suggests that a simple GARCH (1,1) is enough to describe most of the volatility dynamics present in financial time series: an ARCH (p), effectively, is equivalent to a GARCH (1,1) as p tends to infinite, i.e.,

$$\lim_{p \rightarrow \infty} \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 = \beta_0 + \gamma_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (8)$$

There are several different MGARCH models: the various MGARCH models basically provide alternative restrictions on the matrix \mathbf{H} . Many of them were estimated for the purpose of this work and I decided to choose the one which minimized the information criteria.

I concentrated my experimentation on four kinds of MGARCH models: MGARCH-BEKK, MGARCH-BEKK (TARCH), MGARCH-DIAGVECH and MGARCH-DIAGVECH (TARCH). I ignored the MGARCH-CCC (*Constant Conditional Correlations*) given that it was not expected, reasonably, the correlation to stay constant through the years of the analysis.

Generally speaking, TARCH (*Threshold ARCH*) models account for the fact that negative innovations have often greater impact in financial markets' volatility than positive innovations. A GARCH-TARCH (1,1) model can be, therefore, expressed as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (9)$$

where, d_{t-1} is a dummy variable that is equal to zero if $\varepsilon_{t-1} \geq 0$, and equal to one if $\varepsilon_{t-1} < 0$.

The diagonal-Vech MGARCH is built such that each element of the conditional covariance matrix follows a univariate GARCH model. More precisely, this model, as formulated by Bollerslev, Engle and Wooldridge (1988) restricts the matrixes \mathbf{A} and \mathbf{B} as presented in (5) to be diagonal. By consequent, the number of parameters to estimate are fewer than the general case: this restriction reduced the number of estimates to $[N(N+1)/2](1+p+q)$. Under the afore-mentioned restrictions, the general diagonal VEC model for each variance-covariance term can be written following the example of Minović and Simeunović (2008):

$$h_{ij,t} = s_{ij} + \sum_{h=1}^p a_{hij} \varepsilon_{i,t-h} \varepsilon_{j,t-h} + \sum_{h=1}^q b_{hij} h_{ij,t-h} \quad 1 \leq i \leq j \leq k \quad (10)$$

The equation (10) for the DIAGVECH (1,1) model reduces to:

$$h_{ij,t} = s_{ij} + a_{ij} \varepsilon_{i,t-1} \varepsilon_{j,t-1} + b_{ij} h_{ij,t-1} \quad (11)$$

For the sake of simplicity and completeness, here follows the matrix representation for a MGARCH-DIAGVECH (1,1) with $N = 2$. The equation (7) simplifies to:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} + \begin{bmatrix} a_{1,1} & 0 & 0 \\ 0 & a_{2,2} & 0 \\ 0 & 0 & a_{3,3} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1} \varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{1,1} & 0 & 0 \\ 0 & b_{2,2} & 0 \\ 0 & 0 & b_{3,3} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \quad (12)$$

The BEKK model, finally, is a multivariate model in which the elements of the conditional covariance matrix arise from quadratic form, reason why such matrix is always positive-definite under general conditions. The positive-definite parametrization as proposed by Engle and Kroner (1995) is the following:

$$\mathbf{H}_t = \mathbf{C}\mathbf{C}' + \sum_{k=1}^K \sum_{i=1}^q \mathbf{A}_{ik} \varepsilon_{t-i} \varepsilon_{t-1}' \mathbf{A}_{ik}' + \sum_{k=1}^K \sum_{i=1}^p \mathbf{B}_{ik} \mathbf{H}_{t-i} \varepsilon_{t-1}' \mathbf{B}_{ik}' \quad (13)$$

where \mathbf{C} , \mathbf{A}_{ik} and \mathbf{B}_{ik} are $(N \times N)$ matrices. More in particular, the intercept is the product $\mathbf{C}\mathbf{C}'$, where \mathbf{C} is a lower triangular matrix⁴, and without any further assumption $\mathbf{C}\mathbf{C}'$ is positive semi-definite⁵.

The values of the information criteria for each considered model are reported below in **Table VIII**.

⁴ A square matrix is called *lower triangular* if all the entries above the main diagonal are zero.

⁵ A symmetric matrix $\mathbf{A} \in \mathbf{R}^{n \times n}$ is *positive semi-definite* if $x^T \mathbf{A} x \geq 0$ for all x .

