LISBON
SCHOOL OF
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UNIVERSIDADE DE LISBOA

MASTER OF SCIENCE IN
FINANCE

MASTER´S FINAL WORK
DISSERTATION

CAN MODEL-BASED FORECASTS PREDICT STOCK MARKET VOLATILITY USING RANGE-BASED AND IMPLIED VOLATILITY AS PROXIES?

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Abstract

This thesis attempts to evaluate the performance of parametric time series models and RiskMetrics methodology to predict volatility. Range-based price estimators and Model-free implied volatility are used as a proxy for actual ex-post volatility, with data collected from ten prominent global volatility indices. To better understand how volatility behaves, different models from the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) class were selected with Normal, Student-t and Generalized Error distribution (GED) innovations. A fixed rolling window methodology was used to estimate the models and predict the movements of volatility and, subsequently, their forecasting performances were evaluated using loss functions and regression analysis.

The findings are not clear-cut; there does not seem to be a single best performing GARCH model. Depending on the indices chosen, for range-based estimator, APARCH (1,1) model with normal distribution overall outperforms the other models with the noticeable exception of HSI and KOSPI, where RiskMetrics seems to take the lead. When it comes to implied volatility prediction, GARCH (1,1) with Student-t performs relative well with the exception of UKX and SMI indices where GARCH (1,1) with Normal innovations and GED seem to do well respectively. Moreover, we also find evidence that all volatility forecasts are somewhat biased but they bear information about the future volatility.

Keywords: Implied Volatility, Range-based Volatility, GARCH, Forecasting Accuracy, Information Content.

JEL Classifications: C22, C52, C53, C58, C87

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Table of Contents

LIST OF TABLES AND FIGURE

List of tables

List of Figure

1. Introduction

Volatility is one of the most studied topics in modern finance. The ability to correctly forecast future volatility has been the interests of anyone who is involved in the financial market. In Finance, volatility is defined as the fluctuation of asset prices from its mean over a specific period of time, which is commonly calculated as standard deviation of its logarithmic returns.

Although, volatility does not translate the full risk in the market, but it is a good representation of the significant portion of risk that can be quantified. Therefore forecasting volatility is crucial in investment decision-making, valuing derivative products, risk management and portfolio hedging. For example, in the notorious Black-Scholes (1973) equation based on the option-pricing model, volatility of the underlying asset over the life of the option is a fundamental input in the determination of the fair value of the derivative products. However volatility cannot be directly observed, but rather needs to be estimated, which again highlights the importance of being able to accurately forecasting volatility of underlying asset.

As a result, it comes as no surprise that forecasting market volatility has received a great deal of attention in recent times both in academia and among financial practitioners. The aim of this thesis is to evaluate the forecasting accuracy of RiskMetrics and various GARCH-type models against future volatility of ten globally traded equity indices. In this paper, both range estimators based on daily price information and model-free implied volatility, which is said to be the market participant's expected future volatility, are used as

proxies for the actual ex-post volatility in the markets. This paper also investigates the information content of the most accurate model forecasts from each index to see if they indeed contain some additional information about future volatility.

Despite the vast amount of literature on the subject of volatility forecasting, it is rather difficult to draw a unanimous conclusion due to differences in research design in terms of asset classes, countries, sample period, forecasting techniques, forecasting horizon and evaluation measures. This thesis attempts to get a clearer picture of the performance of time series models by incorporating 10 globally traded indices within a common framework.

Many studies attempt to forecast future realized volatility using model-based forecasts such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and others attempt to model volatility by using option implied volatility forecasts. The results are somewhat mixed mainly due to difference in research methodology. Poon and Granger (2003) compiled a detailed literature review on this subject and concluded that overall implied volatility seems to contain more information about future volatility than the time series forecasts and it outperforms all other models. When it comes to time series forecasts, models that take into account volatility asymmetric response to negative and positive news tend to outperform others.

In the work of this thesis, a variety of GARCH-type models and RiskMetrics along with Normal, Student-t and Generalized Error Distribution (GED) innovations are fitted through a fixed rolling window methodology to capture the movement of range-based and model-free implied volatility of 10 international equity indices. Afterwards, each individual model's out-of-sample predicting performance is evaluated based on a number of

forecasting accuracy metrics to see which model tends to outperform the others. As proxies for the actual volatility we use range-based Parkinson (1980) estimator, which is said to be 5 times more efficient than the squared returns (Garman & Klass, 1980). We also considered the implied volatility based on the model-free variance swap concept, which improves on Black-Scholes implied volatility by addressing the constant volatility assumption and symmetric Gaussian return assumption (Siripoulos & Fassas, 2009).

Though the results are mixed, but conclusions can be made. In forecasting volatility using Parkinson (1980) range-based estimator as a proxy, asymmetric GARCH(1,1) model such as APARCH (1,1) with Normal distribution are able to better capture the dynamics of volatility with exception of HSI (China) and KOSPI (Korea) where RiskMetrics excel. This seems to be in line with the conclusion from Poon and Granger (2003) that time series models that allow asymmetric effects perform well overall. However, when it comes to forecast volatility using implied volatility as proxy, simple GARCH (1,1) with Student-t innovation seems to be the one with better performance overall.

The content of this thesis is structured as follows. Section 2 presents relevant literature review of the volatility forecasting using both model-based and model-free implied volatility approaches. Theoretical concepts of volatility are presented in section 3 and section 4 elaborates on the methodologies and data used to carry out this research. The main findings of this research along with analysis are presented in section 5. The main conclusions and future research suggestions are detailed in section 6.

2. Literature Review

There is a wide variety of literature on the topic of volatility forecasting; in which some are focused more on model-based forecasting while others attempt to evaluate the forecasting accuracy of several models with implied volatility.

Earlier studies attempted to capture the dynamics of volatility using the autoregressive moving average model (ARMA) and Box-Jenkins ARMA yield relatively low forecasting accuracy due to many of the characteristics of volatility such as the clustering effect, asymmetric response to shocks, and heteroscedastic nature of the residual terms. (Tsay, 2010) Later on, ARCH model of Engle (1982) was developed to capture clustering effects of volatility and its non-linear dynamics. ARCH model estimates conditional variance as a function using a number of lags of its past squared residuals. One of the shortcomings of ARCH model is it might require many lags in the conditional variance equation to better capture the dynamics of volatility, and it also means many coefficients would have to be estimated. Bollerslev (1986) and Taylor (1986) independently came up with a generalized version of ARCH called GARCH model, which limits the number of estimated parameters and it is said to be more parsimonious than the ARCH model. (Brook, 2008) Subsequently, many variations of ARCH/GARCH models were developed to capture the other stylized facts of volatility such as asymmetric response to shocks.

Akgiray (1989), one of the earliest researches to test the predictability of GARCH model concludes that GARCH consistently outperforms historical volatility and exponential weighting moving average models. Since then there have been numerous

researches testing the predictability of the GARCH model against other time series and implied volatilities mostly in major stock indices and foreign exchange rates. Cumby, Figlewski and Hasbrouck (1993) introduced Exponential GARCH model and concluded that EGARCH outperforms historical volatility model despite the low R^2. Figlewski (1997) concludes that GARCH model's adequate performance is mostly restricted to stock market data and only for short horizon forecasting. There are many other studies that give mixed conclusions. As the methodology used in conducting research varies, the results could also be different. There are many factors that can affect the outcome of the study; such as different loss functions used in the evaluation, different sampling methodology (fixed rolling window estimation or recursive expanding estimation), or even different sample period for different asset could lead to rather different conclusion.

In a model-based volatility comparative research, Brownlees et al. (2011) evaluate the forecast accuracy of the ARCH family models with different horizons and study how the predictability can be affected by factors such as estimation window length, different distribution assumptions and re-estimation frequency for the parameters. The authors include a wide range of asset classes including numerous domestic and international equity indices as well as exchange rates. The models that the authors include in the study are: GARCH (1,1), TARCH, EGARCH, NGARCH, and Asymmetric power ARCH. The loss functions are Quasi-likelihood (QL) and mean square error (MSE) but the authors focused on QL and argued that QL's bias is independent of volatility level while MSE's bias is proportional to the true variance squared. In addition to the daily dividend adjusted log return data on S&P 500 index from 1990 to 2008, the authors also use 10 exchange rates, 9 domestic indices and 9 international indices and the out-of-sample forecasting period spans

from 2001 to 2008 covering period of both low volatility and crisis. First, forecasting performance of S&P 500 is assessed against daily-realized volatility and squared returns over a range of horizon and subsequently, a direct comparison of forecasts from the GARCH models during the full sample period with only the crisis period of fall 2008 are conducted and the results of the out-of-sample QL losses are reported using TARCH (1,1). The results show that asymmetric models such as TARCH model performs relatively well across asset classes, methods and sample periods including period of distress. The authors conclude that use the longest date series available seem to enhance the model performance and weekly parameter re-estimation is ideal to combat the parameter drifting. Innovation distribution such as student–t does not yield any improvement in predictability of the model. For period of extreme high distress such as fall 2008, short horizon forecast such as 1 day ahead forecast is able to capture the dynamic of volatility; the problem lies with long horizon forecasts (multistep forecasts).

Using implied volatility as a forecast of market's expectation of future volatility has also gain popularity. Implied volatility is derived from Black-Scholes' (1973) option pricing formula using the backward induction as all the inputs in the formula can be either observed or computed with the exception of volatility. However, Black-Scholes' implied volatility suffers from discrepancy of volatility smile; where implied volatilities computed from options on the same underlying with the same maturity but different exercise prices yield different results that violate the theory, which states volatility is assumed constant over the life of the option. Many researchers decide to use At-the-money option implied volatility due to its liquidity and large trading volume, which can correct some of the market microstructure concerns. Despite its shortcomings, many articles claim implied

volatility can better predict realized volatility than their time series counterparts. (Lamoureux and Lastrapes, 1993; Vasilellis and Meade, 1996) Since then there are numerous studies on predictability of implied volatility index (original "VIX", now titled VXO) from the Chicago board of Option Exchange (CBOE). Many studies such as Fleming et al. (1995) use the old CBOE's "VIX" implied volatility index based on the options of S&P 100 to forecast true volatility of equity index. Most studies seem to confirm that implied volatility contains crucial information about the future volatility. Fleming et al (1995), in addition to confirming that implied volatility performs better when comparing to first order autoregressive volatility models and also discover the strong inverse and asymmetric relationship between the VXO implied volatility and its S&P 100 market price. Blair et al (2001) report the highest explanatory power of VXO implied volatility to the S&P 100 index and reach the similar conclusion that implied volatility seems to perform better than the model-based counterparts. In addition to Blair et al. (2001), Lamoureux and Lastrapes (1993), Canina and Figlewski (1993), and Fleming et al. (1995) all find implied volatility biases in their forecast of realized volatility.

