LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

# MASTER DEGREE IN

## APPLIED ECONOMETRICS AND FORECASTING

## MASTER'S FINAL WORK

DISSERTATION

# EXTERNAL DEBT AS LONG-RUN EQUILIBRIUM LEVER

MARTINHO DE MATOS SILVESTRE

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**SUPERVISOR:**

ANTÓNIO MANUEL PEDRO AFONSO

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# Abstract

A long standing model – the Solow-Swan – does not provide a suitable answer in the case of a financial crisis. This paper proposes an extension to the standard neoclassical growth model. By incorporating the international capital market, more robust results are found regarding the unsustainability of the debt situation. In this sense, we believe our augmented model gives relevant insights about financial crisis. Progress is made by modeling the propensity to borrow. Our results exclude the idea that external debt is associated with economic development state and Government budget balances. Moreover, we conclude that indebtedness is mostly influenced by private sources.

# Resumo

Um modelo de longa data – Solow-Swan – não fornece uma resposta adequada no caso de uma crise financeira. Este trabalho propõe uma extensão do modelo de crescimento neoclássico padrão. Ao incorporar o mercado internacional de capitais encontramos resultados mais robustos em relação à situação de insustentabilidade da dívida. Neste sentido, acreditamos que nosso modelo dá uma visão relevante sobre a crise financeira. Por fim, modelizamos a propensão ao endividamento. Os nossos resultados excluem a idéia de que a dívida está associada ao estado de desenvolvimento económico e aos saldos orçamentais governamentais. Além disso, podemos concluir que o endividamento é influenciado principalmente por fontes privadas.

# Acknowledgements

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# 1 Introduction

From late 2009, fears of a sovereign debt crisis developed among investors as a result of rising private and government debt levels around the world together with a wave of government debt rating downgrade in some European states. Causes of the crisis varied by country: private debts arising from property bubbles were transferred to sovereign debt as a result of bailouts in the banking system and government post-bubble intervention to stimulate slowing economies. For example in Greece, high public sector wages and pensions commitments were connected to debt increase (Lewis, 2011).

The actual euro zone financial crisis is characterized by the difficulty or impossibility for some countries in the euro area to repay or re-finance their government debt without the assistance from third parties. This crisis has some similarities with previous ones as, in general, all crisis are characterized by the same negative spiral. Normally, there is some event that shake the economic agents trust causing the aforementioned negative spiral. The implications are a self-feed on the economy leading to the need of some sort of economic paradigm shift in order to overcome the crisis.

Neoclassical growth models are one of the main streams of Growth Theory. The theory was driven by the Solow-Swan model which considered constant returns to scale, diminishing marginal productivity of capital, exogenously determined technical progress and substitutability between capital and labor. Economies changed substantially since the publication of this work, making this model unable to capture some actual effects.

External debt is beneficial to the economic growth until the limit of debt sustainability is reached. From that point, the external debt self feeds and consumes the internal savings, lowering the stock of physical and human capital. We consider a small economy open only to international capital market. Thus, the reason of this crisis becomes endogenous (in economic sense) to the model and more robust results are found regarding the unsustainability of the debt situation. In this sense, we believe our augmented model gives relevant insights about financial crisis.

Progress is made by modeling the net borrowing (NB) using real Gross Domestic Product (GDP) growth, real long-term interest rate (RLTIR), debt-to-GDP ratio (GDR) growth and risk-free interest rate (RFIR). Despite the weak results found in linear estimation, we get convincing nonlinear estimates. We did not find empirical evidence supporting the relevance of GDP and GDR growth rate explaining NB. These results exclude the idea that debt is associated with the economic development state and Government budget balances. Moreover, RLTIR has the major effect in NB, showing that indebtedness is mostly influenced by private sources.

Finally, the nonlinear long-run multipliers estimates appear to be very similar to the short-run ones as we expected once our model was developed thinking in the long-run equilibrium. Additionally, the long-run multipliers estimates are all slightly higher in magnitude. This is economically consistent with the existence of some economic feedback contributing with a small effect to the contemporaneous stream.

The remainder of the paper is organized as follows. Section 2 presents a survey of the literature and Section 3 describes the theoretical contextual setting. Empirical application is developed in Section 4 and conclusions regarding the addressed topic are provided in Section 5.

# 2 Literature Survey

Developments have been presented in the aggregate growth theory since the Solow-Swan (1956) appeared. Several authors turned some of the exogenous growth into endogenous. Conlisk (1967) first introduced endogenous technical change into a neoclassical growth model. He added an equation to describe the Human Capital and assumed that both

Fixed Capital and Human Capital grew partly by an endogenous component (dependent on the output level) and by an exogenous component (as in the Solow-Swan model). The main advantage of this model is that the long-run equilibrium also depends on labor variables giving a partial explanation to what previously was considered as exogenous. Several papers about endogenous growth models were later introduced. Romer (1986) defended the increasing returns to scale due to the endogenous accumulation of capital. Lucas (1988) compared three neoclassical models with different sources of endogenous growth and concluded that an economic growth model should consider the Human Capital accumulation by education and mostly by learn-by-doing. This process of learn-by-doing is deeply studied by Arrow (1962) and consists in the productivity increase due to experience. He also pretended to begin the discussion about the international trade and the international worker migrations. These phenomena explained the human capital accumulation and the asymmetry between developed and developing countries.

In this paper, we assume a neoclassic growth model to study the leverage effect of foreign capital in the steady-state *per capita* product. An augmented Solow-Swan model is assumed and focus is given to the savings rate, on the thread of the fixed savings rate models. Proposed a few years later, the Villanueva (1994) model is a variant of the Conlisk (1967) model which develops a formulation that considers the learn-by-doing as a way to raise productivity, concluding three major effects:

- The steady-state growth rate becomes endogenous, and therefore affected by the government policies, in particular, the size of government in relation to GDP must have limits due to increasingly heavy costs of deficits;
- The speed of adjustment increases leading to the conclusion that learning reduces the adjustment time;
- With learn-by-doing, the optimal savings rate should be set at a fraction of capital

income share instead of the hole share as in the Solow-Swan model.

More recently, there has been several attempts to incorporate the external debt and fiscal adjustment in this model, namely Villanueva (2003) considers that the aggregate capital stock is the accumulated sum of domestic saving and net external borrowing. As a result, the steady-state solution is characterized by constants *per capita* capital and external debt to capital ratios. On the other hand, Milbourne (1995) uses an open economy neoclassical growth model to investigate the reasons why only some countries accumulated debt. It is shown that debt stabilization relates to the marginal propensity to consume, the population growth rate and real rate of interest. Finally, Piazza (2010) studies a growth model for a small open economy where decreasing marginal returns to capital appear only after the country has reached a threshold level of development. The observation made is that financial crisis are caused by sudden lending restrictions occurring when international investors feel that the country has entered in decreasing marginal returns, resulting in economy defaults and financial crisis.

Nkoro and KelvinUko (2012) examined the relationship between capital inflows and economic growth using an Error Correction Model for Nigeria and found results supporting that interconnection. On the other hand, Thompson (2008) suggested a growth model considering both foreign and domestic capital, recognizing that a steady-state occurs where both capital and labour ratios are stationary. In the development process, some countries would become perpetual investment hosts depending on their saving propensity, labor growth rates, and foreign investment openness.

In this paper, we consider a small economy open to the international capital market so the investment capacity is dependent on both internal savings rate and foreign capital inflow. We study steady-state *per capita* product considering foreign capital using an EU-15 panel to investigate the variables influence in the countries borrowing behavior.

# 3 Neoclassic growth model

## *3.1 Contextual setting*

A country economy is characterized by transforming resources into goods in order to satisfy its population needs. The productive process can be seen as a function receiving inputs and generating outputs, which is referred to as a production function. Cobb and Douglas (1928) suggest a form for this function that has achieved widely acceptation by the research community

$$Y = AL^{\beta}K^{\alpha}. \tag{3.1}$$

Where:

$Y$ : Total production (the monetary value of all goods produced in a year)

$A$ : Total factor productivity

$L$ : Labour (the total number of person-hours worked in a year)

$K$ : Capital (the monetary worth of all machinery, equipment and building)

$\alpha$ and $\beta$ are the output elasticity of capital and labor, respectively. These values are determined by available technology and are constant over time (at least in developed countries).

The interpretation of the output elasticities is very relevant for the economic analysis. The magnitude of $\alpha$ and $\beta$ translates the responsiveness of the output to input variations. In other words, a 1% increase in labor would lead to approximately an $\alpha \times 100\%$ increase in output. Thus, three different classifications for the production function according to $\alpha$ and $\beta$, are of interest:

$\alpha + \beta = 1$ : The production function has constant returns to scale;

$\alpha + \beta < 1$ : Returns to scale are decreasing;

$\alpha + \beta > 1$ : Returns to scale are increasing.

This theoretical framework has been used by various streams of growth economics. The primary reference in growth economics is the neoclassic paradigm, which attempts to explain long-run economic growth by looking at productivity, capital accumulation, population growth and technological progress. They are better known as exogenous growth models due to the fact that the main source of growth is not the productive factors.