MGARCH DIAGVECH			
Log likelihood	19930.29	Schwarz criterion	-22.40178
Avg. log likelihood	3.755472	Hannan-Quinn criter.	-22.46232
Akaike info criterion	-22.49778		

TABLE VIII - *Information criteria coefficients for all the MGARCH models considered in the analysis*

Source: EViews 10 output

MGARCH DIAGVECH TARCH			
Log likelihood	19890.97	Schwarz criterion	-22.41652
Avg. log likelihood	3.748063	Hannan-Quinn criter.	-22.44971
Akaike info criterion	-22.46916		

MGARCH BEKK			
Log likelihood	19888.28	Schwarz criterion	-22.40078
Avg. log likelihood	3.747555	Hannan-Quinn criter.	-22.43984
Akaike info criterion	-22.46272		

MGARCH BEKK TARCH			
Log likelihood	19906.33	Schwarz criterion	-22.40851
Avg. log likelihood	3.750956	Hannan-Quinn criter.	-22.45342
Akaike info criterion	-22.47973		

As visible, the model which minimizes most of the information criteria is the MGARCH DIAGVECH and, for this reason, I selected this model to continue my analysis. The detail of the whole model specification is given in **Table IX** available in the Annexes section.

A reasonable criterion to check if a given model of the ARCH class is suitable for analysis is to perform diagnostic checking procedures to the residuals of the estimated model. In particular, an empirical practitioner should verify if ARCH effects are still active in the residual series. In case there are ARCH effects, one may perceive the model as adequate to pursue with the econometric analysis.

To detect the absence or the presence of remaining ARCH effects in the residuals, I applied the Portmanteau test with the Cholesky orthogonalization. Under this framework, the Portmanteau Test sets the absence of remaining ARCH effects in the residuals as the null hypothesis. Hence the null favors the application of the model under analysis as it is considered well specified. The alternative hypothesis is more loosely specified as this test as power against a range of possible alternatives, in particular, the presence of ARCH effects. The Portmanteau Test, therefore, allows checking how a model matches with the dataset when there exist many ways in which the model can deviate from the DGP.

The results of the application of a sequence of Portmanteau tests are shown in **Table X**. The results and interpretations of this test show no evidence of any remaining residual ARCH effects in the estimated model.

System Residual Portmanteau Tests for Autocorrelations
 Null Hypothesis: no residual autocorrelations up to lag h
 Orthogonalization: Cholesky (Lutkepohl)
 Date: 03/10/19 Time: 10:43
 Sample: 1/05/2012 2/15/2019
 Included observations: 1769

TABLE X - Test for residual ARCH
 in the selected VAR-MGARCH
 model

Source: EViews 10 output

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	4.063142	0.9072	4.065440	0.9071	9
2	6.746472	0.9921	6.751808	0.9921	18
3	17.36333	0.9218	17.38670	0.9211	27
4	28.99853	0.7898	29.04827	0.7878	36
5	37.53815	0.7774	37.61209	0.7748	45
6	45.88310	0.7761	45.98545	0.7728	54
7	47.67485	0.9242	47.78431	0.9226	63
8	54.01703	0.9439	54.15531	0.9423	72
9	61.07529	0.9518	61.24966	0.9501	81
10	70.87571	0.9319	71.10580	0.9293	90
11	88.60024	0.7639	88.94122	0.7558	99
12	102.1119	0.6415	102.5452	0.6301	108

*The test is valid only for lags larger than the System lag order.
 df is degrees of freedom for (approximate) chi-square distribution

5.4 The conditional correlations and volatilities

Since the estimated VAR-MGARCH model seems to be well specified as argued in section 5.3, the following step was to extract the key variables for the financial contagion analysis following Brexit, namely the conditional correlation estimates for each pair of countries and the conditional standard deviation estimates for each country.

As the conditional correlation graphs suggest in **Figure 12** (on the horizontal axis the years and months included in the sample, whereas on the vertical axis the value of conditional correlations), after the beginning of 2015 the values of all countries analyzed show more turbulence, especially during 2016 and starting from 2018. Moreover, correlations that involve the United Kingdom exhibit a more unstable pattern compared to the correlations between France and Germany.

Conditional Correlation

Cor(FR_LNRET,UK_LNRET)