Despite of its promising results from U.S stock market indices, many international, smaller indices seem to have mixed results. Frennberg and Hanssan (1996) study the Swedish market and find that implied volatility in fact, cannot outperform even the simple autoregressive and random walk model. Australian market study of implied volatility conducted by Brace and Hodgson (1991) yields very inconsistent forecasting outcomes. While Doidge and Wei (1998) find combination of GARCH and implied volatility to be ideal in forecasting the Canadian Toronto index volatility.

Poon and Granger (2003) in a comprehensive volatility forecasting review summarize 93 studies on the matter of volatility forecasting and conclude that overall with mixed results, implied volatility seems to outperform other volatility forecasters that includes historical volatility model, random walk, autoregressive, moving average and exponential weights as well as GARCH/ARCH family models. Moreover, the authors report that time series models that take account of asymmetric response in volatility seems to perform better compared to others, such as EGARCH and TGARCH model.

In a study conducted by Martens and Zein (2004) in which the authors incorporate high frequency intraday data and long memory models to forecast volatility of 3 different asset classes: equity, foreign exchange and commodities. Data sample from S&P 500, Yen/USD, and Sweet crude oil start from beginning of 1994, 1996 and June 1993 respectively span to the end of 2000 from various sources. Implied volatility is calculated using the weighted average of two nearest at-the-money calls and puts and weights are selected where the average exercise price matches the underlying future prices. Realized volatility is calculated using the sum of squared intraday return rather than the standard squared daily return to avoid possible noise. Autoregressive fractionally integrated moving average is used to estimate log-realized volatility in addition to the GARCH (1,1) and recursive expanding rolling estimation method is used with initial in sample period of 500 observations. The loss function of Heteroskedasticity consistent Root Mean Squared Error is computed to evaluate the forecasting performance of the models. Implied volatility outperforms GARCH models and in encompassing analysis, implied volatility also subsumes mostly all the information content. Interestingly, the authors find long memory

model able to compete with implied volatilities and in some cases even outperforms implied volatilities. Both measures contain information that the other does not possess.

In a more recent study, Ryu (2012) investigates the information content and forecasting accuracy of the implied volatility index of KOSPI (South Korea) against RiskMetrics, Black-Scholes' implied volatility and GJR-GARCH models. He argues that since option market of KOSPI ranks highest in terms of trading volume and investor's interest, therefore the implied volatility index extracted from option prices should contain predictive information about future volatility. The implied volatility index of KOSPI (VKOSPI) is computed using the model-free methodology based on the concept of fair value variance swap and does not rely on any option pricing models; its calculation resembles the new VIX index from the S&P 500 U.S equity index. The total sample size contains 2,057 daily observations and using fixed rolling analysis with forecasting horizon of 1, 5, 10, 21, and 63 trading days and finally the results are evaluated using the Mincer-Zarnowitz decomposition of Mean Square Error. Ryu (2012) concludes that implied volatility index (VKOSPI) contains meaningful information about future volatility of KOSPI and it outperforms all other forecasters in predicting realized volatility. Moreover, when the forecasting horizon is 5, 10, and 21 trading days, more than half of the changes in realized volatility can be explained by the VKOSPI index. In studying the relationship between the volatility index and its underlying equity return, the author confirms the asymmetric inverse relationship between the two, which is a well documented in the literature. (French et al., 1987; Schwert, 1989,1990; Fleming 1995)

Shaikh and Padhi (2014) attempt to study the forecasting performance of Indian volatility index along with RiskMetrics GARCH, and GJR-GARCH (1,1) using both

overlapping and non-overlapping sampling procedure with roughly of 6 years span of daily data with forecasting horizon of 1, 5, 10, 22, and 66 days. Indian VIX index uses the same model-free implied volatility methodology as many of the global volatility indices and realized volatility is computed based on the sum of the squared returns. The performance measure is based on the loss functions of RMSE, MAE and Theil's U statistics. For non-overlapping samples with exception of 1- day and 66-day forecasts, implied volatility outperforms other models and following by RiskMetrics. GJR-GARCH (1,1) seems to dominate the overlapping sample methods. Implied volatility also contains more information about the future volatility than the other forecasts, especially in the case of 10-day and 22-day horizon showing the highest adjusted R^2. Interestingly, the authors conclude that based on the univariate and encompassing regression, implied volatility dominates other forecasts and it is unbiased and efficient estimator of the market volatility.

In an international-focused, comparative study of implied, realized and GARCH volatility forecasts conducted by Kourtis et al. (2016), where the authors use 13 global indices from 10 countries with different forecasting horizon (1, 5, 22 days) and under different market conditions (before, during and after crisis of 2008) to study the forecasting performance and information content of model-free implied volatility, random walk, GJR-GARCH and Heterogeneous Autoregressive (HAR) model. Daily realized volatility is calculated using square root of sum of intraday returns collected at 5 min equally spaced intervals and the data spans for roughly 12 years from 2000 to October 2012. Out-of-sample evaluation is illustrated by using the loss functions of RMSE and QLIKE and the results show that for daily horizon forecasts, HAR performs better while for weekly horizon, MFIV adjusted for risk premium (C-MFIV) and HAR are comparable and finally

10

monthly forecasting horizon shows that C-MFIV has the lowest forecasting errors. Separate

out-of-sample evaluation is conducted for pre-crisis, crisis and after crisis period and the

results show that while all models deteriorate during the crisis period, HAR and MFIV-C

are better models compared to the rest in daily and monthly horizon. The authors also find

that HAR has the greatest explanatory power for daily horizon, while C-MIFV contains

more information about future realized volatility for monthly horizon.

3. Theoretical Concepts of Volatility

As volatility cannot be directly observed in the market, but rather needs to be

estimated from market indicators, and as this is the main focus of this thesis, it is instructive

to take a deeper dive into the concepts of volatility and its common features.

3.1 Various Types of Volatility

3.1.1 Historical Volatility

There are many ways of calculating volatility in the financial world. Volatility is a

statistical measure of the variation of the return over time for a given security or equity

index and is usually expressed as the standard deviation of the log returns. Volatility is used

as a form of risk measurement and is calculated using the formula below, where $\hat{\sigma}$ is the

sample standard deviation (volatility), r_t is the return observed and $\bar{r}$ is the mean return.

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \tag{1}$$

In this thesis and as it is standard in the financial econometrics literature, the return r_t is calculated under the continuous compounding framework as log difference of price P at time t and price at the previous period P_{t-1}, which gives the following formula.

$$r_t = \ln \left(\frac{P_t}{P_{t-1}}\right) \tag{2}$$

3.1.2 Realized Volatility

When evaluating the forecasting accuracy of various potential predictors it is crucial to select a good proxy for the true ex post volatility. One option is to use historical volatility, which typically uses daily closing price, but a lot of intraday information could be lost using historical closing prices (Andersen & Belzoni, 2008). Other options are to use squared returns or squared residuals from an ARMA model fitted to r_t but it is well known that these are very noisy estimators for daily variance (Andersen and Bollerslev, 1998). A frequently used proxy is the realized variance, which is calculated as the sum of the squared intraday returns sampled at equal time intervals. (Andersen & Bollerslev, 1998) However, sometimes, intraday price levels could be costly and hard to obtain. As a more viable alternative, we use the range estimator based on daily price information of Parkinson (1980) who concluded that a log function of daily high and low price range is also an unbiased estimator of daily volatility and it is said to be 5 times more efficient than computing daily volatility using the daily closing price (Shu & Zhang, 2006). For these reasons, we use Range-based Estimator as a Proxy for Realized Volatility, which are briefly discussed next.

3.1.3 Range-based Estimator as a Proxy for Actual Volatility

As high-frequency intraday prices may not be readily available, one of the advantages of using price range estimator is that for many assets, daily high, low, opening and close prices are easy to retrieve. We use Parkinson's equation (1980) with daily highest and lowest price, which serves as a proxy for the true volatility in this study and it is calculated as follows:

$$\widehat{\sigma_t^2} = \frac{1}{4 \ln 2} * (H_t - L_t)^2 \tag{3}$$

where H_t and L_t are the highest and lowest price of the t-th trading day, respectively.

3.1.4 Option Implied Volatility as a Proxy for Actual Volatility

Option implied volatility is generally defined as the expected market participant's assessment of future volatility of the underlying asset during the life of that option. It is viewed as a forward looking measure of the volatility, due to the fact that it is based on the prices of the actively traded option observed in the market over the remaining life of that option.

3.1.4.1 Black-Scholes' Implied Volatility

The most well-known implied volatility measure is based on the option-pricing model developed by Black and Scholes (1973). Under Black-Scholes framework, the behavior of the stock price, denoted as S, follows the following Geometric Brownian motion where μ is the drift term (percentage expected rate of return), σ is the diffusion term (percentage standard deviation) of the stock, both are assumed to be constant. The variable dt is change in small period of time t and dz follows a Wiener process $dz = \epsilon\sqrt{dt}$ with $\epsilon \sim N(0,1)$. The left side of the equation (4) below represents the return generated by the

stock for a short period of time and it implies the return of stock is normally distributed

with mean of μdt and variance of $\sigma^2 dt$ (Hull, 2014).

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{4}$$

Based on Ito lemma process, the log stock price follows a generalized Wiener process with

the following characteristics:

$$dln(s) = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dz \tag{5}$$

Where the drift rate is $\mu - \sigma^2/2$ and variance rate is σ^2. It shows that log stock price has a

normal distribution and it infers that the stock price is lognormally distributed. (Poon &

Granger, 2003)

Using the Ito lemma process along with no-arbitrage argument that return of the stock must

be at the risk-free rate lead to the Black-Scholes (1973) differential equation for pricing

derivatives. The inputs for equation to price a call, C and put, P option are r: Risk free rates;

T: time to mature; S: Stock price; K: strike price and σ: Volatility.

$$C = s_0 N(d_1) - Ke^{-rT}N(d_2) \tag{6}$$

$$P = Ke^{-rT}N(-d_2) - S_0 N(-d_1) \tag{7}$$

Where $N(d_x)$ is the function of the cumulative probability distribution function under the

assumption of standard normal distribution.

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{8}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{9}$$

Black-Scholes implied volatility could be extracted through backward induction method

given the price of call or put options can be observed in the marketplace. However, such

calculations involve a number of assumptions. The key assumptions are: stock price follows a geometric Brownian motion with constant mean and volatility, short selling is unrestricted, absence of transaction costs and taxes, absence of arbitrage opportunities and risk-free rate is constant.