The neoclassical growth theory is based on Solow (1956) and Swan (1956). These authors have published simultaneously the same model known as the Solow-Swan. In first instance, the success of this model is due to its parsimony. By explaining *per capita* product the Cobb-Douglas production function turns out to be only expressed through capital *per capita*, so the necessary equations are:

$$y = Ak^{\alpha}, \tag{3.2}$$

$$\Delta k = sf(k) - (n + \delta)k. \tag{3.3}$$

Where $y$ is the *per capita* product, $k$ is the *per capita* capital, $\alpha < 1$ so that production involves decreasing returns to capital, $\Delta k$ represents the variation of the *per capita* physical capital, $s$ is the domestic savings rate, $\delta$ is the physical capital depreciation rate and $n$ represents the population growth rate. So, $sf(k)$ is the aggregate domestic *per capita* saving and $(n + \delta)k$ the aggregate depreciation of *per capita* capital.

Finally, the economy will reach an equilibrium when there is no need of further fluctuations in the productive factors so that $\Delta k = 0$. This equilibrium, also known as the Solow-Swan steady-state, can also be seen graphically in Figure 1 [1]:

$$sf(k) = (n + \delta)k. \tag{3.4}$$

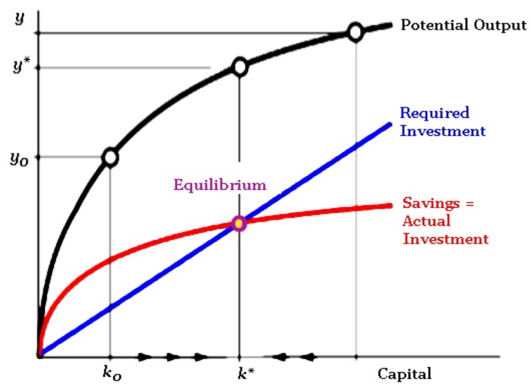



Figure 1: Graphical representation of the Solow-Swan growth model.

The Solow-Swan model is the benchmark for growth analysis by its ability to explain long-run growth without assuming any economic conditions. Conclusion is derived that regardless of a country political orientations its *per capita* product depends on the growth of $A$, referred to as "technical progress" by Solow. It is also one of the major limitations of the model. When analyzing growth policies, this model is not helpful as the source of growth (technical progress) cannot be explained or even rationalized. To overcome such challenge, we must consider a theoretical framework where productivity growth is endogenous, which means that technical progress is considered as dependent of the productive factors of the economy.

[1]This figure was retrieved from: http://jrm-research.blogspot.pt/2007/05/solow-swan-classical-growth-theory.html.

### 3.2 Theoretical Model

Economy can be seen as a function where output is produced with two inputs: physical capital and labor. Furthermore, we assume a Cobb-Douglas (1928) production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{3.5}$$

Where $Y$ represents the output, $K$ is the stock of physical capital, $\alpha < 1$ and $L$ is the labor force. As in the Solow-Swan model we consider the *per capita* product and in the sequel we work with small letters instead of the capitals, so $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. The definition of physical capital variation is kept as:

$$y_t = A_t k_t^{\alpha}, \tag{3.6}$$

$$\Delta k_t = s_t y_t - (\delta_t + n_t) k_t. \tag{3.7}$$

The previous definitions are compliant with the Solow-Swan model assumptions. Since we are working with a small closed economy with $c + s = 1$, where $c$ is the consumption propensity. However, the aim of this paper is to introduce the international capital market, thus considering a small open economy. As a consequence, the investment in the economy depends not only on domestic savings but also on external debt. So we consider the sum of $b$, the borrow propensity, to the savings rate. A country can leverage its economy to have a faster economic growth and convergence. However, the effect can also be negative if the country is over-indebted.

Finally, we can define our equilibrium equation. By first starting with the Solow-Swan model equilibrium:

$$s_t y_t = (\delta_t + n_t) k_t \tag{3.8}$$

We include the international capital market by considering:

$$(b_t + s_t)y_t = (\delta_t + n_t)k_t \iff y_t = \frac{\delta_t + n_t}{b_t + s_t} \times k_t \tag{3.9}$$

After optimal GDP estimation, we model the borrow propensity using both linear and nonlinear models and we give thought to some theoretical behaviors to make its definition. The analysis is made easier by dealing with public and private sector separately:

$$b_t = b_t^{priv} + b_t^{publ}. \tag{3.10}$$

The private sector is more rational than the public one and faces stricter debt restrictions. Naturally, the decision of taking debt is dependent on the interest rate. When facing a given interest rate, one has to decide whether it is profitable to lend or to borrow. The private agents should take external savings whenever it is possible to invest the borrowed funds getting higher income. So, the assumption sets that private sector considers the the difference between real growth rate ($g_t$) and real interest rate ($r_t$) as an indicator of investment profitability.

$$b_t^{priv} = e^{\psi(g_t - r_t)} - 1 \tag{3.11}$$

By contrast, the public sector does not pursue profits which makes it less rational and harder to model. We assume that there are three major event chains:

- When there is no public debt, the public policy makers have to decide whether to borrow or to lend, if the risk-free interest rate is very high, the capital market is very unstable so, as a caution attitude, they avoid entering the market.

$$\lim_{\substack{d_t \to 0 \\ i_t^f \to +\infty}} b_t^{publ} = 0$$

- A very low risk-free interest rate encourages public policy makers to take more debt. This behavior causes debt-to-GDP ratio to grow followed by the risk-prize. This triggers a snowball effect leading to the public debt propensity burst.

$$\lim_{i_t^f \to 0} b_t^{publ} = +\infty$$

- When Debt-to-GDP ratio grows wildly, the financial markets tend to distrust the paying back capacity. This makes the risk prize grow immeasurably, turning that debt unsustainable.

$$\lim_{d_t \to +\infty} b_t^{publ} = +\infty$$

According to this acknowledgments, we propose an *ad hoc* equation to define the public choices of some country debt:

$$b_t^{publ} = e^{\phi d_t} + \frac{1}{e^{\theta i_t^f}} - 1. \tag{3.12}$$

Considering that the debt of any economy is given by the sum of these two components, we reach the final expression for the borrowing behavior:

$$b_t = e^{\psi(g_t - r_t)} + e^{\phi d_t} + \frac{1}{e^{\theta i_t^f}} - 2. \tag{3.13}$$

# 4 Empirical Application

## 4.1 Data

In this paper we use a panel for EU-15 countries[2] covering the period from 1995 to 2012. Considering the analysis, we acknowledge that a longer panel would be more suitable, maybe fifty years instead of the actual eighteen. However, a balanced longer panel is hard to find. On one hand, the excessive deficit procedure, defined by the Maastricht Treaty, was only implemented by European Union in 1994. Only after that was this debt components measurement uniformed for all countries in European Union. On the other hand, Germany was only reunited in 1990, which makes finding data for this country earlier from that date a challenging task.

Our approach stands by estimating steady-state product using both Solow-Swan model and our augmented model. The following AMECO [3] series are used:

**Gross Domestic Product (GDP) at constant prices (OVGD)** refers to the volume level of GDP. Constant price estimates of GDP are obtained by expressing values in terms of a base period. The estimation of this quantity involves dividing GDP at current prices by the price index.

**Net capital stock at constant prices (OKND)** is concerned with the quantity of fixed capital of some economy. Fixed capital refers to any kind of real or physical capital (fixed asset) that is not fully consumed in the production of an output and is contrasted with working capital such as raw materials. To measure it we consider the last value of the same series and sum the difference between

[2]This group includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom.

[3]The annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs.

gross formation and consumption of fixed capital. Initialization of the series is performed using perpetual inventory method (PIM) (Dey-Chowdhury, 2008).

**Total population (NPTN)** measures the number of living humans on each country. This value is based on an annual average using National accounts data on population.

**Consumption of fixed capital (CFC) at current prices (UKCT)** is a term used in national accounts for depreciation of fixed assets. CFC is used rather than "depreciation" to emphasize that fixed capital is used up in the process of generating new output and because, unlike depreciation, it is not valued at historical cost but at current market value.

**Price deflator for gross fixed capital formation (PIGT)** allows us to transform the previous series (CFC) from current prices to constant prices. In our case, we use the year basis 2005. In practice, this index turns the measure of the series from market value to quantity level.

**Gross national saving (USGN)** is calculated as GDP less final consumption expenditure (total consumption). Hence, domestic savings can also be perceived as a choice between consumption today and consumption tomorrow as it is a way to accumulate wealth over time and increase living standards in the future.

**Net lending (+) or borrowing (-) (UBLA)** represents the net sources made available to the rest of the world (if positive) by the economy or received from the rest of the world (if negative). To be consistent with the theoretical approach we consider the symmetric of this variable divided by the GDP and call Net Borrowing (NB) to this transformed variable. The data from Luxembourg had some issues, so we used it only from 2002 to 2012.

In a second stage, to perform some inference about net borrowing, we estimate some models using additional data:

**Real long-term interest rate (ILRV)** represents the return demanded by the long-run investors and is deflated using the GDP price deflator. The series was retrieved from AMECO.

**Government debt-to-GDP ratio** is a measure of a country public debt in relation to its Gross Domestic Product. By comparing what a country owes to what it produces, debt-to-GDP ratio indicates the country ability to pay back its debt. We used a series compiled by the IMF [4] and updated for the recent years as indicated by the authors.