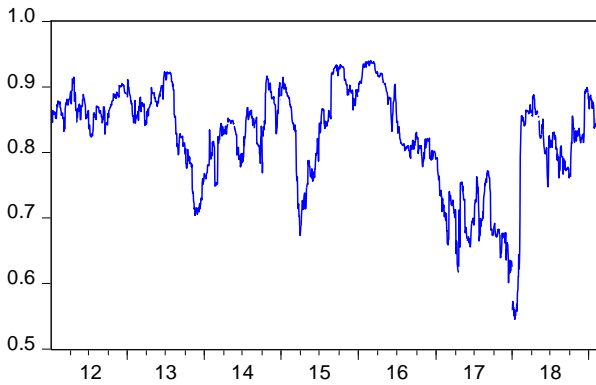
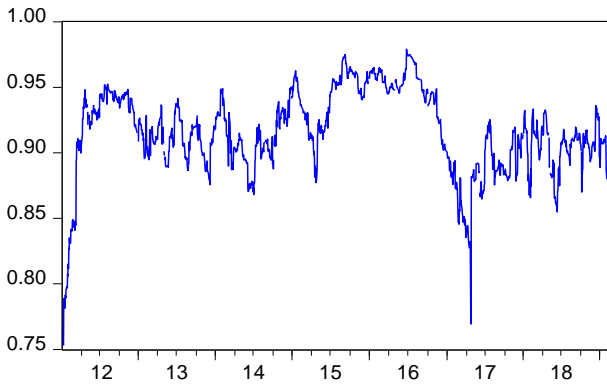


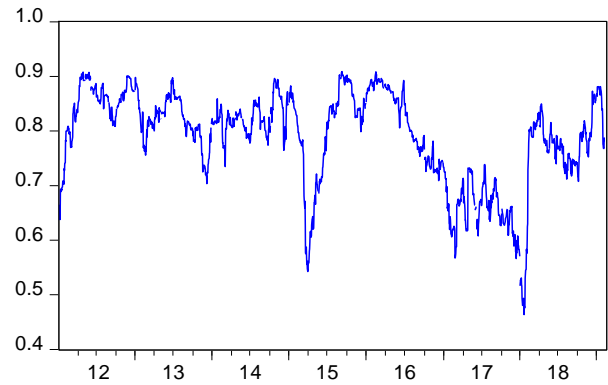
FIGURE 12 - *Conditional correlation estimates from the VAR-MGARCH model for the series of log returns of the United Kingdom, France and Germany*

Source: EViews 10 output

Cor(FR_LNRET,GER_LNRET)



Cor(UK_LNRET,GER_LNRET)



Likewise, the charts of the conditional standard deviations displayed in **Figure 13** (on the horizontal axis the years and months included in the sample, whereas on the vertical axis the value of conditional standard deviations) below show an abrupt increase during the last trimester of 2015 and the first semester of 2016 in all countries, period of time during which the markets had been creating expectations about the Brexit.

Conditional Standard Deviation

Std(FR_LNRET)

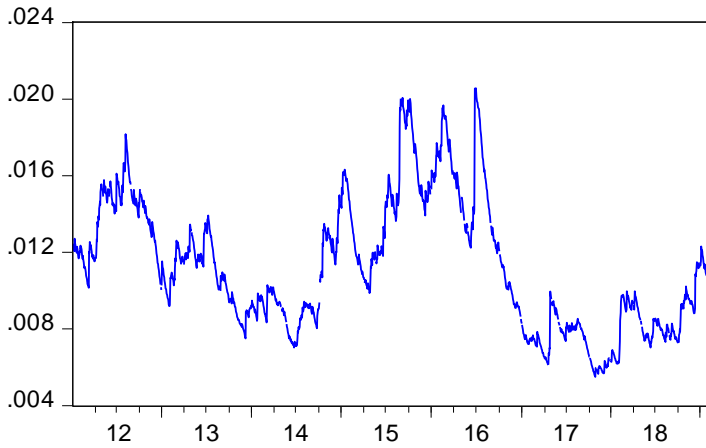
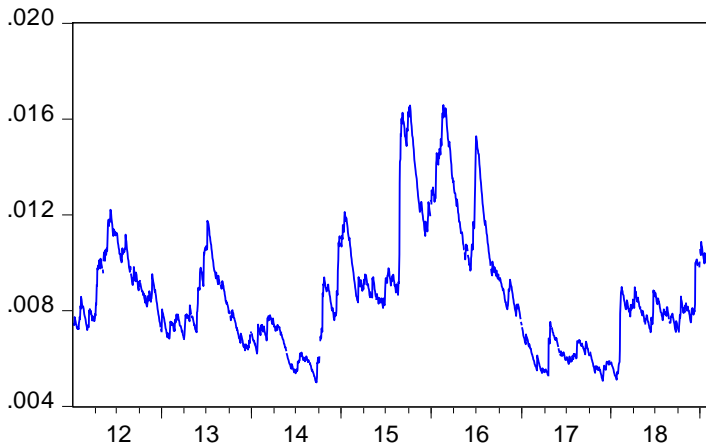