3.1.4.2 Model-Free Implied Volatility (MFIV)

As Black-Scholes option pricing model applies many assumptions in deriving the fair asset price, implied volatility inevitably suffers from model restrictions and assumptions as well. Two of the noticeable shortcomings of the model are the assumption of constant volatility and symmetric Gaussian distribution of the return of the underlying assets assumptions. Volatility smile is one of the problems that arise from the constant volatility assumption. Implied volatilities calculated from options on the same underlying asset with the same maturity with different exercise prices should be the same under the constant volatility assumption. However, in practice, implied volatilities usually are higher for In-the-money and Out-of-money options compared to the At-the-money options (Hull, 2014). Model-free implied volatility (MFIV) improves on Black-Scholes model by addressing the two potential shortcomings. Model-free implied volatility methodology was adopted by the Chicago Board Option Exchange in 2003 for its VIX (CBOE Volatility index) derived based on the S&P 500 option market prices by averaging call and put option prices from a wide range of strike prices. It is constructed to provide market's expectation for implied volatility of S & P 500 index over the next 30 calendar (~22 trading) days and is expressed in annualized percentage format (Chicago Board of Options Exchange, 2003).

Many global exchanges soon based their volatility index computation on the model-free VIX methodology such as Deutsche Borse , Euronext, Swiss exchange etc.

The new VIX model – free implied volatility calculation is based on the concept of over-the-counter variance swaps (Demeterfi et al., 1999a), which are forward contract with no initial cash required. In a variance swap, both parties enter into a contract to swap the realized variance rate between the start of contract to the expiration for a pre-determined variance rate of the underlying asset. According to Hull (2014), computationally, valuing variance swap is easier than volatility swap due to the ease of replicating the variance rate between the start of the contract and expiration with a portfolio of call and put options. The payoff of the swap can be replicated by the payoff strategy of static options portfolio along with delta hedging the underlying asset. This implies that the fair value of the promised claim to pay for the future variation of index return is given by the market of the replicating options portfolio (Neuberger, 1996;Dupire B. 1994, 2004). Since this reasoning does not depend on any model such as Black-Scholes (1973), it is called model-free implied variance and its square roots are called Model-free implied volatility.

The new VIX index introduced in 2003 closely approximates one-month swap rate and is annualized in percentage term. It is calculated based on the quoted prices of a wide variety of Out-of-money European calls and puts written on the underlying S&P 500 equity index (CBOE, 2003).

$$VIX = 100 * \sqrt{\frac{2}{T}\sum \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\{\frac{F}{K_0} - 1\}^2} \qquad (10)$$

T: Time to maturity measured in minutes.

$T = (M_{today} + M_{Settlement_{days}} + M_{Remaining_{days}})/$Minutes in a year

M_{today} = Minutes remaining till midnight today

$M_{Settlement_{days}}$ = Minutes from midnight till settlement day

$M_{Remaining_{days}}$ = Total minutes from today to settlement day

F: Forward option level from the option prices

$$F = Strik + e^{RT} \times (Call\ price - put\ price)$$

K_o : First strike price below the Forward option level

K_i: Out-of-money option strike price – For calls, K_i>F and for puts, $K_i < F$

ΔK_i: Difference between strike prices. Half the difference of strike price above and below K_i

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

R: risk-free interest rate to maturity of the contract

$Q(K_i)$: The midpoint of bid-ask spread for each option with exercise price K_i

In this thesis we use volatility indices computed based on the Model-free variance swap concept called Model-free implied volatility as a proxy for actual volatility.

3.2 Characteristics of Volatility

In order to effectively model future volatility, it is crucial to take some of the empirically documented features of volatility into consideration. These common well-known features are:

- **Volatility Clustering** – Period with high volatility tends to continue with rather high volatility while period with low volatility tends to stay the same. This implies volatility is not constant and it is time varying. All GARCH models capture this

stylized fact and it is one of the main distinctions of GARCH models from simple ARMA models. Therefore in this paper, we focus mainly on GARCH variations of time series forecasts.

- **Persistence of Volatility/Long Memory Effect** - Autocorrelation coefficients of the variance decays very slowly and persists even after up plenty lags. Poon and Granger (2005) in an example, illustrate for S&P 500 volatility of series return during period 1983 – 2003, the autocorrelation coefficient remain significant and positive after 1,000 lags. This suggests that price shocks tend to have a long lasting effect on volatility. The partial autocorrelation of variance tends to be long in extent as well though not nearly as long as the autocorrelation lags. All GARCH models seem to have the capabilities to model this phenomenon while some might perform better than others.

- **Mean Reversion of Volatility** - It is believed that after some period of time, volatility will move back to its average level historically observed.

- **Asymmetric Response to Price Shocks/Leverage Effect** - Volatility tends to respond stronger to a negative shock than a positive shock of same magnitude. This phenomenon is also referred to as the leverage effect. As the price of stock drops, it would increase the debt-to-equity ratio of the company leading to higher leverage of the firm and as a result it would increase the risk of firm's equity, which is reflected by increasing volatility observed in the stock market. In this paper, we include asymmetric GARCH models such as EGARCH; TGARCH and APARCH to capture this stylized fact of volatility.

- **Leptokurtic tail and Negatively Skewed of its Return Distribution** – Return series distribution is said to have higher positive Kurtosis (>3) value meaning heavy tailed with high peak around the mean and a longer left tail compared to its right tail.

- **Weak/Low Autocorrelation in Return Series** – According to the Market efficiency theory, stock prices should already contain past information, therefore models such as ARMA using its past lags rarely add any value to the predictability of the forecasts. Therefore in this thesis, we assume the mean return to be 0.

4. Data and Methodology

4.1 Data Collection and Forecasting Method Description

There are two types of data involved in carrying out this forecasting study. First, daily price range (High, Low, Opening and Close) of 10 major equity indices were collected for the sample period spans from 1/1/2005 to 12/31/2016 using Bloomberg, which roughly equals to 3000 observations with some indices have slightly less observations. Actual daily volatility is estimated using the range-based estimator of Parkinson's equation (3).

Subsequently, the model-free implied volatility levels from 10 indices described in sections 3.1.3.2 were also retrieved based on the same time span from Bloomberg. This ensures consistency of the data from which the estimator for daily actual volatility is based. Model-free implied volatility is based on the prices of the liquid traded options and it represents the expected market future volatility of underlying equity index over the next 30

(22 trading) days and annualized in percentage terms. In order to forecast the volatility at different horizon, rescaling needs to be applied so that it can predict n-day volatility. To rescale the implied volatility, the following formula is used: $MFIV_{t+n|t} = (\sqrt{\frac{n}{250}} IV_t)/100$ where IV_t represents daily closing price of volatility index and $MFIV_{t+n|t}$ is the n day horizon option model-free implied volatility forecast. As index price is expressed in percentage terms, we divide it by 100 to convert to decimal format. Table I illustrates the 10 equity indices used in this study, its implied volatility indices, country of origin and exchanges traded in.

Table I: List of equity indices along with its volatility indices

Equity Index	Volatility Index	Region/Country	Stock Exchanges
SPX (S&P 500)	VIX	US	New York Stock Exchange
NKY (Nikkei 225)	VXJ	Japan	Tokyo Stock Exchange
UKX (FTSE 100)	VFTSE	UK	London Stock Exchange
DAX	V1X	Germany	Frankfurt Stock Exchange
SMI	V3VI	Switzerland	SIX Swiss Stock Exchange
HSI (Hang Seng)	VHSI	China	Hong Kong Stock Exchange
NDX (NASDAQ)	VXN	US	New York Stock Exchange
SX5E (EURO STOXX 50)	V2V	Europe	Multiple Exchanges Europe
KOSPI	VKOSPI	Korea	Korea Stock Exchange
CAC	VCAC	France	Euronext Paris

The entire sample from each index is divided into in-sample estimation period and out-of-sample forecast period. The unknown parameters of the chosen forecasting models are estimated using the fixed rolling window method of 1,000 in-sample observations at a time and are subsequently rolled one step forward while excluding the oldest observation to make new estimation. This procedure is repeated until it goes through the entire sample. For example, the first forecasted value that corresponds to sample sequence 1,001 is estimated through in-sample observation from 1 – 1,000, and subsequently, the second

forecasted value that corresponds to sample sequence 1,002 is estimated by rolling the fixed

window of 1,000 observations one step ahead, which now includes in-sample observations

2 – 1,001 excluding the first observation and including the 1,001 observation. This

procedure is repeated until we reached the last forecasted value for the entire sample.

Hence instead of identifying the time series model for each index with Box-Jenkins

methodology, this thesis takes on a different approach for model selection. A variety of the

most popular GARCH models are selected with various innovation distributions, a fixed

window approach is applied and the goal is to see which GARCH model forecasts the 1-

step ahead volatility best in the period under analysis.

4.2 Volatility Forecasting Models

This section goes in detail to describe the forecasting models used to estimate and

predict true volatility using both Parkinson (1980) Price range estimator and MFIV as

proxies.

4.2.1 GARCH

As briefly mentioned, Engle (1982) developed ARCH to capture the clustering

effect of volatility and its nonlinear nature. ARCH model consists of conditional mean and

conditional variance equation. In the mean equation, μ_t represents its conditional mean

return at time t and a shock/residual term, ε_t. In this thesis, the main focus is to model its

conditional variance not its mean return and we assumed that μ_t=0. It is not unreasonable to

assume the mean return is zero; in fact, many of the considered log-return series have minor

or almost no autocorrelation. Here z_t is a series of independent and identically distributed

random variable takes the mean of 0 and variance of 1 with restriction that $\alpha_0 > 0$, $\alpha_i \geq 0$

for $i = 1, \dots, m$.

$$R_t = \mu_t + \varepsilon_t \,, \varepsilon_t = \sigma_t z_t \,, z_t \sim \text{i.i.d } (0,1) \tag{11}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_m \varepsilon_{t-m}^2 \tag{12}$$

In the variance equation, the ARCH effects come from its dependence on the squared

lagged errors or shocks. It is the shock term that mainly explains the volatility of the asset

return and it's clear to see that larger past squared shocks infer that the σ_t^2 tend to be large

as well (Tsay, 2010). Moreover, variance of the disturbance term is not constant but rather

time varying, which is why ARCH is a good model to explain the heteroscedastic and

clustering features of volatility. However, to fully capture the dynamics of the volatility, it

might need many of its lagged squared residuals in the conditional variance equation and

thus requiring many parameters to estimate.

The GARCH model developed by Bollerslev (1986) and Taylor (1986) overcomes

this limitation and is said to be more parsimonious than ARCH model; it requires fewer

parameters to capture the complete volatility dynamics. The mean equation is the same as

equation (11) and for consistency it is assumed conditional mean return to be zero as well

for this thesis. In the conditional variance equation, the current period variance not only

depends on its previous lagged squared residuals but also its own previous conditional

variance lags. By incorporating its own lagged value into variance calculation it is believed

to capture the volatility persistence characteristics better than ARCH type models. The

GARCH model is described by the following equation:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{13}$$

Where α_i and β_j are the ARCH and GARCH parameters respectively and m, and n

represent number of lags of the squared error and its previous conditional variance

respectively. The restriction applied here $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, $\sum_{i=1}^{\max(m,n)}(\alpha_i + \beta_i) < 1$.

The constraint on parameters α_i and β_i infers that unconditional and conditional variance

of squared residual is positive finite (Tsay, 2010).