**3-month Treasury bill (USA) interest rate** is considered a riskless investment because it is a direct obligation of the United States government and its term is short enough to minimize the risks of inflation and market interest rate changes. This series was published by the Federal Reserve of the USA.

Our empirical approach uses these series as a proxy for our theoretical variables as may be seen in Figure 1 and in Figure 2:

We summarize some descriptive statistics in appendix B. Before we use these series, they must be checked if they are stationary and we resort to a Levin-Lin-Chu unit-root test. The null hypothesis of this test is that the series contains a unit root, and the alternative is that the series is stationary. The Levin–Lin–Chu test assumes a common autoregressive parameter for all panels, so this test does not discriminate if only some countries series contain unit roots. The test involves fitting an augmented Dickey–Fuller regression for each panel (Levin et al., 2002). Test results are presented in Table 3.

[4]See http://www.imf.org/external/pubs/ft/wp/2010/wp10245.pdf

Table 1: Theoretical variables and economic series for steady-state product estimation

| | |
|---|---|
| $y$ | $\frac{\text{GDP at constant prices}}{\text{Total population}}$ |
| $\delta$ | $\frac{\text{CFC at current prices}}{\text{Net capital stock at constant prices}}$ |
| $n_t$ | $\frac{\text{Total population}_t - \text{Total population}_{t-1}}{\text{Total population}_{t-1}}$ |
| $b$ | $-\frac{\text{Net lending (+) or borrowing (-)}}{\text{GDP at constant prices}}$ |
| $s$ | $\frac{\text{Gross national saving}}{\text{GDP at constant prices}}$ |
| $k$ | $\frac{\text{Net capital stock at constant prices}}{\text{Total population}}$ |

Table 2: Theoretical variables and economic series to model Net Borrowing

| | |
|---|---|
| $b$ | $-\frac{\text{Net lending (+) or borrowing (-)}}{\text{GDP at constant prices}}$ |
| $g_t$ | $\frac{\text{GDP at constant prices}_t - \text{GDP at constant prices}_{t-1}}{\text{GDP at constant prices}_{t-1}}$ |
| $r$ | Real long-term interest rate |
| $d$ | Government debt-to-GDP ratio |
| $i^f$ | 3-month Treasury bill (USA) interest rate |

The Levin-Lin-Chu bias-adjusted t statistic for the Government debt-to-GDP ratio (GDR) is 1.958, which is not significant at all the usual significance levels. Therefore, we do not reject the null hypothesis and conclude that the series appear to have a unit root in its autoregressive representation using a 5% dimension test. However, if we consider its growth rate the Levin-Lin-Chu bias-adjusted t statistic is -3.448, which is significant at all the usual significance levels. So, without loss of generality, we used this transformed variable instead of the original. The Levin-Lin-Chu bias-adjusted t statistic for the remaining variables used in our theoretical model (Net borrowing, GDP real growth rate, Real long-term interest rate, Government debt-to-GDP ratio growth rate and 3-month Treasury bill interest rate) are significant at all the usual significance levels except Real long-term interest rate, which is statistically significant considering a 5% dimension test, and conclusion is drawn that all these series are apparently stationary.

Table 3: Levin-Lin-Chu unit root tests

| Variable | N | T | Model | *Adjusted t-statistic* | *p-value* |
|---|---|---|---|---|---|
| NB | 15 | 11 | c | -2.440 | 0.007 |
| DLGDP | 15 | 18 | c | -3.448 | 0.000 |
| RLTIR | 15 | 18 | c | -1.780 | 0.038 |
| GDR | 15 | 18 | c, t | 1.958 | 0.975 |
| DLGDR | 15 | 18 | c | -3.448 | 0.000 |
| RFIR | 15 | 18 | c | -5.666 | 0.000 |

Note that $c$ denominates the panel means, $t$ the time trend, $D$ the difference and $L$ the logaritm.

However, we cannot fully accept these results since this statistical test good properties are only verified for large T (over 50 periods) and fixed N (Hlouskova and Wagner, 2005).

## 4.2 Product per capita steady-state

The steady-state level represents the situation when the destructive forces of capital *per capita* match the investment. From that moment capital *per capita* level should not change substantially, although this level is not very relevant and we may interpret the corresponding product *per capita*. But what happens if a country reaches the steady-state? Is it possible to change this equilibrium level? The answer is straightforwardly positive as only a change in the relevant economic parameters are needed. For example, a permanent increase in the investment rate will cause a shift on the investment curve and the same effect in the steady-state product *per capita*. In other words, the economy is now richer than it was before (Jones, 1998). This happens according to the assumptions of the Solow-Swan model with no international capital market. This ensures that a permanent increase in the investment rate is caused by a permanent increase in the savings rate, since internal savings are the only capital source for investment.

For an instant lets consider how the Solow-Swan steady-state [5] behaves in an insolvent economy. In this case, an economy enters a debt spiral (debt generates even more debt). In the Solow-Swan model external debt is considered as saving (because there cannot be external debt without international capital market) and because of the equivalence between saving and investment, the steady-state will increase disproportionately. This may be seen in Figure 2 especially in Greece, Portugal and Ireland (the economies under external financial aid). In spite of our highlighting, the case where external debt is accumulated the reverse is also true. The economies responsible for funding these countries appear to have penalization on their steady-state level.

Our proposed model[6] deals better with these situations once we consider differently save from external debt. In fact, it may be seen that often when the Solow-Swan steady-state is increasing (because of the external debt accumulation) our results point out an eminent drop. This phenomenon may be clearly seen in the countries subjected to a financial bailout (Greece, Portugal and Ireland). Inversely, in funding economies case (for example Germany), our results are in general higher. If we look to our results more attentively, we see that the financial crisis was preceded by declines in product *per capita* steady-state levels.

[5] $y_t = \frac{\delta_t + n_t}{s_t} k_t$

[6] $y_t = \frac{\delta_t + n_t}{s_t + b_t} k_t$

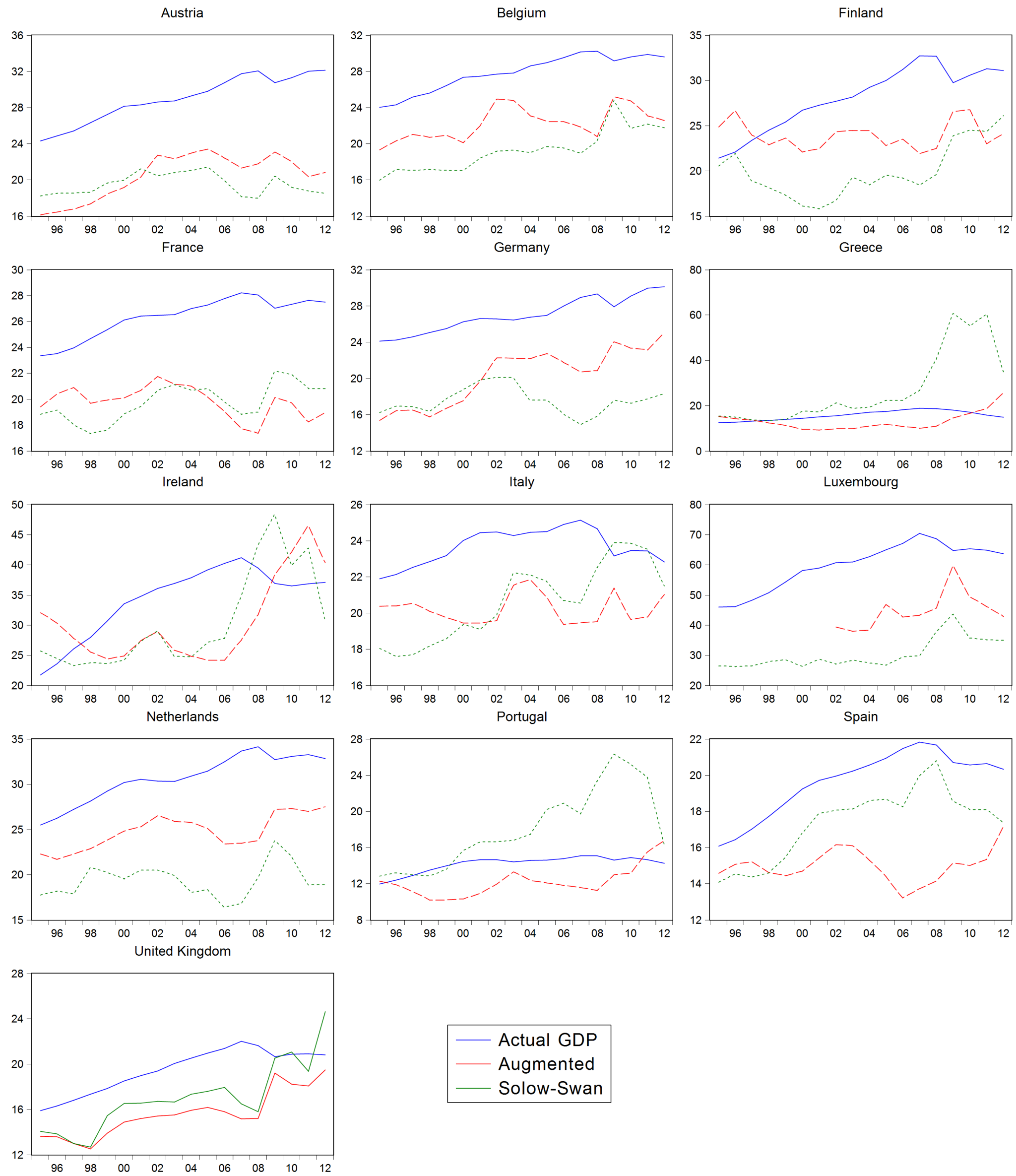



Figure 2: Solow-Swan simple and augmented steady-state *per capita* product level.

