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MASTER'S FINAL WORK

PROJECT

VOLATILITY ADJUSTED MOMENTUM STRATEGY: IMPLEMENTATION AND PERFORMANCE EVALUATION

FILIPE JOÃO DA ASSUNÇÃO JANEIRO

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Abstract

We implement and detail a stock trading strategy, named volatility adjusted momentum strategy (VAMS) that minimizes the negative impact human emotions can instill in traders investment decisions. It withdraws the randomness and subjective interpretation from the portfolio asset selection. Our fully automatized approach takes advantage of a pervasive and well documented financial inefficiency, the so-called: *momentum effect*. By adjusting momentum to volatility and using risk parity in asset portfolio weighting, our findings support that in the long run, investors can actively expect to outperform the S&P500 benchmark. This strategy has the potential of exhibiting higher returns with lower exposure to risk, and presents the capability to diminish greatly the impact of bear markets in portfolio management.

Key words

Momentum, human emotions, risk management, adjusted volatility, risk parity, portfolio management.

Resumo

Implementamos e detalhamos uma estratégia de negociação em ações, de nome estratégia de momentum com volatilidade ajustada, que minimiza o impacto negativo que as emoções humanas podem incutir aos traders nas suas decisões de investimento. Remove a aleatoriedade e interpretação subjetiva na seleção dos ativos constituintes do portfólio. A nossa abordagem totalmente automatizada tira proveito de uma generalizada e bem documentada ineficiência financeira, o *efeito momentum*. Ao ajustar o momentum à volatilidade e usando paridade de risco no peso a atribuir aos activos no portfólio, os nossos resultados suportam que a longo prazo, os investidores podem ativamente esperar obter melhor performance que o mercado de referência S&P500. Esta estratégia tem o potencial de exibir maiores retornos com menor exposição ao risco, e apresenta a capacidade de diminuir consideravelmente as grandes viragens para terreno negativo do mercado bolsista, na gestão de portfólios.

Palavras chave

Momentum, emoções humanas, gestão de risco, volatilidade ajustada, paridade de risco, gestão de portfólios.

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Finally, I want to make the most sincere gratitude to all those who throughout my life, helped and supported me, but unfortunately as consequence of life vicissitudes are no longer part of it.

Risk comes from not knowing what you are doing.

by Warren Buffett

Contents

Acronyms

ATR average true range. 21, 22

EMH efficient market hypothesis. 4, 8, 9

SMA(100) 100 days simple moving average. 13–16

SMA(200) 200 days simple moving average. 13, 16

SMA(50) 50 days simple moving average. 13

SMMA smoothed moving average. 21

VAMS volatility adjusted momentum strategy. ii, v, vi, 1–3, 12–15, 18, 23–29, 31, 33–35

VaR value-at-risk. 26–28, 34, 35

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Chapter 1

Introduction

A significant amount of people around the world invest in some kind of financial asset. Stock ownership is definitely among one of the most preferable choices. As noticed by Gallup polls[1], in 2015 more than half of all Americans owned stocks in their personal portfolio, having this amount achieved a 65% score in 2007. Given this information, a question immediately pops-up: how well is expected an average individual investor to perform?

In a world continuously more complex, competitive and demanding, finding evidences that investors can beat the market may not be very promising. Barber and Odean (2011) provide powerful insights about how in theory, investors hold well diversified portfolios and trade infrequently, looking to minimizing all type of costs, but in practice, they behave rather differently. The authors document individuals performance being poor, underperforming standard benchmarks, and usually selling winning investments while holding losing investments, the *disposition effect*, firstly documented by Shefrin and Statman (1985)[2]. Human emotions seem to play a massive role in investment decisions, largely contributing towards a poor performance. Overconfidence, limited attention, seeking pleasure, failure to diversify are also other behaviors displayed by individual investors while investing.

This project aims at increasing the individual investors' chances of succeed in stock investments. In order to accomplish this milestone, we will detail and implement an automatized trading strategy (VAMS) that overcomes the human emotion factor in the decision investing process. As individuals have a limited amount of attention they can assign to the investment process, we present a systematic approach based

[1] http://www.gallup.com

[2] See also: Odean (1998).

on a exact set of rules. This way investors can head their attention, eliminating overreactions that would occur from noise and irrelevant information.