4.2.2 RiskMetrics methodology

J. P. Morgan introduced RiskMetrics ™ methodology to calculate Value-at-risk,

another risk measure. RiskMetrics assumes that the continuous daily return follows a

Normal distribution and it can be shown to be a restricted GARCH (1,1) process without

drift term and α typically around 0.94 (Tsay, 2010). In this thesis we did not fixed the alpha

term and expressed the RiskMetrics in the following way:

$$\mu_t = 0, \ \sigma_t^2 = \alpha\sigma_{t-1}^2 + (1 - \alpha)r_{t-1}^2, \ \ 1 > \alpha > 0 \qquad (14)$$

4.2.3 GJR-GARCH/TGARCH

In GARCH and IGARCH model, positive price shocks and negative price shocks

are weighted equally in calculating the conditional variance; however, this does not capture

the asymmetric effects of volatility response to shocks. The GJR-GARCH, also known as

Threshold GARCH, proposed by Glosten, Jagannathan and Runke (1993) overcomes this

shortcoming by introducing a dummy type variable that takes 0 or 1 value when dealing

with non-negative or negative shocks, respectively. The restrictions that α_i, γ_i, and β_i are

non-negative parameters still apply. Mean equation is same as equation (11) mentioned

above and conditional variance equation is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 D_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{15}$$

Where D_{t-1} is an indicator for the negative shocks due to that $D_{t-i} = \begin{cases} 1, & \varepsilon_{t-i} < 0 \\ 0, & \varepsilon_{t-i} \geq 0 \end{cases}$

It is easy to see that a positive shock leads to $\alpha_i \varepsilon_{t-i}^2$ in the squared residual term, whereas a negative shock leads to a greater impact on the squared residual term with $(\alpha_i + \gamma_i) \varepsilon_{t-i}^2$. Hence we include this model to capture volatility asymmetry.

4.2.4 Exponential GARCH

Exponential GARCH developed by Nelson (1991) also captures the asymmetric response of volatility to positive and negative price shocks. The conditional variance of the model would always be positive even if the coefficients might be negative.

$$\ln(\sigma_t^2) = \alpha_0 + \frac{\alpha_1 |\varepsilon_{t-1}| + \gamma_1 \varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 \ln(\sigma_{t-1}^2) \tag{16}$$

The parameter γ_i is an indicator that captures the asymmetric effect of the price shocks. When the price shock is positive, the residual term of the log conditional variance becomes $\alpha_i(1 + \gamma_i)|z_{t-i}|$, whereas when the shock is negative, it changes to $\alpha_i(1 - \gamma_i)|z_{t-i}|$ and γ_i is a negative coefficient. This is another popular asymmetric model, which we will use in modeling the realized volatilities.

4.2.5 APARCH

Asymmetric Power ARCH (APARCH) can also be referred to as APGARCH introduced by Ding, Granger and Engle (1993) to capture the asymmetric effects. In addition, it nests other variations of GARCH models. Its free roaming parameters δ is able to flexibly capture the characteristics of volatility compared to other GARCH specifications.

$$\sigma_t^\delta = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \gamma_1\varepsilon_{t-1})^\delta + \beta_1\sigma_{t-1}^\delta \qquad (17)$$

The power parameter δ can either be estimated or imposed, while γ can capture the

leverage effect. In this thesis we don't restrict the power parameter and let Eviews estimate

this parameter based on the maximum likelihood method. APARCH equation can reduce to

the standard GARCH model when restricting the parameters $\delta = 2, \gamma = 0$, or becomes

GJR-GARCH when $\delta = 2, 0 \leq \gamma \leq 1$ (Hentschel, 1995).

4.3 Statistical Distributions

Since GARCH type models are not linear in nature, ordinary least square would not

be appropriate when estimating non-linear models; instead. Maximum likelihood (ML)

method is used instead to estimate the parameters of GARCH type models. In order to use

ML method, a likelihood function needs to be stated, which is a joint probability density

function and by maximizing the function with respect to the parameters, it finds the most

likely values of the parameters in question given the data set. Before using the ML method

to estimate parameters, a distributional assumption of the error term needs to be specified.

In the work of this thesis, Normal, Student-t and Generalized Error distribution are used in

estimate the parameters for the various GARCH models. Recall in equation (12), z_t, the

error term is assumed to be i.i.d with constant mean and variance; the three distribution

assumptions aforementioned apply to that innovation term.

4.3.1 Normal/Gaussian Distribution

Under the Normal/Gaussian distribution, the density function is defined as:

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \qquad (18)$$

Normal distribution is the most common assumption and it is often criticized for its inability to capture the heavily kurtosis characteristics exhibits in the financial time series.

4.3.2 Student-t Distribution

Student-t innovation is described by the following probability density function:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)} \tag{19}$$

Where v represents the number of degrees of freedom and Γ denotes the gamma function with the following feature $\Gamma(x) = \int_0^\infty y^{x-1}e^{-y}dy$. Student-t distribution converges to normal distribution as the numbers of degree of freedom increase.

4.3.3 Generalized Error Distribution

GED distribution is less restrictive than the normal distribution assumption and it can take many forms depending on its degrees of freedom.

$$f(z) = \frac{v\,e^{-\frac{1}{2}\left|\frac{z-\mu}{\lambda}\right|^v}}{\lambda 2^{\left(1+\frac{1}{v}\right)}\Gamma\left(\frac{1}{v}\right)} \tag{20}$$

Where Γ (.) is a gamma function and $\lambda = \left[2^{\left(-\frac{2}{v}\right)}\Gamma\left(\frac{1}{v}\right)/\Gamma\left(\frac{3}{v}\right)\right]^{\frac{1}{2}}$. The distribution can be reduced to Gaussian by restricting $v = 2$ and it has heavy kurtosis when $v < 2$.

4.4 Forecasting Evaluation

In order to make inference about which forecasting model performs best in predicting the future actual volatility using range-based estimator and model-free implied volatility as proxies, standard evaluation criteria will be introduced. These evaluation criteria will determine the forecasting accuracy of the 1-day ahead out-of-sample forecast

against the observed value (range-based estimator and MFIV) by measuring the distance of the observed from the forecasted.

4.4.1 Root Mean Square Error

Root Mean Square Error (RMSE) is used to measure the deviation of the forecasted value from the observed value. The lower the RMSE, it means the smaller the deviation, which implies the better forecasting accuracy.

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_{t+1} - \hat{\sigma}_{t+1|t})^2} \tag{21}$$

Where n is the total number of out-of-sample volatility forecasts, σ_{t+1} is the actual volatility at time t+1 and $\hat{\sigma}_{t+1|t}$ is the forecasted volatility for day t+1 with origin at time t. Recall that we used both range-based estimator and model-free implied volatility as proxies for the unobserved actual volatility (σ_{t+1}).

4.4.2 Mean Absolute Error

Mean Absolute Error (MAE) is another loss function used to evaluate the forecasting accuracy of the predicted model by measuring the average of the absolute deviation of the predicted value from the realized value. It is said to be less sensitive to outliers compared to the RMSE (Hyndman & Koehler, 2006).

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\sigma_{t+1} - \hat{\sigma}_{t+1|t}\right| \tag{22}$$

4.4.3 Mean Absolute Percentage Error

Mean Absolute Percentage Error is another quality measure of the deviation of the forecasted value from the observed value expressed in percentage term.

$$MAPE = \frac{100}{n} \sum_{t=1}^{n} \left| \frac{\sigma_{t+1} - \hat{\sigma}_{t+1|t}}{\sigma_{t+1|t}} \right| \tag{23}$$

4.4.4 Theil Inequality Coefficient

Another measure of the forecasting error is called Theil Inequity Coefficient developed by Henri Theil is known commonly as the Theil-U1 statistics.

$$U_1 = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_{t+1} - \hat{\sigma}_{t+1|t})^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\sigma_{t+1}^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}\widehat{\sigma_{t+1|t}^2}}} \tag{24}$$

Intuitively, the numerator is the RMSE and the denominator is the sum of individual forecasted and realized spreads which acts as standardization of RMSE. $0 \leq U_1 \leq 1$, where $U_1 = 0$ means the best forecast without any forecasting errors.

4.4.5 Information Content Univariate Regression

To further assess the forecasting performance of the chosen time series models, the "best" forecasting models from each category (Range-based and MFIV) are selected based on the loss functions and are used in estimating the Mincer-Zarnowitz regression using OLS for one-day ahead forecasts. This approach was recommended by numerous literatures for assessing the information content of chosen model and at the same time for testing the bias of the forecasts. (Poon & Granger, 2003; Prokopczuk & Simen, 2014; Siriopoulos & Fassas, 2009; Shaikh &Padhi, 2014)

$$Vol_{t+k|t} = \alpha + \beta \, \widehat{F_{t+k|t}} + e_t \tag{25}$$

Where $Vol_{t+k|t}$ denotes the range-based estimator or option implied volatility over k day horizon, while $\hat{F}_{t+k|t}$ represents k-day ahead forecast from the winning forecasting model, α is the constant term and ε_t is the error term. If the chose model indeed contains some

28

information about the future range-based estimator or implied volatility, then its coefficient β should be different from zero, statistically significant and $\bar{R}^2$, adjusted R-square should also be substantial. Using the same equation can also test the biasness of the time series model; the forecasting model is said to be unbiased estimator of the future volatility if $\alpha = 0$ and $\beta = 1$ jointly. Finally, to account for the heteroscedastic nature of volatility and serial correlation problem, Newey and West (1987) procedure is used in the OLS regression by correcting its standard error. (Kourtis et al., 2016)

5. Empirical Results

5.1 Descriptive Statistics

The asymmetric and negative relationship between the changes in implied volatility index and its underlying equity index for many stock markets is a well-documented fact in the literature (Giot, 2005; Fleming et al., 1995). Usually negative price movement follows by a larger increase in the volatility than the positive price movement of the same magnitude. To visualize this relationship graphically, daily closing prices of equity index for the full sample period is plotted along with its volatility index levels for the 10 equity indices in Appendix, Figure 1. As the graphs clearly show, there seems to be a negative correlation between the equity index levels and its model-free implied volatility levels and for many of the indices there also seems to be asymmetric relationship between the two.

Table III illustrates the descriptive statistics of the daily range-based series using the Parkinson (1980) price range estimator and its daily scaled model-free implied volatility series of the sample period 1/3/2005 – 12/30/2016 for the 10 stock indices. Across all indices, the range-based time series and its implied volatility series exhibit positive

29

skewness and large excess kurtosis. As a result, Jarque-Bera test of the normality is rejected

for all instances even at 1% significance level.

5.2 Out-of-Sample Forecast Evaluation

Using the 4 loss functions (RMSE, MAE, MAPE and Theil's U Statistic)

aforementioned, the forecasting accuracy of the various GARCH (1, 1) type models with

Normal, Student-t and GED innovations are scrutinized against range-based estimator and

implied volatility as proxies of ex-post volatility and the findings are summarized in the

table below. More detailed outputs of EViews test results are included in the appendix,

Table IV and V where model with the lowest forecasting errors for each loss function is

highlighted in grey. The model with best 1-day ahead out-of-sample forecasting

performance is assessed with the values of RMSE, MAE, MAPE and Theil's U Statistics.