At last, we need to point out some limitations of our results. On one hand, for Sweden and Denmark we could not get valid results. Somehow for these countries net borrowing is higher (and with symmetric signals) than gross savings. It appears that both countries lend more than what effectively they can save (during the recent years). We think that this may be the consequence of financial arbitrage. The practical consequence of this behavior in our model is negative product *per capita* steady-state values. On the other hand, it is clear that the observed GDP is almost always higher than the steady-state values. We experienced the extension of our data base to cover the period from 1960 to 2012 and concluded that the inversion happened gradually and roughly halfway. We think this is motivated by the restrictive hypotheses of the model. For example, we consider a closed economy but these countries have faced a growing integration since 1951 (with the Paris Treaty that marked the beginning of European Coal and Steel Community). This integration phenomenon brings productivity gains and contributed to the economic development. All these gains are ignored by our model due to the assumption of closed economy considered.

## *4.3 Estimation*

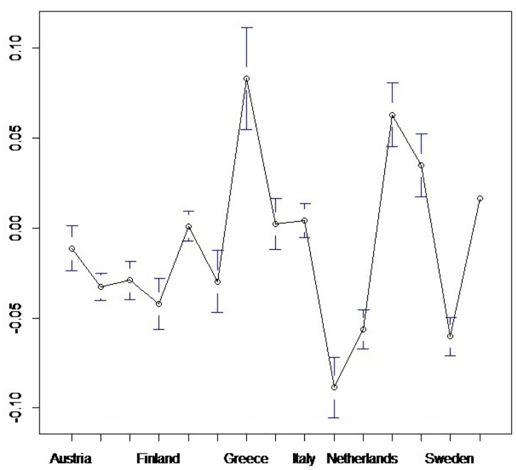
In this section, we study the effect of the variables, presented previously, in borrow propensity using both linear and nonlinear models. We start by using a simple static linear model:

$$\mathrm{NB}_{it} = \beta_0 + \beta_1 \mathrm{DLGDP}_{it} + \beta_2 \mathrm{RLTIR}_{it} + \beta_3 \mathrm{DLGDR}_{it} + \beta_4 \mathrm{RFIR}_{it} + u_{it}. \tag{4.1}$$

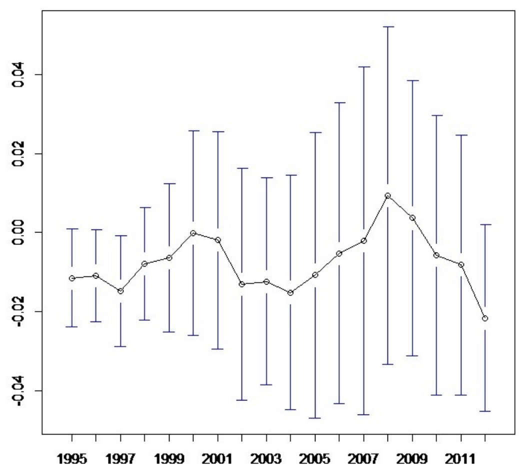
Where NB denotes Net Borrowing, DLGDP the Gross Domestic Product real growth rate, RLTIR the Real Long-Term Interest Rate, the DLGDR the Government debt-to-GDP ratio growth rate and RFIR the Risk-Free Interest Rate:

$$u_{it} = c_i + \lambda_t + v_{it}. \tag{4.2}$$

Equation 4.2 considers a two way error component, where $c_i$ captures the individual heterogeneity. This affects the interest rates charged to each country and so the amount of interest paid. On the other hand, there are some phenomena whose impact may be considered equal and constant (in time) to all these countries. For example, the global situation expectations or even some European Central Bank decisions. If we consider that financial credibility is probably correlated with DLGDR and DLGDP, then the random effects estimator would get inconsistent estimates. As a consequence, a consistent estimator should be used such as the fixed effects estimator. Confirmation of these hypotheses are checked later in this paper using proper statistical tests.




(a) Across countries




(b) Across years

Figure 3: Heterogeneity of net borrowing from 1995 to 2012.

In Figure 3a, we may see the graphical representation of the heterogeneity effects across countries while Figure 3b represents heterogeneity effects across time.

Regarding the heterogeneity across countries, two points must be stressed: there are huge gaps between countries net borrowing averages; and the individual variance is small. The combination of these two characteristics indicate that the individual heterogeneity exists and may be approximately constant.

In Figure 3b, the conclusions are not so straightforward. Firstly, in spite of existing some year mean fluctuation, the variation amplitude is small (the difference between the higher average and the lower one is just around 0.03). Moreover, there is high variation in each year which lead us to believe that the heterogeneity is much more intense in country rather than in time, but no definite conclusions can be based only on this information.

Table 4: Static model for net borrowing (NB) using time dummies

| | POLS | FE | RE |
|---|---|---|---|
| DLGDP | -0.246 | -0.260 | -0.265 |
| | (0.282) | (0.170) | (0.166) |
| RLTIR | 0.254 | -0.179* | -0.144 |
| | (0.219) | (0.094) | (0.095) |
| DLGDR | 0.041 | 0.035* | 0.032 |
| | (0.076) | (0.017) | (0.021) |
| Constant | -0.019 | 0.002 | -0.001 |
| | (0.016) | (0.014) | (0.012) |
| Observations | 263 | 263 | 263 |

Robust standard errors in parentheses.
$***p < 0.01$, $**p < 0.05$, $*p < 0.1$

In Table 4, we compiled the static linear estimation results for Pooled Ordinary Least Squares (POLS) estimator, Fixed Effects Estimator (FE) and Random Effects Estimator (FE). The obtained standard errors are robust to individual heterogeneity. This adjustment is motivated by the the possibility of errors being correlated within cluster (in our case, the clusters are the countries) (Cameron et al., 2011). In spite of random effects estimator being a FGLS, which means that it operates on the covariance matrix, we used that same adjustment. We did this because the inference using random effects estimator needs the Assumptions 4.3 and 4.4[7]:

$$E\left(\underline{v}_i\underline{v}_i'|\underline{\underline{x}}_i, c_i\right) = \sigma_v^2 I_T, \tag{4.3}$$

[7]Where $\sigma$ denotes the standard error, $I$ the identity matrix, the underline a vector and $\underline{v}'$ the transpose of $\underline{v}$.

$$E\left(c_i^2 | \underline{\underline{x}}_i\right) = \sigma_c^2. \tag{4.4}$$

However, the failure of these assumptions does not cause inconsistency and it is very useful to conduct inference without worrying about them. On one hand, $E\left(\underline{u}_i \underline{u}_i' | \underline{\underline{x}}_i\right)$ may not be constant so that $E\left(\underline{u}_i \underline{u}_i' | \underline{\underline{x}}_i\right) \neq E\left(\underline{u}_i \underline{u}_i'\right)$. On the other hand, $E\left(\underline{u}_i \underline{u}_i'\right)$ may not have the RE structure: the idiosyncratic errors ($v_{it}$) may have variance that changes over time, or they could be serially correlated. Thus, by making robust analysis we eliminate possible problems if the conditions do not hold. In addition, no error is introduced when using robust standard errors, even if the previous assumptions are verified (Wooldridge, 2002).

To deal with temporal heterogeneity, we consider the inclusion of time dummies. By doing so, it is not possible to include the variable RFIR since this variable is constant on the individuals. In addition, RFIR and temporal mean evolution are very much alike. So we use this variable as a trend, capturing the temporal heterogeneity. Because we believe that this variable is economically relevant to the model, it is better to include RFIR instead of time dummies. Conversely, this variable has more variability than time dummies and only requires the estimation of one parameter which brings more efficiency to the estimation. Following this idea, the same static model is estimated including RFIR instead of the time dummy variables and results are summarized in Table 5.

Static models usually have serial correlation, which we overcome by estimating a dynamic model with the inclusion of a dependent variable lag. Although the inclusion of the $\mathrm{NB}_{it-1}$ might mitigate the serial correlation problem, it also induces endogeneity in the model, due to the dependence of both $\mathrm{NB}_{it}$ and $\mathrm{NB}_{it-1}$ on $c_i$.