The strategy under analysis has been proposed by Clenow (2015), having in mind the discovery of Jegadeesh and Titman (1993). One of the strongest and most pervasive financial phenomena: the *momentum effect*. As acknowledge by Fama and French (2008) *momentum is the premier market anomaly*. Their research observes that the abnormal returns associated with momentum are pervasive, and we want to capitalize it too.

Momentum investing is about buying stocks that are moving up. When price is increasing, we buy with the expectation that the price continues to increase. We should remember that it is impossible to dissociate returns from risk. Risk is one of the most important features to be taken in consideration when investing. Therefore, we seek to minimize risk by adjusting momentum to volatility and by using risk parity[3] in asset portfolio weighting.

The main contribution of this work to the literature is reinforcing the potential of automatized quantitative momentum investment strategies[4], providing the obliteration of human emotions in investment decisions, and consequently enabling the expectation of possible enduring higher performance. Specially the capability of stanch bear markets negative impacts. Passive investment strategies have gather many supporters in the last years[5], but are unable to present such capability[6]. VAMS has the potential of achieving higher returns when compared to passively investing in the S&P500 benchmark, while minimizing risk and drawdowns[7].

Our study also contributes to consolidate Clenow (2015) empirical findings, as well as going further in results analysis, in particular in terms of risk-adjusted returns,

[3]Strategy based on targeting equal risk levels across the various components of an investment portfolio.

[4]See Gray and Carlisle (2012) and Gray (2014). Foltice and Langer (2015) show that momentum trading profits can be attained by retail investors.

[5]Malkiel (2003) strongly supports passive investment management in all markets even if markets are less than fully efficient.

[6]Passively investing in the S&P500 in the 2008/09 bear market would have achieved a -37% 2008 yearly absolute return.

[7]A drawdown is the peak-to-trough decline during a specific recorded period of an investment. High drawdowns can have huge impact on portfolio value.

value-at-risk analysis, trading costs stress test and out-of-sample analysis. These corroborate the previously suggested hypothesis, where VAMS outperforms the benchmark.

The remain of this study is organized in the following away: In chapter 2 we present a literature review, covering 3 core topics: investors performance and behavior, the momentum effect and risk parity weighting. Then, in chapter 3, VAMS is fully presented and explained. Next, in chapter 4 we exhibit the data and methodology, where we explain VAMS implementation process. Outcomes and performance can be found in chapter 5, as well as stress tests and an out-of-sample process. Finally a conclusion ends our main work structure, where our major findings are summarized.

Chapter 2

Literature Review

In this literature review we summarize the three most significant topics covered in this project.

2.1 Investors performance and behavior

According to several studies, the regular individual investor usually struggles in the investment industry, consistently underperforming the benchmark. This supports efficient market hypothesis (EMH)[1] pretensions. The EMH claims that one should not be expected to outperform the market consistently. Meyer et al. (2012) show strong evidence of this possibility, having found that about 89% of individual investors have negative results, getting beaten by the market when pursuing an active strategy[2]. Odean (1999) shows evidence of individual investors systematically earning subpar returns before costs, and broaden trading frequency when facing their poor performance. Heavy losses are also documented by Barber et al. (2009). It states that the aggregate portfolio of individual investors in Taiwan suffers an annual performance penalty of 3.8 percentage points. Similar bad results are achieved by Barber and Odean (2000). Their analysis of returns earned on common stock investments shows an underperformance of 1.1% annually when compared to a value-weighted market index. This research also provides interesting insights about why bad results are attended. The poor performance can be traced to the costs associated with high level of trading. The authors believe that this overtrading can be at least partly explained by a simple behavioral bias: People are *overconfident*, and overconfidence

[1]Developed independently by Fama (1965) and Samuelson (1965), this idea has been applied extensively to theoretical models and empirical studies of financial securities prices.