Table II: Forecasting Performance Summary

Equity Index	"Best" Models (Range Volatility)	"Best" Models (Implied Volatility)
SPX	EGARCH (Normal)	GARCH (Student-t)
UKX	APARCH (Normal)	GARCH (Normal), IGARCH (Normal) – RM
DAX	APARCH (Normal)	GARCH (Student-t)
HSI	IGARCH (Normal) – RM, IGARCH (GED)	GARCH (Student-t), APARCH (Student-t)
KOSPI	IGARCH (Normal) – RM	GARCH (Student-t)
NDX	APARCH (Normal)	GARCH (Student-t)
NKY	APARCH (Student-t)	GARCH (Student-t)
SMI	APARCH (Student-t)	GARCH (GED)
SX5E	APARCH (Normal)	GARCH (Student-t)
CAC	APARCH (Normal)	GARCH (Student-t)

As the 4 loss functions were designed differently in measuring the deviation from

the observed value, it is not common to one model that uniformly beats the others across all

criteria. For instance, RMSE is said to be scale-dependent measure and is more sensitive to

the large deviation than the sum of smaller deviations even if the total deviations are the

same, while MAE is also scale-dependent, but it is considered to be less sensitive to larger deviation in comparison to RMSE. The method of selecting the best model(s) of each time series is based on at least two or more loss functions.

Based on table II, in forecasting range estimator the results are rather mixed; generally, asymmetric GARCH type model (1,1) such as APARCH (1,1) seem to adequately capture the dynamic of the future volatilities across 7 of 10 indices. This coincides with Poon and Granger's (2003) conclusion that models account for volatility asymmetry generally performs well. Within these indices, when taking into consideration of distribution variations, normal distribution can explain 5 of 7 indices while Student-t innovation explains the other two indices namely NKY (Japan) and SMI (Switzerland). This result is in line with another research conducted by Brownless et al. (2011), where the authors found Student-t innovation generally did not yield any improvement in the performance of the models across a wide variety of asset classes. It seems that NKY and SMI are the exceptions, which suggests that realized volatility obtained using range-based estimator exhibits heavier tails compared to the other indices chosen. RiskMetrics methodology using IGARCH (1,1) normal distribution as a proxy gains competitive edge in forecasting realized volatility of HSI (Hong Kong) and VOSPI (Korea) indices. On the other hand, when forecasting realized volatilities using model-free implied volatilities across 10 indices, simple symmetric GARCH (1,1) seems to perform well overall. This result suggests that market participants seem to weight positive and negative price movements symmetrically in their perceived future risk. When drilling down to the different distribution assumptions, Student-t innovation performs relatively well for 8 of 10 indices, normal and GED are suitable for UKX (U.K) and SMI (Switzerland) respectively.

5.3 Information Content Regression Result

The best forecasting model from each category (Range-based estimator and MFIV) of each index is then selected in the univariate regression analysis using ordinary least square (OLS) method based on equation 25 to see if they also contain some information about the future volatility in addition to their forecasting accuracy. To account for heteroscedasticity and autocorrelations of residuals of the regression, Newey West correction of standard error procedure is used in EViews with automatic selection of lag length option.

Before running the univariate regression, Augmented Dickey-Fuller Unit Root test is done on all the time series to eliminate the uncertainty of possible spurious regression problem with non-stationary time series. All time series presented in the regression are able to reject the null hypothesis of a unit root at less than 5% with the exceptions of implied volatility time series of HSI and RiskMetrics model for HSI, where the rejections of unit root are at 8.5% and 7.2% respectively.

In Appendix, Table VI and VII illustrate the results of the univariate regression analysis on range-based estimator and model-free implied volatility respectively. Both constant coefficient (α) and slope coefficient (β) with its corresponding T-statistics in parentheses of each model are reported along with $\bar{R}^2$. It is evident that all slope coefficient estimates are different from 0 and statistically significant at less than 1%, which provides evidence that all forecasts bear some information about the future volatility.

Finally, using the same regression can also test the unbiasness of the time series forecasts through the joint hypothesis α=0 and β=1 using Wald F-statistic in EViews. The

results show that at every instance, the null hypothesis can be rejected at less than 1%. This provides statistical evidence that all volatility forecasts presented in this study are somewhat biased, which is in line with the conclusion from Jiang and Tian (2005) and Andersen et al. (2007b) that most volatility forecasts seem to be somewhat biased.

6 Conclusions and Future Research

The main purpose of this thesis is to investigate the forecasting performance of various GARCH models with different innovation assumptions (Normal, Student-t and GED) against the range-based estimator and implied volatility as proxies for realized volatilities in a more global context by using 10 frequently traded equity indices and its volatility indices. In this study, Parkinson (1980) range-based estimator is used to represent the ex-post "realized" volatility due its unbiasness and it is believed to be 5 times more efficient as the squared return measure according to Garman and Klass (1980). On the other hand, model-free implied volatility index from each exchange is used to represent market's expected future volatility, which is another proxy for ex-post volatility. As many prior researches state that implied volatility contains important information about future ex-post volatility and it is the markets best guess of what will happen in the near future. Model-free implied volatility does not depend on any econometrics model and it does not subject its calculation to any distribution assumption of the stock prices and returns unlike the popular Black-Scholes equation. In order to assess the forecasting error of each model overtime, fixed rolling window method is employed with 1,000 observations for each in-sample estimation covering over 10 years of data (1/3/2005 – 12/30/2016) and the accuracy of each

forecast is evaluated with the four loss functions (RMSE, MAE, MAPE and Theil's U statistic).

Although the results are not clear-cut, there are some patterns observed. Depending on the index, in forecasting the ex-post volatility, overall asymmetric model such as APARCH (1,1) with Normal innovations is the one with best performance with the exceptions of NKY (Japan) and SMI (Switzerland) where Student-t innovation performs better. RiskMetrics methodology excels in forecasting ex-post volatility of HSI (China) and KOSPI (Korea) index. As volatility has many characteristics, some characteristics might be more pronounced in one index over the others. For instance, it seems that heavy kurtosis such as Student-t can describe NKY and SMI relatively well, which implies that realized volatility from these two indices might be more heavy-tailed comparing to others. The results from forecasting implied volatility are more consistent in terms of the type of GARCH model that can capture the market's expected volatility across 10 indices; a simple GARCH (1,1) with Student-t innovation seems to perform relatively well with exceptions of UKX (U.K) and SMI (Switzerland) where Normal and GED seem to perform better. This result from forecasting implied volatility seems to imply that market participants seem to weight positive and negative price shocks equally in their perceived future expected risk. Finally, through univariate regression analysis, it shows that all the winner models bear some information about future volatility (range-based estimator and MFIV). The regression analysis also suggests that all the forecasts are somewhat biased, which is in line with many other prior researches that suggest many volatility forecasts are biased. Fleming (1995) concludes that while unbiasness is a good property, it is not the most crucial property if the degree of biasness can be pinpointed and corrected.

Although this thesis provides a good overview of the forecasting ability of various GARCH (1,1) models across 10 global equity indices, it can be extended to include higher order GARCH (m, n) models and long memory time series models proposed in the recent literature such as the Fractional Integrated GARCH (FIGARCH) and Heterogeneous Autoregressive Model (HAR) to forecast future volatility. In a study conducted by Martens and Zein (2004), the authors conclude that long memory models such as the Fractional Integrated GARCH model dramatically change the result of the contest between time series model and implied volatilities. Kourtis et al. (2016) report the success of using Heterogeneous Autoregressive model (HAR) in one-day ahead forecasts for realized volatilities compared to GJR-GARCH model. In addition of incorporating more variety of the time series models to forecast the future volatility, the sample period in question could also be divided into different sub-periods, for example, to analyze how the various model perform during period of high distress such as late 2008 – 2009.

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Appendices

Figure 1:Graph of equity index level along with its volatility index level (1/3/2005 - 12/30/2016)

SX5E_V2X

CAC_VCAC
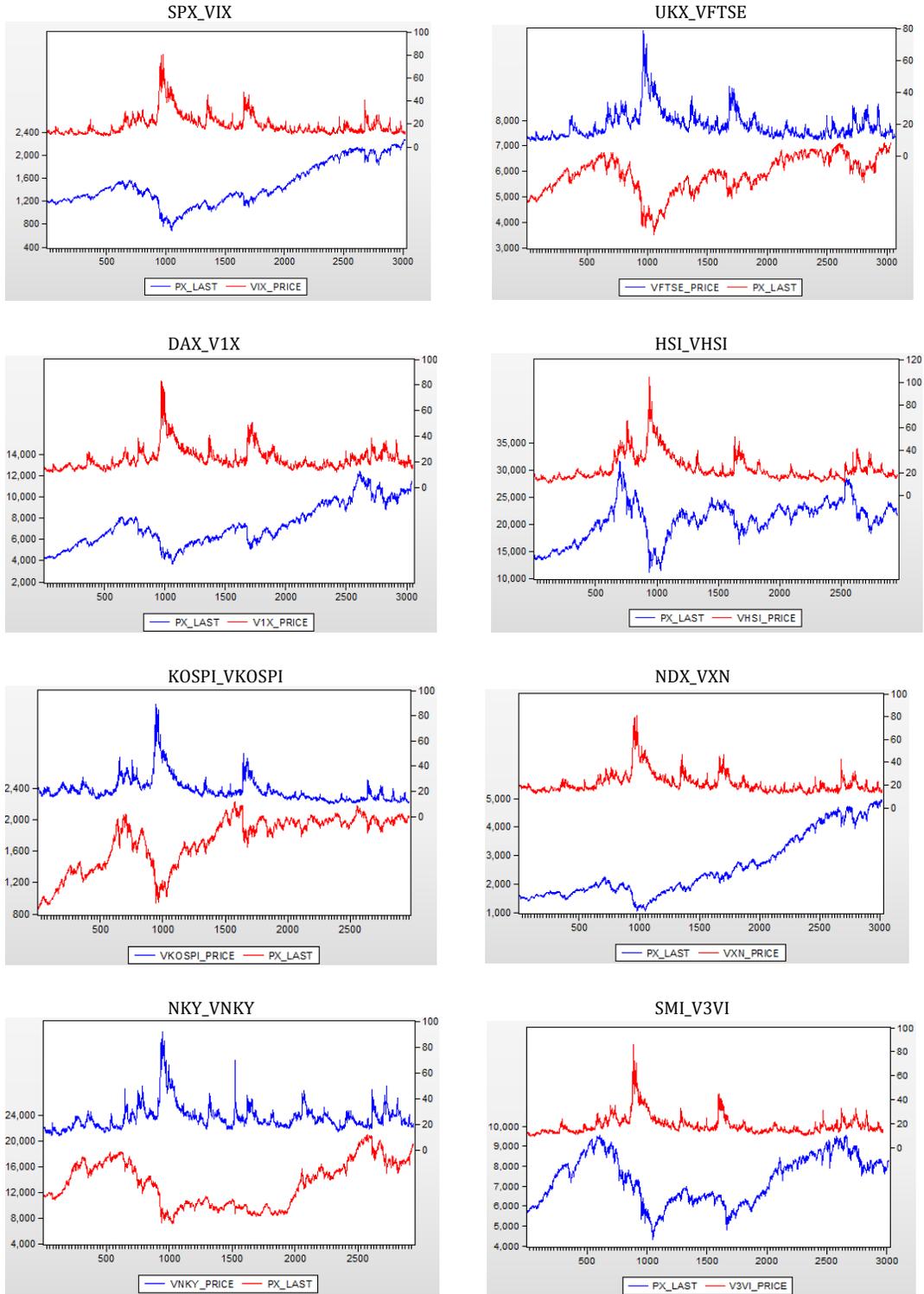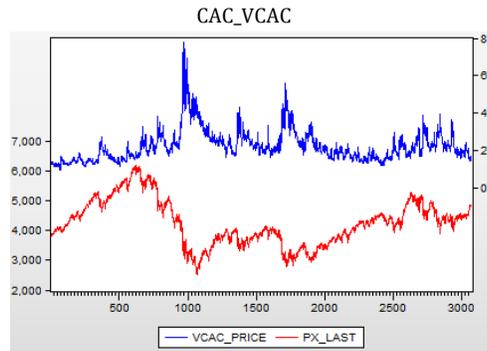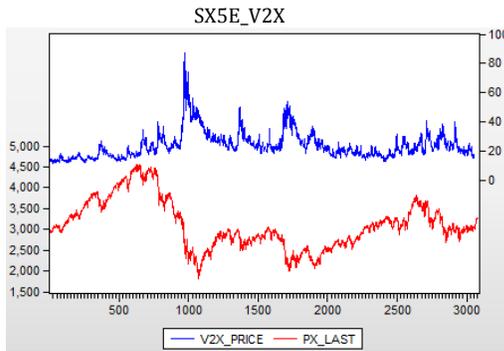
Table III: Descriptive Statistics of range estimator and implied volatility