To solve the endogeneity, Arellano and Bond (1991) suggested starting by the differentiation of the variables, removing the fixed individual effect that was causing the

Table 5: Static model for net borrowing (NB) using RFIR

| | POLS | FE | RE |
|---|---|---|---|
| DLGDP | -0.217 | -0.211** | -0.217** |
| | (0.165) | (0.0899) | (0.0870) |
| RLTIR | 0.141 | -0.227* | -0.198 |
| | (0.222) | (0.126) | (0.129) |
| DLGDR | 0.0450 | 0.0471*** | 0.0444*** |
| | (0.0618) | (0.0125) | (0.0158) |
| RFIR | 0.260 | 0.212 | 0.216 |
| | (0.169) | (0.191) | (0.185) |
| Constant | -0.0152 | -0.00354 | -0.00646 |
| | (0.0166) | (0.00805) | (0.0182) |
| Observations | 263 | 263 | 263 |

Robust standard errors in parentheses.
$***p < 0.01$, $**p < 0.05$, $*p < 0.1$

endogeneity. Unfortunately, the estimation of the model with the differences brings endogeneity between $\Delta \text{NB}_{it-1}$[8] and $\Delta v_{it}$ because both depend on $v_{it-1}$. To solve this the authors recommend the use of $\text{NB}_{it-2}, \ldots, \text{NB}_{i1}$ as instruments for $\Delta\text{NB}_{it-1}$. Therefore the number of instruments to be used will grow with T and the estimator will become more efficient while its bias worsens.

Blundell and Bond (1995) came up with a better solution by noticing that if the correlation between the dependent variable lags was high or the individual heterogeneity very heavy then the instruments used by Arellano and Bond would become weak. To overcome this situation Blundell and Bond (1995) suggest the introduction of even more instruments and not only estimate the differences model but also the model in levels. In the second equation, $\text{NB}_{it-1}$ would use $\Delta\text{NB}_{it-1}, \ldots, \Delta\text{NB}_{i1}$ as instruments [9].

[8]Where $\Delta$ denotes the time difference.

[9]For further information about GMM and system GMM estimation see Roodman (2009).

We consider the following dynamic model:

$$\begin{aligned}\mathrm{NB}_{it} = \delta\mathrm{NB}_{it-1} + \beta_1\mathrm{DLGDP}_{it} + \beta_2\mathrm{DLGDP}_{it-1} + \beta_3\mathrm{RLTIR}_{it} + \beta_4\mathrm{RLTIR}_{it-1} \\ +\beta_5\mathrm{DLGDR}_{it} + \beta_6\mathrm{DLGDR}_{it-1} + \beta_7\mathrm{RFIR}_{it} + \beta_0 + u_{it}\end{aligned}$$

The estimation results according to Arellano and Bond and Blundell and Bond are summarized in Table 6. Once again we estimated robust standard error for every model. Using this procedure, the standard errors are consistent in the presence of any pattern of heteroskedasticity and serial correlation within clusters. In the case of two-step estimation, the standard covariance matrix is already robust in theory, but typically yields downward biased standard errors due to the optimal weighting matrix estimation using first-step residuals. In this case, the Windmeijer (2005) correction should be applied.

At last, we get to the nonlinear model. Recalling the model presented in Section 3.2:

$$\mathrm{NB}_{it} = \beta_1 + e^{\beta_2\mathrm{DLGDP}_{it}+\beta_3\mathrm{RLTIR}_{it}} + e^{\beta_4\mathrm{DLGDR}_{it}} + e^{\beta_5\mathrm{RFIR}_{it}} + c_i + v_{it} \qquad (4.5)$$

In order to avoid making parameters restrictions, we consider the split of $\psi$ into $\beta_2$ and $\beta_3$. No limitation is introduced as the estimator may not be consistent with the restriction $\beta_2 = -\beta_3$. This formulation allows a different impact from both variables, which makes economic sense. The central question in nonlinear panel data models is how to control for presence of individual effect $c_i$. In opposition to the linear models, the *within* transformation does not wipe out the individual heterogeneity. Thus, we thought of including individual dummies to capture the individual fixed effects. However, the inclusion of individual dummy might create the *incidental parameter* problem, especially if we pretend to enlarge the country group (what is more feasible than increase the temporal range). From that point, we decided not to include individual dummies. Also

Table 6: Dynamic model for net borrowing (NB)

| | AB 2step | BB 2step | FE | POLS |
|---|---|---|---|---|
| L.NB | 1.341*** | 0.923*** | 0.846*** | 0.987*** |
| | (0.133) | (0.095) | (0.026) | (0.015) |
| DLGDP | 0.204* | 0.358* | 0.029 | 0.014 |
| | (0.116) | (0.217) | (0.034) | (0.039) |
| L.DLGDP | 0.192** | 0.228** | 0.103* | 0.079 |
| | (0.096) | (0.092) | (0.056) | (0.057) |
| RLTIR | -0.037 | 0.061 | -0.135*** | -0.129** |
| | (0.062) | (0.161) | (0.040) | (0.052) |
| L.RLTIR | -0.210*** | -0.277 | -0.241*** | -0.183*** |
| | (0.077) | (0.195) | (0.046) | (0.047) |
| DLGDR | 0.056 | 0.112* | 0.042*** | 0.031** |
| | (0.039) | (0.068) | (0.014) | (0.012) |
| L.DLGDR | 0.018 | 0.025*** | 0.015** | -0.001 |
| | (0.020) | (0.008) | (0.006) | (0.011) |
| RFIR | 0.217 | 0.022 | 0.220*** | 0.225** |
| | (0.161) | (0.170) | (0.073) | (0.081) |
| Constant | | -0.009*** | -0.001 | 0.000 |
| | | (0.003) | (0.002) | (0.002) |
| Observations | 233 | 248 | 248 | 248 |
| Instruments | 10 | 16 | | |

Note that "*L.*" denotes the lag of the variable. AB represents the Arellano and Bond estimator while BB the Blundell and Bond estimator. Robust standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$

assuming a static formulation may lead to serial correlation, to mitigate these problems a dynamic nonlinear model is estimated according Equation 4.6, and considering robust standard errors (to accommodate heteroscedasticity and serial correlation).

$$\mathrm{NB}_{it} = \beta_1 + \delta \mathrm{NB}_{it-1} + e^{\beta_2 \mathrm{DLGDP}_{it} + \beta_3 \mathrm{RLTIR}_{it}} + e^{\beta_4 \mathrm{DLGDR}_{it}} + e^{\beta_5 \mathrm{RFIR}_{it}} + c_i + v_{it} \quad (4.6)$$

In order to estimate the parameters for the nonlinear model we use Nonlinear Least Squares (NLS). This estimator plays the same role as OLS but for nonlinear models.

There is no closed-form solution to a nonlinear least squares problem. Instead, numerical algorithms are used to find the value of the parameters which minimize the residuals. Then, the parameters are refined iteratively, that is, the values are obtained by successive approximation (Davidson and MacKinnon, 1993). Nonlinear estimation using Panel Data is not fully developed, so we did not found a suitable program to estimate RE and FE or even nonlinear two stages least squares (NL2SLS). To prevent misleading results we decided to use a pooled estimator instead. Therefore, endogeneity problems may arise as in the linear case. Hence, we must be careful regarding the estimation conclusions. The results summarized in Table 7 were achieved using nonlinear least squares after 4 iterations.

Table 7: Nonlinear model for net borrowing (NB)

| | Static | Dynamic |
|---|---|---|
| L.NB | | 0.984*** |
| | | (0.017) |
| DLGDP | -0.329* | 0.018 |
| | (0.157) | (0.042) |
| RLTIR | 0.319* | -0.284*** |
| | (0.181) | (0.055) |
| DLGDR | 0.028 | 0.028* |
| | (0.055) | (0.013) |
| RFIR | 0.566* | 0.223** |
| | (0.266) | (0.097) |
| Const. | -3.021*** | -3.000*** |
| | (0.014) | (0.003) |
| Observations | 150 | 150 |
| R-squared | 0.921 | 0.073 |

Robust standard errors in parentheses.
$***p < 0.01$, $**p < 0.05$, $*p < 0.1$

## *4.4 Discussion of the results*

In this section, we analyze the validity of all estimates shown in the previous section following the same sequence. Because RE assumes $corr(c_i, \underline{x}_{it}) = 0$, which is a strong assumption, we have to test if this assumption is plausible by performing a Hausman specification test. This test assumes that we can estimate a consistent estimator whether or not the null is true and an efficient (and consistent) estimator under the null, but inconsistent otherwise. The assumption that one of the estimators is efficient (i.e., has minimal asymptotic variance) is strict and it is violated, for example, if your model is somehow misspecified.

In essence, this test compares the estimates of fixed effects and random effects estimator using the chi2 statistic (Hausman, 1978) [10].

$$m_1 = \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)' \left[Var\left(\hat{\beta}_{FE}\right) - Var\left(\hat{\beta}_{RE}\right)\right]^{-1} \left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right) \overset{a}{\sim} \chi^2_{(K)} \tag{4.7}$$

If RE fails to be more efficient than FE then nothing ensures that $Var\left(\hat{\beta}_{FE}\right) - Var\left(\hat{\beta}_{RE}\right)$ is positive, resulting in a negative statistic test which is not admissible as chi2 distribution is strictly positive. Moreover, even if the assumption is satisfied, there may be a "small sample" problem with the Hausman test. This may happen because under the previous assumptions, $Var\left(\hat{\beta}_{FE}\right) - Var\left(\hat{\beta}_{RE}\right)$ is a consistent estimator of $Var\left(\hat{\beta}_{FE} - \hat{\beta}_{RE}\right)$, but is not necessarily positive definite "in finite samples". Unfortunately, our analysis is in this category meaning we had to opt for a robust test.