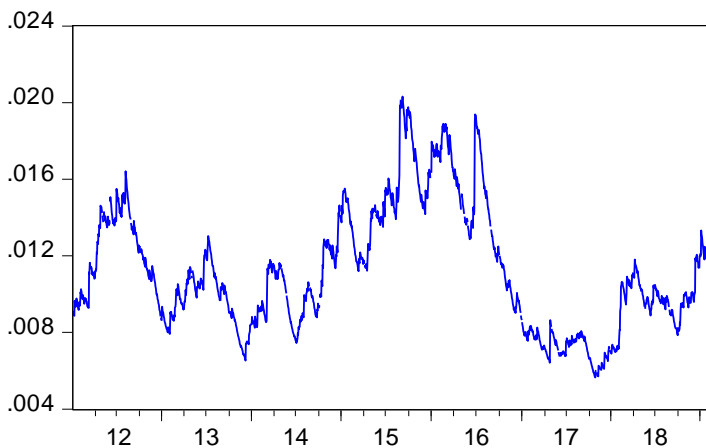
FIGURE 13 - Conditional standard deviation estimates from the VAR-MGARCH model for the series of log returns of the United Kingdom, France and Germany

Source: EViews 10 output

Std(UK_LNRET)



Std(GER_LNRET)



5.5 The test

With the conditional correlations and standard deviations obtained from the VAR-MGARCH model selected in Section 5.3 it was possible to implement a test to check the occurrence of financial contagion due to the Brexit shock.

I ran a test based on a dummy variable equal to 1 on all dates presented in section 3 and 0 on the remaining days. In order to verify whether or not there were significant differences between periods with more density of information related to the Brexit, I ran for both the conditional correlations and the conditional standard deviations an OLS regression in which the independent variables were the Brexit dummy and a constant: if the Brexit-related information is significant for the behavior of the conditional correlations and standard deviations throughout time, then the t-statistic referred to the coefficient of the dummy variable must be statistically significant. The model, thus, takes the following specification:

$$cond_corr_t = constant + \delta Brexit_dummy_t + error_t \quad (14)$$

Tables XI until **XVI** show the EViews outputs of the application of this test to the conditional correlations and standard deviations. In all EViews outputs the name *Value* identifies the Brexit dummy variable.

Starting with the three conditional correlations, the test in all cases resulted in a statistically significant coefficient for the dummy variable, at 1% level. Hence, I found strong statistical evidence that the events related to the Brexit had a relevant impact, either an increase or a decrease, in the interdependences among these financial markets during the sample period.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.043938	0.012042	-3.648705	0.0003
C	0.823182	0.001856	443.6384	0.0000
R-squared	0.007478	Mean dependent var		0.822139
Adjusted R-squared	0.006916	S.D. dependent var		0.077378
S.E. of regression	0.077110	Akaike info criterion		-2.286027
Sum squared resid	10.50661	Schwarz criterion		-2.279834
Log likelihood	2023.991	Hannan-Quinn criter.		-2.283739
F-statistic	13.31305	Durbin-Watson stat		0.029532
Prob(F-statistic)	0.000271			

TABLE XI - Estimation of OLS regression for the conditional correlation between the United Kingdom and France through the Brexit dummy

Source: EViews 10 output

In the case of the correlation between the United Kingdom and France (**Table XI**), the dummy variable is significant for any level of significance and has negative coefficient. Hence, evidence is found that the occurrence of Brexit events reduces the correlation between the countries compared to the pattern in “normal” periods.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.042335	0.013402	-3.158868	0.0016
C	0.789629	0.002065	382.3778	0.0000
R-squared	0.005615	Mean dependent var		0.788624
Adjusted R-squared	0.005053	S.D. dependent var		0.086035
S.E. of regression	0.085818	Akaike info criterion		-2.072054
Sum squared resid	13.01337	Schwarz criterion		-2.065861
Log likelihood	1834.732	Hannan-Quinn criter.		-2.069766
F-statistic	9.978448	Durbin-Watson stat		0.025935
Prob(F-statistic)	0.001611			

TABLE XII - Estimation of OLS regression for the conditional correlation between the United Kingdom and Germany through the Brexit dummy

Source: EViews 10 output

Likewise, in the case of Germany and the United Kingdom (**Table XII**), the dummy coefficient was statistically significant and show evidence of a decrease in the correlation value following a Brexit event.

Finally, in the case of indirect effects of Brexit upon France and Germany (**Table XIII**), the dummy variable linked to the Brexit shock kept being significant (at the 1% level of significance) with a negative coefficient.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.013496	0.005068	-2.662889	0.0078
C	0.916257	0.000781	1173.318	0.0000
R-squared	0.003997	Mean dependent var		0.915936
Adjusted R-squared	0.003433	S.D. dependent var		0.032508
S.E. of regression	0.032452	Akaike info criterion		-4.016950
Sum squared resid	1.860940	Schwarz criterion		-4.010756
Log likelihood	3554.992	Hannan-Quinn criter.		-4.014662
F-statistic	7.090977	Durbin-Watson stat		0.041798
Prob(F-statistic)	0.007818			

TABLE XIII- Estimation of OLS regression for the conditional correlation between France and Germany through the Brexit dummy

Source: EViews 10 output

By applying the same approach to the conditional standard deviations, I checked the statistical significance of the Brexit dummy for the conditional standard deviations.