SPX_VIX	PK_VOL	VIX1
Mean	0.007747	0.012225
Median	0.005999	0.010381
Maximum	0.065486	0.050937
Minimum	0.001207	0.006230
Std. Dev.	0.006363	0.005907
Skewness	3.600757	2.480602
Kurtosis	23.17167	11.13996
Jarque-Bera	57746.16	11438.57
Probability	0.000000	0.000000

UKX_VFTSE	PK_VOL	VFTSE1
Mean	0.008515	0.012122
Median	0.006875	0.010564
Maximum	0.064581	0.049768
Minimum	0.001400	0.005755
Std. Dev.	0.006027	0.005426
Skewness	3.025790	2.331841
Kurtosis	18.36100	11.17321
Jarque-Bera	34436.20	11186.97
Probability	0.000000	0.000000

DAX_V1X	PK_VOL	V1X1
Mean	0.009511	0.014239
Median	0.007962	0.012848
Maximum	0.066910	0.052639
Minimum	0.000864	0.007368
Std. Dev.	0.006440	0.005515
Skewness	2.677921	2.328785
Kurtosis	15.11698	11.02875
Jarque-Bera	22318.55	10955.88
Probability	0.000000	0.000000

HSI_VHSI	PK_VOL	VHSI1
Mean	0.008158	0.014998
Median	0.006526	0.012478
Maximum	0.105983	0.065959
Minimum	0.001589	0.006868
Std. Dev.	0.005973	0.007104
Skewness	4.346506	2.070407
Kurtosis	42.82799	8.436248
Jarque-Bera	204890.6	5757.614
Probability	0.000000	0.000000

42

	KOSPI_VKOSPI	
	PK_VOL	VKOPSI1
Mean	0.007844	0.013310
Median	0.006239	0.011770
Maximum	0.095138	0.056478
Minimum	0.001462	0.006457
Std. Dev.	0.005851	0.006098
Skewness	4.519293	2.599290
Kurtosis	42.06828	12.66749
Jarque-Bera	199395.3	14940.21
Probability	0.000000	0.000000

	NDX_VXN	
	PK_VOL	VXN1
Mean	0.008727	0.013586
Median	0.007150	0.011751
Maximum	0.068524	0.051001
Minimum	0.001669	0.007185
Std. Dev.	0.006115	0.005554
Skewness	3.401815	2.587490
Kurtosis	22.57394	12.12037
Jarque-Bera	54054.33	13841.41
Probability	0.000000	0.000000

	NKY_VNKY	
	PK_VOL	VNKY_1
Mean	0.008067	0.015833
Median	0.006586	0.014753
Maximum	0.082658	0.057973
Minimum	0.001389	0.006967
Std. Dev.	0.006011	0.006207
Skewness	4.100579	2.563174
Kurtosis	32.22934	12.71043
Jarque-Bera	112974.4	14780.11
Probability	0.000000	0.000000

	SMI_V3VI	
	PK_VOL	V3VI1
Mean	0.007345	0.011798
Median	0.005951	0.010398
Maximum	0.094066	0.054473
Minimum	0.001293	0.005844
Std. Dev.	0.005201	0.005043
Skewness	3.992814	2.778342
Kurtosis	38.73910	14.22627
Jarque-Bera	165843.6	19404.02
Probability	0.000000	0.000000

	SX5E_V2X	
	PK_VOL	V2X1
Mean	0.009784	0.014929
Median	0.008139	0.013751
Maximum	0.071425	0.055348
Minimum	0.000000	0.007336
Std. Dev.	0.006595	0.005776
Skewness	2.624952	1.967203
Kurtosis	14.82794	8.897416
Jarque-Bera	21323.45	6399.656
Probability	0.000000	0.000000

	CAC_VCAC	
	PK_VOL	VCAC1
Mean	0.009314	0.014044
Median	0.007735	0.012924
Maximum	0.055616	0.049366
Minimum	0.001409	0.005845
Std. Dev.	0.006109	0.005320
Skewness	2.290436	1.960153
Kurtosis	11.15772	9.115705
Jarque-Bera	11204.20	6754.642
Probability	0.000000	0.000000

Table IV: Forecasting performance of GARCH-type models on range estimator.

SPX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004803	0.003738	70.96408	0.234929
F_APARCH_SE	0.004736	0.003704	71.12876	0.232788
F_APARCH_T_SE	0.004887	0.003793	71.46807	0.237679
F_EGARCH_G_SE	0.004771	0.003770	72.48087	0.234420
F_EGARCH_SE	0.004698	0.003735	72.57100	0.232067
F_EGARCH_T_SE	0.106197	0.023428	284.9719	0.901658
F_GARCH_G_SE	0.005178	0.004018	79.24697	0.251206
F_GARCH_SE	0.005103	0.003979	78.97213	0.248797
F_GARCH_T_SE	0.005291	0.004099	80.51506	0.254772
F_IGARCH_G_SE	0.005180	0.003848	72.87652	0.253490
F_IGARCH_SE	0.005245	0.003892	73.67186	0.255755
F_IGARCH_T_SE	0.005648	0.004202	81.83659	0.271329
F_TGARCH_G_SE	0.005041	0.003854	73.06195	0.244311
F_TGARCH_SE	0.004972	0.003826	73.29110	0.242025
F_TGARCH_T_SE	0.005119	0.003901	73.46773	0.246884

UKX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004321	0.003365	53.36247	0.207567
F_APARCH_SE	0.004310	0.003357	53.28851	0.207238
F_APARCH_T_SE	0.004331	0.003373	53.38314	0.207923
F_EGARCH_G_SE	0.004336	0.003410	54.44262	0.208086
F_EGARCH_SE	0.004319	0.003395	54.21418	0.207520
F_EGARCH_T_SE	0.009505	0.003612	56.66408	0.398776
F_GARCH_G_SE	0.004630	0.003616	58.84426	0.219995
F_GARCH_SE	0.004635	0.003615	58.74877	0.220049
F_GARCH_T_SE	0.004644	0.003633	59.18373	0.220437
F_IGARCH	0.004598	0.003504	54.82557	0.221068
F_IGARCH_G_SE	0.004631	0.003528	55.23262	0.222519
F_IGARCH_T_SE	0.005475	0.003996	63.51723	0.254268
F_TGARCH_G_SE	0.004378	0.003426	54.78423	0.209387
F_TGARCH_SE	0.004362	0.003411	54.58697	0.208847
F_TGARCH_T_SE	0.004404	0.003447	55.04590	0.210286

DAX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.005586	0.004567	65.43357	0.218612
F_APARCH_SE	0.005513	0.004525	65.29828	0.216507
F_APARCH_T_SE	0.005672	0.004621	65.95422	0.221017
F_EGARCH_G_SE	0.005577	0.004587	66.15472	0.218785
F_EGARCH_SE	0.005501	0.004540	66.01436	0.216542
F_EGARCH_T_SE	0.005681	0.004659	66.80110	0.221805
F_GARCH_G_SE	0.005984	0.004918	72.88006	0.231761
F_GARCH_SE	0.005948	0.004898	72.84851	0.230684
F_GARCH_T_SE	0.006046	0.004968	73.54837	0.233466
F_IGARCH_G_SE	0.005988	0.004781	68.14522	0.232698
F_IGARCH_SE	0.006023	0.004832	69.30688	0.233657
F_IGARCH_T_SE	0.007613	0.005586	78.58917	0.281782
F_TGARCH_G_SE	0.005730	0.004636	66.78650	0.223336
F_TGARCH_SE	0.005645	0.004591	66.75282	0.220901
F_TGARCH_T_SE	0.005831	0.004706	67.45754	0.226259

HSI

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.006885	0.005800	100.0192	0.305896
F_APARCH_SE	0.006819	0.005753	99.59236	0.304049
F_APARCH_T_SE	0.006947	0.005853	100.7733	0.307633
F_EGARCH_G_SE	0.006804	0.005747	99.33265	0.303446
F_EGARCH_SE	0.006752	0.005716	99.18518	0.301998
F_EGARCH_T_SE	0.011303	0.005992	101.1515	0.445502
F_GARCH_G_SE	0.006847	0.005764	100.0569	0.304915
F_GARCH_SE	0.006799	0.005734	99.82213	0.303558
F_GARCH_T_SE	0.006887	0.005810	100.8287	0.305972
F_IGARCH_G_SE	0.006598	0.005420	93.06146	0.298341
F_IGARCH_SE	0.006594	0.005421	93.20645	0.298288
F_IGARCH_T_SE	0.007100	0.005713	98.60801	0.315453
F_TGARCH_G_SE	0.006722	0.005679	98.92103	0.301246
F_TGARCH_SE	0.006672	0.005646	98.65229	0.299775
F_TGARCH_T_SE	0.006778	0.005732	99.74922	0.302910