The random effects estimator uses the additional orthogonality conditions that the regressors are uncorrelated with the group-specific error $c_i$, so Arellano (1993) and Wooldridge (2002) propose an overidentifying restrictions test. Significance of this test means that there are signs of serial correlation and evidence that the instruments are not exogenous (Sargan, 1958). The results are summarized in Table 8:

The Sargan-Hansen statistic is 24.548 which is significant at all usual significance levels. The null hypothesis is rejected so the individual effect should be endogenous. A

[10]Where a hat refers to an estimator, $Var$ to the variance, $\chi^2_{(K)}$ to a Chi-square distribution with $K$ degrees of freedom and $\overset{a}{\sim}$ stands for distribution asymptotic convergence.

Table 8: Specification test (FE vs RE)

| Sargan-Hansen statistic | DF of chi2 | Prob. |
|---|---|---|
| 24.548 | 3 | 0.000 |

Where DF means Degrees of Freedom.

by-product result is the rejection of the null hypothesis of Hausman specification test. A consistent estimator is only achieved by having fixed instead of random effects.

The next step is to test for serial correlation of the errors. In spite of the natural correlation due to individual effect, which is constant over time, the problematic is the serial correlation of the idiosyncratic error ($v_{it}$). Wooldridge (2002) suggests a test with a good size evidence and power properties in reasonable sample sizes. The test null hypothesis states that there is no serial correlation in the original model. The residuals from the regression of the first-difference variables should have a serial correlation of -0.5 $[corr\left(\Delta u_{it}, \Delta u_{it-1} = -0.5\right)]$. Given this observation, the procedure regresses the residuals $\hat{u}_{it}$ from the regression with first-differenced variables on their lags and tests that the coefficient on the lagged residuals is equal to -0.5 (Drukker, 2003).

Table 9: Wooldridge test for serial correlation in panel data

| F-statistic | DF numerator | DF denominator | Prob. |
|---|---|---|---|
| 42.64 | 1 | 14 | 0.000 |

Where DF means Degrees of Freedom.

The results summarized in Table 9 show that the F-statistic is 42.64. This statistic is significant at all the usual significance levels. Therefore, we reject the null hypothesis and conclude that apparently the idiosyncratic errors are serial correlated. Nevertheless, the serial correlation affects the efficiency but not the consistency. The estimate is valid as long as standard errors are not used. Another option is to use a robust estimator, however it is better to insert some dynamics.

In sequence, we added one lag of the dependent variable and estimated FE and

POLS. In the dynamic model context, these estimators suffer from *dynamic panel bias* as seen before (Nickel, 1981). We use these inconsistent estimators because while FE is downwards-biased, POLS is upwards-biased for the autoregressive parameter (Blundell and Bond, 1995). This way, we expect from the consistent estimate $\hat{\delta}$ to be between the estimates of FE and POLS. To choose between AB and BB, we should look at $\delta$ estimate. Both estimators show high values for this parameter, the strength of the instruments used by AB estimator can be questioned.

Arellano-Bond AR test looks for serial correlation in the difference residuals. A residual transformation is used to reduce the error from the unobserved and perfectly serial correlated $c_i$. AR(1) is expected in first differences because both $\Delta v_{it}$ and $\Delta v_{it-1}$ share the $v_{it-1}$ term. So, focusing on AR(2) test, if we find evidence in the differences, the error (in level) is serially correlated of first order. In this case, serial correlation indicates that lags of the dependent variable (and any other variables used as instruments that are not strictly exogenous), are in fact endogenous, thus bad instruments. The results of Arellano-Bond test for AR in first differences are compiled in Table 10. The *p-values* of AR(1) test are all near null as expected (statistically significant). However, the *p-values* for AR(2) test are all high meaning that the z-statistics are not significant regardless the significance level considered. The non-rejection of the null hypothesis translates in not having statistical evidence against the absence of serial correlation. Thus, we do not have statistical basis to suspect of the instruments quality for any of the estimators.

The results of Sargan-Hansen test of overidentification restrictions are summarized in the same table (Table 10). The null hypothesis of this test validates the group of instruments used while the alternative states that some of the instruments may be endogenous. The Sargan statistic uses the 1 step non-robust covariance matrix which makes this test vulnerable to individual heterogeneity and serial correlation. In contrast,

Table 10: Statistical tests to the dynamic model

| | | AB 2step | BB 2step |
|---|---|---|---|
| Arellano-Bond test for AR(1) | z-stat | -3.033*** | -2.837*** |
| | Prob. | 0.002 | 0.005 |
| Arellano-Bond test for AR(2) | z-stat | -0.068 | -0.405 |
| | Prob. | 0.946 | 0.686 |
| Sargan test of overid. restrictions | chi2 | 2.836 | 11.560 |
| | Prob. | 0.242 | 0.116 |
| Hansen test of overid. restrictions | chi2 | 4.031 | 5.625 |
| | Prob. | 0.133 | 0.584 |

$***p < 0.01$, $**p < 0.05$, $*p < 0.1$

the Hansen statistic is robust to these problems because it uses the two-step or corrected covariance matrices. In spite of Hansen test robustness, it can be greatly weakened by instrument proliferation. Thus, it is convenient to study both statistics together. In our particular case, we used relatively few instruments which allows us to ignore Sargan test. The results confirm our suspicions about the strength of the AB instruments. The Hansen statistic for BB is 5.625 while for AB is only 4.031. Despite instruments of both estimators appear to be valid, the non-rejection of the null hypothesis in BB instruments is larger.

## *4.5 Marginal effects*

After an estimation, a marginal effects analysis is imposed. A marginal effect of an independent variable is the partial derivative, with respect to that variable, of the prediction function. For linear models, often this analysis is straightforward, however, the same is not true for nonlinear models.

Firstly, we should interpret the signals of the estimates. Naturally, if the estimate do not have the expected signal (given by the economic theory) we should distrust the estimation results. According to Section 3.2, we only expect Real Long-Term Interest Rate (RLTIR) and Risk-Free Interest Rate (RFIR) coefficients to be negative. However, RFIR coefficient estimates are always positive. We think this is due to the trend role that this variable eventually takes. An alternative cause is a bad choice of the economic series. The others estimates differ a lot from model. Looking at static estimates, we concluded that none of them gathers plausible results. Naturally, these results are due to misspecification regarding the dynamic effects. Thus, our analysis focus only dynamic models, namely BB, FE, POLS and the nonlinear one.

Regarding marginal effects analysis, we approach the impact (or short-run) and the equilibrium (or long-run) multipliers. The former measures the contemporaneous effect while the lather is related to the impact of a permanent change in an independent variable. As an example, for the model $y_t = \delta y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + u_t$, the Short-Run Multiplier (SRM) and Long-Run Multiplier (LRM) are given by:

$$SRM = \beta_1 \qquad SRM1step = \delta\beta_1 + \beta_2 \qquad LRM = \frac{\beta_1 + \beta_2}{1-\delta}$$

We estimated both multipliers for all linear models. The results concerning SRM are presented in Table 11.

Table 11: Short-run multipliers for linear dynamic models

| | BB 2step | FE | POLS |
|---|---|---|---|
| DLGDP | 0.358 | 0.029 | 0.014 |
| RLTIR | 0.061 | -0.135 | -0.129 |
| DLGDR | 0.112 | 0.042 | 0.031 |
| RFIR | 0.022 | 0.220 | 0.225 |

The estimated short-run multipliers are not very strong. On one hand, the signal of RLTIR impact is not coincident with economic theory. On the other hand, we expected a higher importance for Government Debt-to-GDP Ratio growth rate (DLGDR), according to these results we estimate that, in average, an increase of one percentage point in the DLGDR will cause NB to grow by 0.00112 while the average estimated effect of the same variation of GDP growth rate is more than tree times. As we did not obtain coherent SRM estimates for all variables, we decided to estimate short-run multipliers 1 step ahead. The results may be seen in Table 12.

Table 12: Short-run multipliers 1 step ahead for linear dynamic models

| | BB 2step | FE | POLS |
|---|---|---|---|
| DLGDP | 0.558 | 0.127 | 0.093 |
| RLTIR | -0.221 | -0.355 | -0.310 |
| DLGDR | 0.129 | 0.051 | 0.029 |
| RFIR | 0.020 | 0.186 | 0.222 |

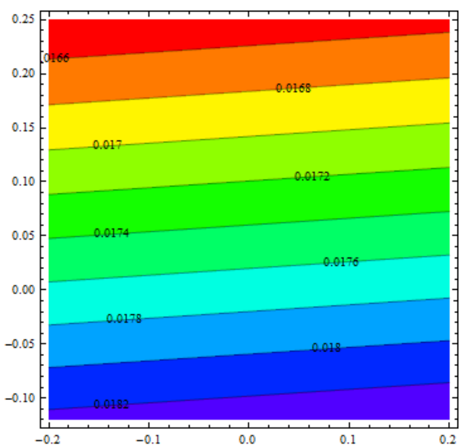
The results concerning impacts signs are more credible. However, Gross Domestic Product real growth rate (DLGDP) impact magnitude is still much heavier than the one of the other independent variables. Notice that DLGDP impact is twice the RLTIR and the triple of DLGDR. Considering these findings, governments have incentives to increase their debt to produce economic growth of at least one third of the magnitude increment. Moreover, a GDP growth rate impact of such magnitude leads to some tendency for countries in very effervescent expansionary phases to accumulate debt at a rapid rate. Despite previous results, we did not find plausible results to LRM accordingly to the economic theory. Due to high autoregressive estimates, the LRM came extraordinarily high. As an example, we estimate that, on average, a GDP growth rate increase of 1pp leads to an increment of 0.076 on Net Borrowing (NB). Bear in mind that NB is the Net Borrowing-to-GDP ratio, which means an increase in GDP leads automatically to a decrease of the ratio (once the denominator is growing). These excessive long-run multiplier estimates are probably caused by incomplete dynamical specification or some kind of wrong functional form. In this sense, the nonlinear model might mitigate the problem.