In this case, the model to estimate is:

$$cond_std_dev_t = constant + \delta Brexit_dummy_t + error_t \quad (15)$$

For France (**Table XIV**) the inclusion of the Brexit dummy results in a statistically significant coefficient at 1% significance level, suggesting that volatility shifts on these two stock market indexes could be explained by Brexit-related announcements.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.001657	0.000526	-3.149933	0.0017
C	0.011317	8.10E-05	139.6382	0.0000
R-squared	0.005584	Mean dependent var		0.011278
Adjusted R-squared	0.005021	S.D. dependent var		0.003377
S.E. of regression	0.003368	Akaike info criterion		-8.547778
Sum squared resid	0.020046	Schwarz criterion		-8.541585
Log likelihood	7562.510	Hannan-Quinn criter.		-8.545490
F-statistic	9.922076	Durbin-Watson stat		0.018651
Prob(F-statistic)	0.001660			

TABLE XIV - Estimation of OLS regression for the conditional standard deviation for France through the Brexit dummy

Source: EViews 10 output

For Germany (**Table XV**), the dummy variable is statistically significant at 2% level as shown in the table below.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.001216	0.000499	-2.437480	0.0149
C	0.011253	7.68E-05	146.4443	0.0000
R-squared	0.003351	Mean dependent var		0.011224
Adjusted R-squared	0.002787	S.D. dependent var		0.003198
S.E. of regression	0.003193	Akaike info criterion		-8.654405
Sum squared resid	0.018018	Schwarz criterion		-8.648212
Log likelihood	7656.821	Hannan-Quinn criter.		-8.652117
F-statistic	5.941310	Durbin-Watson stat		0.014256
Prob(F-statistic)	0.014888			

TABLE XV - Estimation of OLS regression for the conditional standard deviation for Germany through the Brexit dummy

Source: EViews 10 output

As for the United Kingdom (**Table XVI**), instead, the inclusion of the Brexit information through the dummy does not show statistical significance: it may be the case that the Brexit information may be endogenous in the originating country resulting in not statistically significant coefficients.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
VALUE	-0.000492	0.000382	-1.285198	0.1989
C	0.008708	5.89E-05	147.7592	0.0000
R-squared	0.000934	Mean dependent var		0.008697
Adjusted R-squared	0.000368	S.D. dependent var		0.002450
S.E. of regression	0.002449	Akaike info criterion		-9.184946
Sum squared resid	0.010600	Schwarz criterion		-9.178753
Log likelihood	8126.085	Hannan-Quinn criter.		-9.182658
F-statistic	1.651735	Durbin-Watson stat		0.009772
Prob(F-statistic)	0.198891			

TABLE XVI - *Estimation of OLS regression for the conditional standard deviation for the United Kingdom through the Brexit dummy*

Source: EViews 10 output

As shown in all the results above, the coefficients of the dummy variables for the conditional standard deviations and correlations were significant (with the exception of the conditional standard deviation for the United Kingdom), but negative: the negative sign can be explained by the fact that, throughout the period considered in the analysis, time intervals with information suggesting that the Brexit could result in an orderly process were followed by time intervals with information suggesting that the Brexit would be just a disastrous process; the continuously and chaotically alternating flows of positive and negative information may have caused, if considered as a whole, the sign of the coefficient of the dummy variable to turn (slightly) negative. That is why, besides, I decided to create a dummy variable with unitary value just on remarkable dates instead of letting it assume unitary value from one date on (e.g., from the referendum date on): after the referendum, for instance, there were periods of relative calmness with events that could let us forecast a EU-friendlier resolution and, anyway, I also tried letting the dummy assume unitary value from the referendum date on, but it always resulted in statistically insignificant coefficient for the dummy variables in both estimations of equations (14) and (15). Finally, the explanation given in this analysis for the negative estimated coefficients seem to be sound given that the negative sign would not disappear even if I added the first lag of conditional correlations and standard deviations in the right-handed side of equations (14) and (15), respectively. Such seesawing pattern, that may have affected the dummy coefficients, can be better understood by looking at the next results that follow.