KOSPI

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.005421	0.004393	85.21155	0.281393
F_APARCH_SE	0.005379	0.004358	84.68654	0.280042
F_APARCH_T_SE	0.005486	0.004446	86.12685	0.283544
F_EGARCH_G_SE	0.005515	0.004503	87.45341	0.284836
F_EGARCH_SE	0.005473	0.004467	86.89817	0.283520
F_EGARCH_T_SE	0.005579	0.004559	88.44587	0.286942
F_GARCH_G_SE	0.005421	0.004388	85.87131	0.283045
F_GARCH_SE	0.005425	0.004386	85.77831	0.283252
F_GARCH_T_SE	0.005450	0.004420	86.55684	0.283969
F_IGARCH_G_SE	0.005270	0.004101	78.65619	0.280123
F_IGARCH_SE	0.005273	0.004079	78.03848	0.279845
F_IGARCH_T_SE	0.006022	0.004499	87.86449	0.310363
F_TGARCH_G_SE	0.005562	0.004526	88.41332	0.286878
F_TGARCH_SE	0.005481	0.004445	86.91932	0.283971
F_TGARCH_T_SE	0.005583	0.004543	88.76145	0.287509

NDX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.005479	0.004375	74.75826	0.251588
F_APARCH_SE	0.005432	0.004349	74.78030	0.250135
F_APARCH_T_SE	0.005547	0.004423	75.27582	0.253722
F_EGARCH_G_SE	0.005511	0.004442	76.70067	0.252359
F_EGARCH_SE	0.005466	0.004417	76.66050	0.250945
F_EGARCH_T_SE	0.005570	0.004486	77.20785	0.254235
F_GARCH_G_SE	0.005737	0.004590	81.08343	0.262247
F_GARCH_SE	0.005724	0.004582	81.00241	0.261818
F_GARCH_T_SE	0.005792	0.004640	81.97644	0.263908
F_IGARCH_G_SE	0.005725	0.004390	74.92920	0.263873
F_IGARCH_SE	0.005730	0.004396	75.26207	0.263978
F_IGARCH_T_SE	0.006262	0.004804	83.43897	0.282582
F_TGARCH_G_SE	0.005480	0.004407	76.44465	0.250683
F_TGARCH_SE	0.005436	0.004389	76.65189	0.249305
F_TGARCH_T_SE	0.005549	0.004454	76.97851	0.252796

NKY

Evaluation statistics				
Forecast	RMSE	MAE	MAPE	Theil
F_APGARCH_G_SE	0.007879	0.006714	124.4068	0.331888
F_APGARCH_SE	0.007901	0.006730	124.8646	0.332645
F_APGARCH_T_SE	0.007857	0.006696	124.0339	0.331277
F_EGARCH_G_SE	0.007980	0.006832	126.7469	0.334866
F_EGARCH_SE	0.007983	0.006834	126.7492	0.335080
F_EGARCH_T_SE	0.007963	0.006815	126.5250	0.334376
F_GARCH_G_SE	0.008329	0.007156	133.3491	0.344430
F_GARCH_SE	0.008340	0.007149	133.0412	0.344585
F_GARCH_T_SE	0.008296	0.007137	133.2114	0.343946
F_IGARCH_G_SE	0.008190	0.006861	125.2929	0.341895
F_IGARCH_SE	0.008169	0.006829	124.7374	0.341294
F_IGARCH_T_SE	0.008609	0.007098	130.0809	0.354575
F_TGARCH_G_SE	0.008210	0.006905	127.7522	0.341138
F_TGARCH_SE	0.008212	0.006902	127.6665	0.341335
F_TGARCH_T_SE	0.008197	0.006893	127.6338	0.340586

SMI

Evaluation statistics				
Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004795	0.003593	66.45296	0.250459
F_APARCH_SE	0.004829	0.003628	67.24769	0.251920
F_APARCH_T_SE	0.004802	0.003586	66.14649	0.250601
F_EGARCH_G_SE	0.004862	0.003595	66.28596	0.253560
F_EGARCH_SE	0.004897	0.003618	66.78599	0.255026
F_EGARCH_T_SE	0.004881	0.003597	66.19463	0.254250
F_GARCH_G_SE	0.005027	0.003821	72.40254	0.261004
F_GARCH_SE	0.005080	0.003860	73.13406	0.262820
F_GARCH_T_SE	0.005052	0.003842	72.78256	0.261841
F_IGARCH_G_SE	0.005113	0.003750	68.74678	0.266901
F_IGARCH_SE	0.005065	0.003704	68.07788	0.265135
F_IGARCH_T_SE	0.005643	0.004009	73.82555	0.288069
F_TGARCH_G_SE	0.005108	0.003687	67.90392	0.263452
F_TGARCH_SE	0.005156	0.003736	68.89579	0.265170
F_TGARCH_T_SE	0.005108	0.003675	67.57256	0.263502

SX5E

Evaluation statistics				
Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.005965	0.004751	NA	0.222708
F_APARCH_SE	0.005939	0.004744	NA	0.222034
F_APARCH_T_SE	0.005997	0.004767	NA	0.223556
F_EGARCH_G_SE	0.006012	0.004797	NA	0.225167
F_EGARCH_SE	0.005957	0.004766	NA	0.223701
F_EGARCH_T_SE	0.007407	0.004913	NA	0.269686
F_GARCH_G_SE	0.006596	0.005279	NA	0.243145
F_GARCH_SE	0.006603	0.005282	NA	0.243285
F_GARCH_T_SE	0.006627	0.005310	NA	0.243898
F_IGARCH_G_SE	0.006616	0.005132	NA	0.244844
F_IGARCH_SE	0.006627	0.005152	NA	0.245179
F_IGARCH_T_SE	0.008118	0.005968	NA	0.286717
F_TGARCH_G_SE	0.006313	0.004957	NA	0.233503
F_TGARCH_SE	0.006262	0.004931	NA	0.232093
F_TGARCH_T_SE	0.006353	0.004985	NA	0.234566

CAC

Evaluation statistics				
Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.005906	0.004811	66.67668	0.229022
F_APARCH_SE	0.005865	0.004791	66.71812	0.227862
F_APARCH_T_SE	0.005950	0.004836	66.80292	0.230336
F_EGARCH_G_SE	0.005964	0.004871	67.72136	0.231529
F_EGARCH_SE	0.005902	0.004834	67.51065	0.229809
F_EGARCH_T_SE	0.006014	0.004891	67.67585	0.232962
F_GARCH_G_SE	0.006474	0.005254	75.53364	0.247611
F_GARCH_SE	0.006480	0.005258	75.58284	0.247709
F_GARCH_T_SE	0.006502	0.005284	75.96402	0.248397
F_IGARCH_G_SE	0.006446	0.005072	70.32019	0.247903
F_IGARCH_SE	0.006453	0.005095	70.91801	0.248177
F_IGARCH_T_SE	0.008133	0.005965	83.66885	0.297587
F_TGARCH_G_SE	0.006262	0.004986	68.53332	0.240564
F_TGARCH_SE	0.006190	0.004949	68.46929	0.238498
F_TGARCH_T_SE	0.006306	0.005017	68.78352	0.241835

Table V: Forecasting performance of GARCH-type models on model-free implied volatility.

SPX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.003185	0.002576	21.48193	0.128044
F_APARCH_SE	0.003168	0.002557	21.12054	0.127896
F_APARCH_T_SE	0.003164	0.002559	21.49265	0.126605
F_EGARCH_G_SE	0.003370	0.002657	21.74147	0.136000
F_EGARCH_SE	0.003337	0.002618	21.22818	0.135257
F_EGARCH_T_SE	0.104872	0.021886	131.8082	0.858129
F_GARCH_G_SE	0.002936	0.002348	19.02746	0.117248
F_GARCH_SE	0.002954	0.002359	19.01976	0.118454
F_GARCH_T_SE	0.002886	0.002297	18.64333	0.114551
F_IGARCH_G_SE	0.003320	0.002756	23.35139	0.133529
F_IGARCH_SE	0.003292	0.002721	23.00742	0.132013
F_IGARCH_T_SE	0.003615	0.002945	25.50668	0.143175
F_TGARCH_G_SE	0.003391	0.002713	22.24385	0.135303
F_TGARCH_SE	0.003355	0.002679	21.79327	0.134318
F_TGARCH_T_SE	0.003398	0.002712	22.33296	0.135013

UKX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004446	0.003415	26.98958	0.183814
F_APARCH_SE	0.004374	0.003362	26.52601	0.180976
F_APARCH_T_SE	0.004506	0.003461	27.38814	0.186148
F_EGARCH_G_SE	0.004367	0.003343	26.30035	0.180386
F_EGARCH_SE	0.004306	0.003294	25.86869	0.178056
F_EGARCH_T_SE	0.009444	0.003581	28.13941	0.347089
F_GARCH_G_SE	0.003676	0.002802	22.13470	0.150536
F_GARCH_SE	0.003662	0.002795	22.10055	0.149857
F_GARCH_T_SE	0.003673	0.002799	22.12939	0.150283
F_IGARCH	0.003558	0.002894	23.64792	0.147187
F_IGARCH_G_SE	0.003607	0.002923	23.88393	0.149136
F_IGARCH_T_SE	0.004274	0.003292	27.81690	0.171593
F_TGARCH_G_SE	0.004460	0.003379	26.46730	0.183650
F_TGARCH_SE	0.004400	0.003338	26.10640	0.181362
F_TGARCH_T_SE	0.004514	0.003418	26.85023	0.185624

DAX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.003341	0.002620	17.63310	0.112433
F_APARCH_SE	0.003281	0.002552	17.07800	0.110745
F_APARCH_T_SE	0.003371	0.002660	17.95924	0.113014
F_EGARCH_G_SE	0.003378	0.002605	17.39936	0.113902
F_EGARCH_SE	0.003283	0.002520	16.73039	0.111030
F_EGARCH_T_SE	0.003449	0.002663	17.86847	0.115813
F_GARCH_G_SE	0.002645	0.002079	13.87281	0.088222
F_GARCH_SE	0.002668	0.002084	13.78608	0.089078
F_GARCH_T_SE	0.002637	0.002073	13.87500	0.087706
F_IGARCH_G_SE	0.002861	0.002352	16.54757	0.095690
F_IGARCH_SE	0.002863	0.002348	16.50917	0.095631
F_IGARCH_T_SE	0.004612	0.003044	21.50418	0.147931
F_TGARCH_G_SE	0.003433	0.002624	17.34402	0.115129
F_TGARCH_SE	0.003367	0.002553	16.72545	0.113296
F_TGARCH_T_SE	0.003480	0.002665	17.69559	0.116248