Lastly, we focus on marginal effects in the nonlinear model and by definition they are not constant, instead they differ according to the value of the independent variables.

Table 13: Long-run multipliers ahead for linear dynamic models

| | BB 2step | FE | POLS |
|---|---|---|---|
| DLGDP | 7.610 | 1.710 | 1.216 |
| RLTIR | -2.808 | -4.883 | -4.052 |
| DLGDR | 1.783 | 0.744 | 0.387 |
| RFIR | 0.288 | 2.857 | 2.922 |

We evaluate the SRM for DLGDP. Due to the assumed nonlinear specification, this effect will not only depend on the level of DLGDP but also on RLTIR. In Figure 4 we depict the graphical representation of the SRM considering both DLGDP and RLTIR in a range between its minimum and its maximum observed value [11].



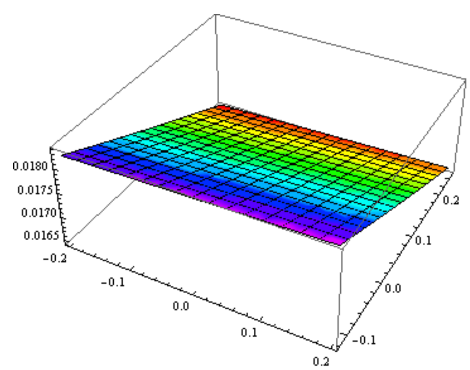




Figure 4: Nonlinear DLGDP short-run multiplier.

Notice that, DLGDP short-run multiplier is positive and quite constant due to the magnitude of its parameter estimate. In practice, this result indicate us that GDP growth has little influence on NB. This result is acceptable since the positive effect generated by the incentive to borrow is somehow countered by the fact that NB is a ratio where GDP is the denominator.

Concerning the RLTIR short-run multiplier the negative effect is much more volatile, because of the magnitude of the estimated parameter. The other independent variables

[11] To check these values look at Table 14

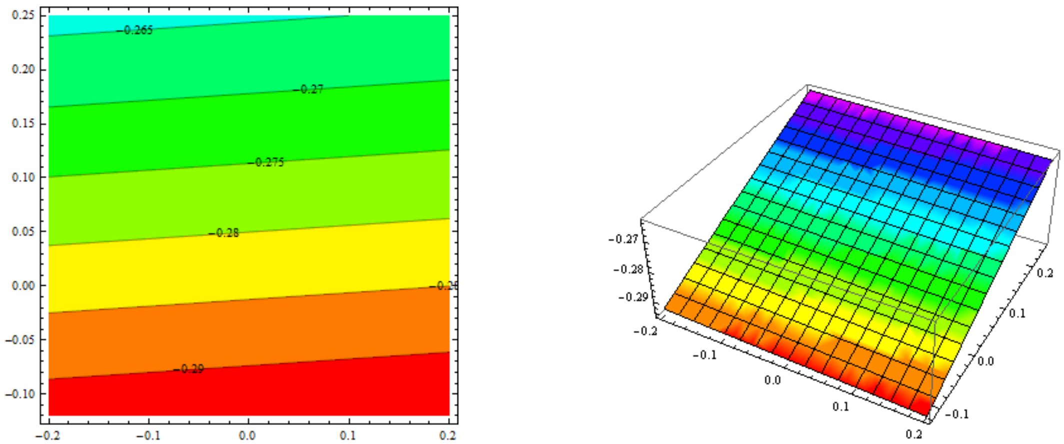



Figure 5: Nonlinear RLTIR short-run multiplier.

were included in the model in such way that SRM only depends on the values of themselves.

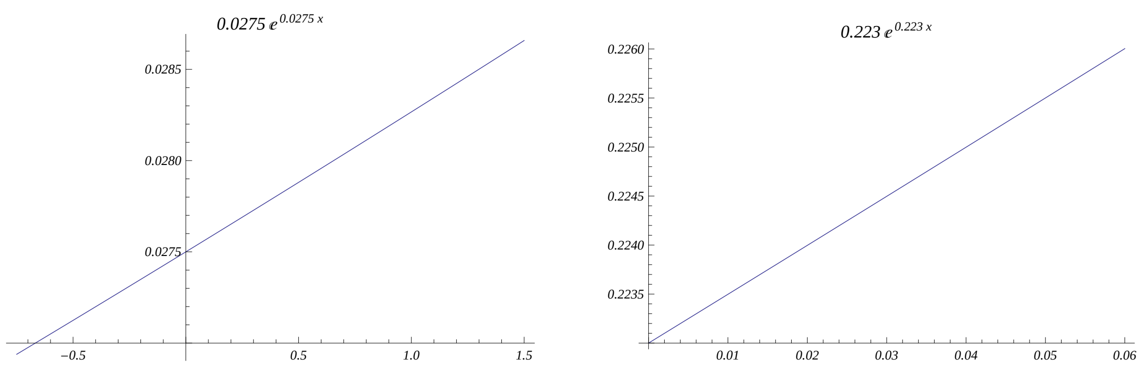



Figure 6: Nonlinear DLGDR and RFIR short-run multiplier.

To ease the analysis and comparison, we draw both graphs in Figure 6. Interesting to point out that we do not find a high influence of DLGDR in NB, which means that debt accumulation is mostly due to private sector. In relation to RFIR, we find a reasonable result that this variable influence is very constant and always positive, which may be due to the trend behavior associated with it. At last, it is important to highlight that the temporal range is short and only covers the economic cycle recessive branch. In this sense, the estimates are less accurate than if we had the whole cycle.

Finally, concerning to the long-run equilibrium, we omit the graphs as the obtained results are identical. However, commenting on these results, as discussed in Section 3.2, we did not expect significant differences between the short-run and long-run multipliers once our model was developed thinking in the long-run equilibrium. Additionally, the long-run multipliers estimates are all slightly higher in magnitude. This is economically consistent with the existence of some economic feedback contributing with a small effect to the contemporaneous stream.

# 5 Conclusions

The focus of this paper is the link between growth and financial crisis. The Solow-Swan model highlights the savings or investment ratio as an important determinant of short-run economic growth. The Solow-Swan assumptions establish an equivalence between savings and investment ratio. However, if we consider the existence of a financial international market, this link is no longer possible.

We propose a Solow-Swan augmented model considering external debt as an investment booster. Thus, the reason of the current financial crisis becomes endogenous (in economic sense) to the model and more robust results are found regarding the unsustainability of the debt situation. In fact, it may be seen that mostly when the Solow-Swan steady-state is increasing (because of the external debt accumulation) our results point out an eminent drop. Conversely, in funding economies case (for example Germany) our results are higher in general. Likewise, the financial crisis was preceded by declines in product *per capita* steady-state levels. The main limitations of our model are the inability to deal with financial arbitrage or any kind of technical progress.

Additionally, we model the propensity to borrow by using Gross Domestic Product real growth, real long-term interest rate, debt-to-GDP ratio growth and risk-free interest rate. Such formulation provides insights about the reasons and the consequences of this financial crisis. Firstly, the autoregressive coefficient estimate is very high which points out to explosive long-run multipliers in the linear model. Our results concerning nonlinear estimation exclude the idea that debt is associated with economic development state and with government budget balances. Moreover, we conclude that indebtedness is mostly influenced by private sources. Therefore, government should reinforce its regulation and supervision role in order to prevent *too big to fail* issues. On the other hand, once government budget balances do not seem to influence long-run growth, it is wise to control public expenditure to prevent investment *crowding out*. At last, long-run multipliers appear to be very similar to the short-run multipliers. This is consistent with the theoretical framework presented in Section 3.2. In addition, this model allows the dependence between the marginal effects and variables levels, which is economically very appealing.

Further improvements can be accomplished. On one hand, it would be useful to enlarge the temporal range since we focus our analysis only on a recessive phase of the economic cycle. On the other hand, we only relaxed one of the Solow-Swan assumptions and better results may be achieved if the proposed model addresses more assumptions. Namely, it would be very interesting to introduce the international cooperation (as it happens in EU) in the model. Finally, the nonlinear specification may be strengthened if we replace GDP growth rate by another economic performance variable. Also, other models may be used, for example the Error Corrector Model as in Filho et al. (2005).