In order to corroborate the results just presented and detail the Brexit events that caused a significant variation on the analyzed variables, I calculated the averages for correlations and standard deviations for sub-samples delimited by Brexit-related key events. The results are shown in **Tables XVII** and **XVIII**.

<i>Average conditional standard deviations</i>			
Interval	UK	FR	GER
05/01/2012-22/01/2013	0,0091	0,0135	0,0120
23/01/2013-21/05/2014	0,0078	0,0104	0,0098
22/05/2014-10/10/2014	0,0058	0,0085	0,0091
13/10/2014-06/05/2015	0,0094	0,0127	0,0128
07/05/2015-22/02/2016	0,0120	0,0158	0,0163
23/02/2016-22/06/2016	0,0124	0,0153	0,0156
23/06/2016-13/07/2016	0,0140	0,0192	0,0182
14/07/2016-18/12/2016	0,0073	0,0090	0,0089
19/12/2017-27/02/2018	0,0065	0,0074	0,0081
28/02/2018-25/06/2018	0,0080	0,0085	0,0102
26/06/2018-06/12/2018	0,0079	0,0084	0,0095
07/12/2018-15/02/2019	0,0102	0,0111	0,0119

TABLE XVII - *Average conditional standard deviations for time sub-samples for the United Kingdom, France and Germany*

Source: Personal elaboration from EViews 10 output

As can be seen in **Table XVII**, the values of the interval average conditional standard deviations as from the 7th May 2016 until the 13th July 2016 rise remarkably and rapidly touching, at the end of this run, levels 50% higher than the levels of the pre-Brexit calmness (the first interval in the table) in all the three cases. The events triggering such abrupt increase are David Cameron's general elections victory with a manifesto for a referendum on the United Kingdom's stay in the European Union (7th May 2015), the Brexit referendum (22nd June 2016) and pro-Brexit Theresa May's government's birth (13th July 2016).

After the shock represented by the Brexit referendum, the values rapidly plummet and, on average, start descending until the interval between the 28th February 2018, when the European Commission publishes the draft of the Withdrawal Agreement between the European Union and the United Kingdom, and the 28th June 2018, when the European Union Withdrawal Act becomes Act of Parliament after receiving the Royal Assent: the values grow fast by 23%, 15% and 26% respectively for the UK, France and Germany as compared to the previous interval.

Finally, a decreasing pattern is again observed until the last interval considered in the table: starting from the 7th December 2018, when the Northern Ireland's backstop issue is revealed, the standard deviations for each series start again rearing up to levels similar to those of the periods around the referendum and about 30% higher than the values in the immediately earlier interval.

As for the average conditional correlations, a similar pattern can be detected (**Table XVIII**).

The pre-crisis level in the first interval shows high correlation due to the intrinsically strong economic interdependence between the pairs of countries.

In the sub-samples prior to the formation of David Cameron's government (7th May 2015), the trends seem to slightly swing. Then, after Cameron's victory, passing through the Brexit referendum, until Theresa May's new pro-Brexit governments all the three correlations hit the highest levels: in the cases of the correlations UK-Germany and UK-France the peak is reached immediately before the referendum, maybe because of the expectations created by the market about the result of the vote, whereas in the case of the correlation between France and Germany, the peak is reached exactly after the referendum date.

To follow, as in the case of the conditional standard deviations previously seen, after the drop registered once that Theresa May's government is formed (13th July 2017), a new rise in the level of correlations between the United Kingdom and Germany and the United Kingdom and France occurs between the 28th February 2018 and the 26th June 2018, day on which the Royal Assent turns the Withdrawal Act into law. Nevertheless, such increase is not found in the France-Germany correlation, which remains stable.

Lastly, after the Northern Irish border issue is raised, a new peak is reached in the last interval in all the series.

<i>Average conditional correlations</i>			
Interval	GER-UK	FR-UK	FR-GER
05/01/2012-22/01/2013	0,8464	0,8702	0,9131
23/01/2013-21/05/2014	0,8150	0,8303	0,9110
22/05/2014-10/10/2014	0,8159	0,8259	0,8966
13/10/2014-06/05/2015	0,7792	0,8381	0,9283
07/05/2015-22/02/2016	0,8371	0,8763	0,9507
23/02/2016-22/06/2016	0,8703	0,9060	0,9522
23/06/2016-13/07/2016	0,8537	0,8546	0,9735
14/07/2016-18/12/2016	0,6929	0,7365	0,9020
19/12/2017-27/02/2018	0,6164	0,6707	0,9080
28/02/2018-25/06/2018	0,7962	0,8416	0,8985
26/06/2018-06/12/2018	0,7631	0,8070	0,9062
07/12/2018-15/02/2019	0,8422	0,8715	0,9087

TABLE XVIII - *Average conditional correlations for time sub-samples for the United Kingdom, France and Germany*

Source: Personal elaboration from EViews 10 output

All the empirical evidence just described both for the correlations and standard deviations matches with the basic points presented at the beginning of the analysis about the relationships among volatility, correlation and financial contagion: extremely high levels of volatility that spill over from one country to another causing a general and contemporary increase in volatility for other countries, higher than average levels of correlation in more markets at the same

time in concurrence with high levels of volatility, abrupt and sensitive changes in correlations and volatilities in absolute terms in a relatively brief period of time.