HSI

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.002406	0.001748	11.91390	0.083705
F_APARCH_SE	0.002462	0.001792	12.06031	0.085880
F_APARCH_T_SE	0.002385	0.001721	11.79412	0.082766
F_EGARCH_G_SE	0.002456	0.001803	12.31630	0.085689
F_EGARCH_SE	0.002500	0.001824	12.29909	0.087417
F_EGARCH_T_SE	0.009076	0.001973	12.90468	0.287124
F_GARCH_G_SE	0.002476	0.001758	11.48286	0.086280
F_GARCH_SE	0.002525	0.001782	11.55542	0.088172
F_GARCH_T_SE	0.002431	0.001716	11.17759	0.084572
F_IGARCH_G_SE	0.002723	0.002109	14.56469	0.096042
F_IGARCH_SE	0.002748	0.002121	14.58075	0.096946
F_IGARCH_T_SE	0.003210	0.002343	16.51137	0.111673
F_TGARCH_G_SE	0.002640	0.001915	12.67826	0.092465
F_TGARCH_SE	0.002678	0.001940	12.74093	0.093991
F_TGARCH_T_SE	0.002622	0.001884	12.48810	0.091645

KOSPI

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.003043	0.002115	16.56958	0.126090
F_APARCH_SE	0.002969	0.002076	16.30701	0.123314
F_APARCH_T_SE	0.003090	0.002146	16.77697	0.127600
F_EGARCH_G_SE	0.002916	0.002057	16.29472	0.120336
F_EGARCH_SE	0.002840	0.002010	15.94545	0.117465
F_EGARCH_T_SE	0.002967	0.002093	16.54467	0.122032
F_GARCH_G_SE	0.002402	0.001800	14.29263	0.099971
F_GARCH_SE	0.002412	0.001809	14.38195	0.100373
F_GARCH_T_SE	0.002380	0.001776	14.07255	0.098889
F_IGARCH_G_SE	0.002780	0.002265	19.02148	0.117354
F_IGARCH_SE	0.002743	0.002250	19.02119	0.115657
F_IGARCH_T_SE	0.003561	0.002560	21.92673	0.146678
F_TGARCH_G_SE	0.003029	0.002087	16.19886	0.124867
F_TGARCH_SE	0.002952	0.002045	15.87485	0.122130
F_TGARCH_T_SE	0.003051	0.002103	16.29262	0.125617

NDX

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.003196	0.002551	19.18577	0.120656
F_APARCH_SE	0.003130	0.002483	18.56824	0.118434
F_APARCH_T_SE	0.003218	0.002577	19.45911	0.121083
F_EGARCH_G_SE	0.003232	0.002531	18.93356	0.121726
F_EGARCH_SE	0.003163	0.002466	18.35347	0.119365
F_EGARCH_T_SE	0.003252	0.002553	19.16289	0.122163
F_GARCH_G_SE	0.002736	0.002131	15.59969	0.102903
F_GARCH_SE	0.002734	0.002128	15.54855	0.102886
F_GARCH_T_SE	0.002707	0.002098	15.34216	0.101537
F_IGARCH_G_SE	0.003116	0.002526	19.35355	0.117993
F_IGARCH_SE	0.003107	0.002514	19.24203	0.117595
F_IGARCH_T_SE	0.003490	0.002742	21.62752	0.129878
F_TGARCH_G_SE	0.003205	0.002502	18.51208	0.120604
F_TGARCH_SE	0.003127	0.002435	17.89819	0.117911
F_TGARCH_T_SE	0.003216	0.002513	18.66620	0.120623

NKY

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APGARCH_G_SE	0.003684	0.002841	17.25728	0.118897
F_APGARCH_SE	0.003673	0.002821	17.09375	0.118481
F_APGARCH_T_SE	0.003711	0.002871	17.45755	0.119855
F_EGARCH_G_SE	0.003598	0.002750	16.82612	0.115779
F_EGARCH_SE	0.003593	0.002750	16.82662	0.115636
F_EGARCH_T_SE	0.003625	0.002770	16.94223	0.116718
F_GARCH_G_SE	0.003557	0.002579	15.27099	0.113185
F_GARCH_SE	0.003620	0.002628	15.58227	0.115096
F_GARCH_T_SE	0.003527	0.002549	15.06592	0.112458
F_IGARCH_G_SE	0.003919	0.003074	19.15056	0.125603
F_IGARCH_SE	0.003980	0.003133	19.55788	0.127657
F_IGARCH_T_SE	0.004327	0.003178	20.02903	0.137274
F_TGARCH_G_SE	0.003982	0.002966	17.73765	0.127186
F_TGARCH_SE	0.003985	0.002966	17.74262	0.127294
F_TGARCH_T_SE	0.003998	0.002977	17.80650	0.127681

SMI

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004436	0.003238	27.27959	0.199145
F_APARCH_SE	0.004399	0.003205	26.94652	0.197300
F_APARCH_T_SE	0.004498	0.003274	27.64463	0.201746
F_EGARCH_G_SE	0.004611	0.003274	27.67843	0.206619
F_EGARCH_SE	0.004615	0.003254	27.46534	0.206563
F_EGARCH_T_SE	0.004659	0.003301	27.97689	0.208565
F_GARCH_G_SE	0.004372	0.002995	24.56663	0.195099
F_GARCH_SE	0.004407	0.003000	24.61601	0.196082
F_GARCH_T_SE	0.004380	0.002998	24.63792	0.195173
F_IGARCH_G_SE	0.004587	0.003337	28.06720	0.205525
F_IGARCH_SE	0.004578	0.003311	27.77933	0.205674
F_IGARCH_T_SE	0.004961	0.003591	30.57423	0.218113
F_TGARCH_G_SE	0.004932	0.003426	28.73979	0.218778
F_TGARCH_SE	0.004915	0.003398	28.47260	0.217495
F_TGARCH_T_SE	0.004969	0.003451	28.99560	0.220425

SX5E

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.004875	0.003867	24.16332	0.155116
F_APARCH_SE	0.004812	0.003806	23.67937	0.153287
F_APARCH_T_SE	0.004900	0.003894	24.42200	0.155726
F_EGARCH_G_SE	0.005047	0.003950	24.63841	0.161007
F_EGARCH_SE	0.004949	0.003894	24.21353	0.158250
F_EGARCH_T_SE	0.006522	0.004077	25.32664	0.203090
F_GARCH_G_SE	0.004225	0.003167	18.91710	0.133019
F_GARCH_SE	0.004257	0.003185	18.96538	0.133974
F_GARCH_T_SE	0.004201	0.003140	18.79371	0.132068
F_IGARCH_G_SE	0.004391	0.003460	21.43350	0.138669
F_IGARCH_SE	0.004373	0.003449	21.30546	0.138074
F_IGARCH_T_SE	0.005842	0.004179	26.88385	0.177291
F_TGARCH_G_SE	0.005133	0.003909	24.28732	0.162029
F_TGARCH_SE	0.005060	0.003852	23.80970	0.160003
F_TGARCH_T_SE	0.005163	0.003933	24.51728	0.162718

CAC

Evaluation statistics

Forecast	RMSE	MAE	MAPE	Theil
F_APARCH_G_SE	0.003585	0.002802	18.82425	0.119715
F_APARCH_SE	0.003517	0.002738	18.29742	0.117639
F_APARCH_T_SE	0.003612	0.002827	19.04592	0.120430
F_EGARCH_G_SE	0.003778	0.002897	19.33995	0.126266
F_EGARCH_SE	0.003697	0.002842	18.93869	0.123889
F_EGARCH_T_SE	0.003833	0.002921	19.51789	0.127883
F_GARCH_G_SE	0.003137	0.002266	14.59733	0.103515
F_GARCH_SE	0.003170	0.002278	14.62368	0.104562
F_GARCH_T_SE	0.003111	0.002251	14.52978	0.102571
F_IGARCH_G_SE	0.003341	0.002618	17.74350	0.110778
F_IGARCH_SE	0.003341	0.002609	17.62450	0.110787
F_IGARCH_T_SE	0.005103	0.003371	23.53807	0.162092
F_TGARCH_G_SE	0.003944	0.002928	19.33359	0.130645
F_TGARCH_SE	0.003852	0.002855	18.72158	0.127905
F_TGARCH_T_SE	0.003970	0.002954	19.57931	0.131325

Table VI: Univariate regression of Mincer-Zarnowitz to test information content of model on range-based estimator.

	APARCH (Normal)	APARCH (Student-t)	EGARCH (Normal)	IGARCH (Normal) - RM
SPX				
α			-0.000 (-0.68)	
β			0.743 (22.04)	
$\bar{R}^2$			0.508	
UKX				
α	9.25E-05 (0.242)			
β	0.805 (20.35)			
$\bar{R}^2$	0.423			
DAX				
α	-0.000 (-0.27)			
β	0.771 (18.63)			
$\bar{R}^2$	0.438			
HSI				
α				0.002 (5.32)
β				0.458 (13.80)
$\bar{R}^2$				0.261
KOSPI				
α				0.001 (3.16)
β				0.522 (9.36)
$\bar{R}^2$				0.307
NDX				
α	0.000 (0.91)			
β	0.661 (15.34)			
$\bar{R}^2$	0.393			
NKY				
α		-5.56E-05 (-0.10)		
β		0.562 (12.26)		
$\bar{R}^2$		0.230		
SMI				
α		0.000 (0.47)		
β		0.690 (8.42)		
$\bar{R}^2$		0.357		
SX5E				
α		0.000 (0.823)		
β		0.733 (19.07)		
$\bar{R}^2$		0.403		
CAC				
α	0.000 (1.13)			
β	0.693 (17.60)			
$\bar{R}^2$	0.401			

Table VII: Univariate regression of Micer-Zarnowitz to test information content of model on implied volatility.

	GARCH (Student-t)	GARCH (Normal)	GARCH (GED)
SPX			
α	0.003 (7.57)		
β	0.855 (19.11)		
$\bar{R}^2$	0.818		
UKX			
α		0.003 (3.51)	
β		0.843 (8.06)	
$\bar{R}^2$		0.529	
DAX			
α	0.003 (5.88)		
β	0.895 (22.52)		
$\bar{R}^2$	0.799		
HSI			
α	0.001 (2.05)		
β	0.980 (16.01)		
$\bar{R}^2$	0.823		
KOSPI			
α	0.001 (1.58)		
β	1.028 (12.52)		
$\bar{R}^2$	0.845		
NDX			
α	0.003 (4.82)		
β	0.892 (16.60)		
$\bar{R}^2$	0.787		
NKY			
α	0.005 (4.67)		
β	0.753 (8.51)		
$\bar{R}^2$	0.572		
SMI			
α			0.009 (10.55)
β			0.246 (3.31)
$\bar{R}^2$			0.09
SX5E			
α	0.006 (6.61)		
β	0.700 (10.12)		
$\bar{R}^2$	0.499		
CAC			
α	0.020 (5.59)		
β	3.559 (12.81)		
$\bar{R}^2$	0.659		