# References

Arellano, M. (1993). On the testing of correlated effects with panel data. *Stata Journal*, 59:87–97.

Arellano, M. and Bond, S. (1991). Some tests of specification for panel data: Monte carlo evidence and an application to employment equations. *Review of Economic Studies*, 58:277–297.

Arrow, K. (1962). The economic implications of learning-by-doing. *Review of Economic Studies*, 29:155–73.

Baltagi, B. (2008). *Econometric analysis of panel data*. John Wiley.

Blundell, R. and Bond, S. (1995). Initial conditions and moment restrictions in dynamic panel-data models. *Journal of Econometrics*, 87:115–143.

Cameron, A., Gelbach, J., and Miller, D. (2011). Robust inference with multi-way clustering. *Journal of Business and Economic Statistics*, 29:238–249.

Cobb, C. and Douglas, P. (1928). A theory of production. *American Economic Review*, 18:139–165.

Conlisk, J. (1967). A modified neoclassical growth model with endogenous technical change. *Southern Economic Association*, 34:199–208.

Davidson, R. and MacKinnon, J. (1993). *Estimation and Inference in Econometrics*. Oxford University Press, ISBN 0-19-506011-3.

Dey-Chowdhury, S. (2008). Methods explained: Perpetual inventory method (pim). *Economic and Labour Market Review*, 2:48–52.

Drukker, D. (2003). Testing for serial correlation in linear panel-data models. *Stata Journal*, 3:168–177.

Filho, M., Silva, R., and Diniz, E. (2005). The empirics of the solow growth model: long-term evidence. *Journal of Applied Economics*, 8:31–51.

Frisch, R. and Waugh, F. (1993). Partial time regressions as compared with individual trends. *Econometrica*, 1:387–401.

Greene, W. (2003). *Econometric Analysis*. Prentice Hall, 5th edition.

Hausman, J. (1978). Specification tests in econometrics. *Econometrica*, 46:1251–1271.

Hlouskova, J. and Wagner, M. (2005). The performance of panel unit root and stationarity tests: Results from a large scale simulation study. Technical Report dp0503, Universitaet Bern, Departement Volkswirtschaft.

Jones, C. (1998). *Introduction to Economic Growth*. Norton, ISBN 0-393-97174-0.

Levin, A., Lin, C., and Chu, C. (2002). Unit root tests in panel data: asymptotic and finite-sample properties. *Journal of Econometrics*, 108:1–24.

Lewis, M. (2011). *Boomerang – Travels in the New Third World*. Norton, ISBN 978-0-393-08181-7.

Lucas, R. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22:3–42.

Milbourne, R. (1995). Growth, capital accumulation and foreign debt. *Economica*, 64:1–13.

Nickel, S. (1981). Biases in dynamic models with fixed effects. *Econometrica*, 49:1417–1426.

Nkoro, E. and KelvinUko, A. (2012). Foreign capital inflows and economic growth in nigeria: An empirical approach. *Asian Journal of Empirical Research*, 2:149–161.

Piazza, R. (2010). Growth and crisis, unavoidable connection? *IMF Staff Papers*.

Romer, P. (1986). Increasing returns and long-run growth. *The Journal of Political Economy*, 94:1002–1037.

Roodman, D. (2009). How to do xtabond2: An introduction to difference and system gmm in stata. *The Stata Journal*, 9:86—-136.

Sargan, J. (1958). The estimation of economic relationships using instrumental variables. *Econometrica*, 26:393–415.

Solow, R. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 70:65–94.

Swan, T. (1956). Economic growth and capital accumulation. *The Economic Record*, 32:334–361.

Thompson, H. (2008). Economic growth with foreign capital. *Review of Development Economics*, 12:694–701.

Villanueva, D. (1994). Openness, human development and fiscal policies: Effects on economic growth and speed of adjustment. *IMF Staff Papers*, 41:1–29.

Villanueva, D. (2003). External debt, capital accumulation and growth. *SMU-SESS Discussion Papers Series in Economics and Statistics*.

Windmeijer, F. (2005). A finite sample correction for the variance of linear efficient two-step gmm estimators. *Journal of Econometrics*, 126:25–51.

Wooldridge, J. (2002). *Econometric analysis of cross section and panel data*. The MIT Press.

# Appendices

## A Panel Data Framework

Panel data typically refer to data containing time series observations of a number of individuals. Observations in panel data involve at least two dimensions: a cross-sectional dimension, indicated by subscript $i$, and a time series dimension, indicated by subscript $t$. We consider $N$ individuals and $T$ time periods. This kind of data can be used, at least under certain assumptions, to obtain consistent estimators in the presence of omitted variables (and endogeneity).

Most of the panel data applications utilize a one-way error component model for the disturbances:

$$u_{it} = c_i + v_{it} \tag{A.1}$$

Where $c_i$ denotes the *unobservable* individual specific effect and $v_{it}$ denotes the remainder disturbance. The $v_{it}$ change across $t$ and $i$ so are called *idiosyncratic errors* (Baltagi, 2008). But it is also possible to consider other disturbances compositions:

$$u_{it} = c_i + \lambda_t + v_{it} \tag{A.2}$$

Where $\lambda_t$ represents the *unobservable* time effect on the dependent variable. The basic unobserved effects model for the cross section observation $i$ can be written as:

$$y_{it} = \underline{x}_{it}\underline{\beta} + c_i + v_{it}, \quad t = 1, 2, ..., T \quad i = 1, 2, ..., N \tag{A.3}$$

Before the estimation of the model coefficients, a choice about $c_i$ is needed as it may be a *fixed* or a *random* effect to be estimated. In modern econometric terms, a random effects framework is synonymous with zero correlation between the observed explanatory variables and the unobserved effect: $cov(\underline{x}_{it}, c_i) = \underline{0}$. In contrast, when we assume a fixed effects framework it means that we allow some dependence between the unobserved effect $c_i$ and the observed explanatory variables $\underline{x}_{it}$ (Wooldridge, 2002).

When using fixed effects, it is possible to estimate the coefficients by ordinary least squares. The only issue is how to overcome the endogeneity due to the individuals heterogeneity (individual unobserved effect $c_i$). A possible solution is to include individual dummies (least squares dummy variable) or by using least squares on the regression of $y^*_{it} = (y_{it} - \bar{y}_{i.})$ on the same transformation of $\underline{x}_{it}$ (within estimator). It can be proved by employing the Frisch and Waugh (1993) theorem that these alternative methods reach the same results. By differentiation we can eliminate the individual unobserved effect of the variables. However, this procedure induces serial correlation into the resulting disturbance. Summarizing, the fixed effects approach has the advantage of allowing the correlation between $c_i$ and $\underline{x}_{it}$ but the estimation of such large number of coefficients leads to the loss of degrees of freedom. Despite this estimator being consistent to $\underline{\beta}$, it will not be efficient (Greene, 2003). Note that there is no consistency guarantee of the estimator of $c_i$ since it only uses $T$ observations (and $T$ might be small).

The random effects model is a generalized linear model which can be estimated by two step feasible GLS. On the first step we consider different combinations of the residual variances from the linear model (with no effects, with group mean regression or with dummies). Thereafter, FGLS is carried out by using the first step estimated variance to mimic the variance of the GLS estimator of $(y_{it} - \theta_i \bar{y}_{i.})$ on the same transformation of $\underline{x}_{it}$, where $\theta_i = 1 - \sqrt{\frac{\sigma_v^2}{T\sigma_\alpha^2 + \sigma_v^2}}$. The main advantage of this model is the inclusion of only one parameter, benefiting the estimator efficiency. Nevertheless, the hypothesis of no correlation between $c_i$ and $\underline{x}_{it}$ is likely to fail, turning the estimator inconsistent (Greene, 2003).

# B Data and variables in analysis

Table 14: Descriptive statistics

| Variable | | | Mean | Std. Dev. | Min | Max | Obs, |
|---|---|---|---|---|---|---|---|
| Net Borrowing | NB | overall | -.0075 | .0519 | -.1464 | .1801 | N = 263 |
| | | between | | .0466 | -.0887 | .0833 | n = 15 |
| | | within | | .0278 | -.0904 | .0893 | T-bar = 17.53 |
| GDP real growth rate | DLGDP | overall | .0216 | .0281 | -.0854 | .1150 | N = 270 |
| | | between | | .0104 | .0080 | .0506 | n = 15 |
| | | within | | .0263 | -.0908 | .0859 | T = 18 |
| Real long-term interest rate | RLTIR | overall | .0288 | .0261 | -.0414 | .2350 | N = 270 |
| | | between | | .0071 | .0135 | .0459 | n = 15 |
| | | within | | .0252 | -.0262 | .2178 | T = 18 |
| Government debt-to-GDP ratio | GDR | overall | .6583 | .2935 | .0607 | 1.6541 | N = 270 |
| | | between | | .2660 | .0961 | 1.1268 | n = 15 |
| | | within | | .1410 | .3560 | 1.2842 | T = 18 |
| 3-month Treasury bill interest rate | RFIR | overall | .0290 | .0208 | .0005 | .0582 | N = 270 |
| | | between | | 0 | .0290 | .0290 | n = 15 |
| | | within | | .0208 | .0005 | .0582 | T = 18 |