6. Conclusions

The decision by the United Kingdom of rejecting the membership in the European Union was a shock from many points of view. My objective with this work was to verify whether it was also a financial shock capable of producing contagion to the Euro area. In order to do so, I selected France and Germany as the representative Euro terms of comparison for the analysis. Then, after homogenizing the series of the log-returns of the British, French and German stock indexes and checking that they were suitable for an econometric analysis, I estimated a VAR-MGARCH model to extract the conditional correlations and standard deviations: this way, it was possible to model the effects of interdependence and heteroskedasticity between countries and through time. Finally, by using a dummy variable approach which tracked the most relevant Brexit-related information, I tested the relationships between the conditional correlations and standard deviations with the main Brexit events: the test showed evidence of statistical significance of the Brexit news. Moreover, the analysis of the time averages of conditional correlations and standard deviations corroborated the result of the test. The analysis, therefore, showed evidence of financial contagion in the Eurozone caused by the Brexit.

However, this work must be considered as a humble attempt to understand one of the most complicated process of our current days. The present paper, in effect, has several limitations: in the sample period and stock markets that could be expanded and, mainly, the construction of the conditional correlations and standard deviations which could be estimated through the specification of a more complex, and maybe complete, model. More in detail, the VAR estimation could include exogenous variables and the MGARCH variance equation could be better modelled by including, for instance, the dummy variable as potential regressor. Even though it is likely the Brexit to be a contagion case, the contrary may have some evidence.

Further research on this topic could, for instance, take into account more stock markets and a wider period of time as well as try resorting to the most recent econometric models to get more robust evidence.

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Annexes

TABLE VII - *Estimation of a VAR(1) model for the series of log returns of the United Kingdom, France and Germany*

Vector Autoregression Estimates

Date: 03/02/19 Time: 11:52

Sample (adjusted): 1/05/2012 2/15/2019

Included observations: 1769 after adjustments

Standard errors in () & t-statistics in []

	FR_LNRET	UK_LNRET	GER_LNRET
FR_LNRET(-1)	0.071296 (0.06885) [1.03548]	0.108557 (0.05276) [2.05760]	0.117824 (0.06871) [1.71492]
UK_LNRET(-1)	-0.053494 (0.05793) [-0.92339]	-0.041602 (0.04439) [-0.93718]	-0.106628 (0.05781) [-1.84455]
GER_LNRET(-1)	-0.061485 (0.06356) [-0.96729]	-0.081465 (0.04871) [-1.67257]	-0.041024 (0.06343) [-0.64678]
C	0.000242 (0.00027) [0.90775]	9.86E-05 (0.00020) [0.48291]	0.000293 (0.00027) [1.10361]
R-squared	0.001589	0.002473	0.002545
Adj. R-squared	-0.000108	0.000777	0.000850
Sum sq. resids	0.221552	0.130083	0.220600
S.E. equation	0.011204	0.008585	0.011180
F-statistic	0.936493	1.458550	1.501180
Log likelihood	5437.368	5908.348	5441.174
Akaike AIC	-6.142869	-6.675351	-6.147173
Schwarz SC	-6.130482	-6.662964	-6.134786
Mean dependent	0.000235	9.54E-05	0.000298
S.D. dependent	0.011203	0.008588	0.011184
Determinant resid covariance (dof adj.)		4.73E-14	
Determinant resid covariance		4.70E-14	
Log likelihood		19614.42	
Akaike information criterion		-22.16215	
Schwarz criterion		-22.12499	
Number of coefficients		12	

Source: EViews 10 output

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TABLE IX- Estimation of a MGARCH-DIAGVECH model for the series of log returns of the United Kingdom, France and Germany

Source: EViews 10 output

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.067079	0.056356	1.190269	0.2339
C(2)	-0.054114	0.045981	-1.176876	0.2392
C(3)	-0.047199	0.051157	-0.922616	0.3562
C(4)	0.000639	0.000212	3.007233	0.0026
C(5)	0.088045	0.043001	2.047511	0.0406
C(6)	-0.055303	0.036682	-1.507622	0.1317
C(7)	-0.046719	0.040713	-1.147512	0.2512
C(8)	0.000366	0.000167	2.184683	0.0289
C(9)	0.098262	0.056412	1.741869	0.0815
C(10)	-0.100791	0.047117	-2.139187	0.0324
C(11)	-0.016729	0.052243	-0.320211	0.7488
C(12)	0.000745	0.000217	3.427513	0.0006

Variance Equation Coefficients				
C(13)	2.72E-06	5.80E-07	4.687490	0.0000
C(14)	1.69E-06	3.73E-07	4.526994	0.0000
C(15)	2.34E-06	5.06E-07	4.624650	0.0000
C(16)	1.62E-06	4.01E-07	4.036914	0.0001
C(17)	1.64E-06	3.81E-07	4.308369	0.0000
C(18)	2.43E-06	5.66E-07	4.305076	0.0000
C(19)	0.046428	0.006047	7.678462	0.0000
C(20)	0.045855	0.005824	7.873784	0.0000
C(21)	0.042240	0.005678	7.439150	0.0000
C(22)	0.046424	0.006294	7.375422	0.0000
C(23)	0.042908	0.005819	7.373631	0.0000
C(24)	0.042061	0.006144	6.846040	0.0000
C(25)	0.924376	0.009626	96.03326	0.0000
C(26)	0.924601	0.009545	96.86617	0.0000
C(27)	0.930058	0.009170	101.4270	0.0000
C(28)	0.926489	0.010452	88.64032	0.0000
C(29)	0.927621	0.009979	92.95512	0.0000
C(30)	0.932706	0.009657	96.58482	0.0000

t-Distribution (Degree of Freedom)				
C(31)	8.162898	0.842240	9.691886	0.0000

Log likelihood	19930.29	Schwarz criterion	-22.40178
Avg. log likelihood	3.755472	Hannan-Quinn criter.	-22.46232
Akaike info criterion	-22.49778		

Equation: FR_LNRET = C(1)*FR_LNRET(-1) + C(2)*UK_LNRET(-1) + C(3)*GER_LNRET(-1) + C(4)

R-squared	0.000212	Mean dependent var	0.000235
Adjusted R-squared	-0.001487	S.D. dependent var	0.011203
S.E. of regression	0.011212	Sum squared resid	0.221857
Durbin-Watson stat	2.014965		

Equation: UK_LNRET = C(5)*FR_LNRET(-1) + C(6)*UK_LNRET(-1) + C(7)*GER_LNRET(-1) + C(8)

R-squared	0.001152	Mean dependent var	9.54E-05
Adjusted R-squared	-0.000546	S.D. dependent var	0.008588
S.E. of regression	0.008591	Sum squared resid	0.130256
Durbin-Watson stat	1.998200		

Equation: GER_LNRET = C(9)*FR_LNRET(-1) + C(10)*UK_LNRET(-1) + C(11)*GER_LNRET(-1) + C(12)

R-squared	0.000749	Mean dependent var	0.000298
Adjusted R-squared	-0.000950	S.D. dependent var	0.011184
S.E. of regression	0.011190	Sum squared resid	0.220998
Durbin-Watson stat	2.014766		

Covariance specification: Diagonal VECM
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)
M is an indefinite matrix
A1 is an indefinite matrix
B1 is an indefinite matrix*

Transformed Variance Coefficients				
M(1,1)	2.72E-06	5.80E-07	4.687490	0.0000
M(1,2)	1.69E-06	3.73E-07	4.526994	0.0000
M(1,3)	2.34E-06	5.06E-07	4.624650	0.0000
M(2,2)	1.62E-06	4.01E-07	4.036914	0.0001
M(2,3)	1.64E-06	3.81E-07	4.308369	0.0000
M(3,3)	2.43E-06	5.66E-07	4.305076	0.0000
A1(1,1)	0.046428	0.006047	7.678462	0.0000
A1(1,2)	0.045855	0.005824	7.873784	0.0000
A1(1,3)	0.042240	0.005678	7.439150	0.0000
A1(2,2)	0.046424	0.006294	7.375422	0.0000
A1(2,3)	0.042908	0.005819	7.373631	0.0000
A1(3,3)	0.042061	0.006144	6.846040	0.0000
B1(1,1)	0.924376	0.009626	96.03326	0.0000
B1(1,2)	0.924601	0.009545	96.86617	0.0000
B1(1,3)	0.930058	0.009170	101.4270	0.0000
B1(2,2)	0.926489	0.010452	88.64032	0.0000
B1(2,3)	0.927621	0.009979	92.95512	0.0000
B1(3,3)	0.932706	0.009657	96.58482	0.0000

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