[2]Goetzmann et al. (2005) exhibit most investors could have improved the performance of their portfolios by simply investing in one of the many available passive index funds.

leads to too much trading[3].

Individual investors are not alone in the harsh path of struggling to outperform the market. Some studies, such as Grinblatt and Titman (1989) and Kosowski et al. (2006) argue that positive alpha generation can be achieved by institutional managers. However, the overall performance is very weak mainly due to transaction fees and costs. Fama and French (2010) conclude that funds results are close to the market portfolio, but high costs of active management give lower returns to investors[4].

The performance numbers are overwhelming for mutual funds. According to the end of 2015 SPIVA report[5], 66.11% of large-cap managers underperformed the benchmark, S&P500, during the past one-year period. Over the 5- and 10-year investment horizons, 84.15% and 82.14% of large-cap managers, respectively, failed to deliver incremental returns over the benchmark. The hedge fund industry also presents disappointing results. According to HFRX Global Hedge Fund Index[6], over the last past 5 years hedge funds achieved a mere annualized return of 1.49%, while the S&P500 presented a 10.49% annualized return[7].

Human emotions are determinant in investors behavior, constantly deteriorating their performance. Shiv et al. (2005) demonstrate the existence of a *dark side* in emotions during the decision making process. Depending on the circumstances, moods and emotions can play a disruptive role in decisions. This aspect achieves further importance when emotions are present in risk-taking decisions, such as financial investments, as shown by Loewenstein et al. (2001), Mellers et al. (1999)

[3]Further in overconfidence: Moore and Healy (2008) argue that people tend to overestimate their performance with overplacement of performance relative to others. Statman et al. (2006) state overconfident investors as being the most prone to sensation seeking, and to trade more frequently. Carhart (1997) shows that Mutual funds managers trade too often, having the consequence of realizing poor performance outcomes.

[4]Since Carhart (1997), it is largely admitted that actively managed mutual funds do not produce alpha on average and that performance persistence is not verified.

[5]SPIVA® U.S. Scorecard, End-Year 2015. `http://www.spindices.com`

[6]`https://www.hedgefundresearch.com`

[7] According to Lack (2012), hedge funds poor performance is the combination of a lack of transparency, high fees and transaction costs. The author goes further and provides evidences that the hedge funds industry might be a wealth transference mechanism, systematically moving money from investors to hedge-fund managers.

and later by Slovic et al. (2007). They support a very important perception: when risk exists, lack of emotional reactions may lead to more advantageous decisions.

The amount of constraint human emotions can induce on investment decisions can acquire huge proportions. As previously referred, investors have the tendency to hold losing investments too long and sell winning investments too soon, the so called *disposition effect*. Barber et al. (2007) find a strong evidence of this effect for individual investors, who are nearly four times as likely to sell a winner rather than a loser. Odean (1998) supports the importance of this phenomena in his findings. He points out that investors realize their gains in a 50% higher rate than their losses, and this difference is not justified by informed trading, rational belief in mean-reversion, transactions costs, or motivated by a desire to rebalance portfolios. Chen et al. (2007) and Choe and Eom (2009) suggest that institutional investors also suffer from the disposition effect.

Another emotional effect present in individual investors is denoted as *Reinforcement Learning*. Odean et al. (2009) propose that investors are driven by a desire to limit the degree of regret they experience in association with unsuccessful trades, and increase feelings of pride and satisfaction associated with successful trades[8]. The authors report that among investors, there was a significant bias towards repurchasing more stocks previously sold at gain than those sold at loss, as if they were repeating the action that brought pleasant experience while avoiding those that brought unpleasant experience.

Chasing the action is also a determinant emotional effect to take into consideration. Engelberg and Parsons (2011) find that individual investors are more likely to trade an S&P500 subsequent to an earnings announcement if that announcement is covered in the investor's local newspaper. Barber and Odean (2008) argue that attention greatly influences individual investor purchase decisions. Investors face a huge search problem when choosing stocks to buy. Rather than searching systematically, many investors may consider only stocks that first catch their attention.

[8]Similar findings are presented by Jiao (2015).

Risk averse investors should hold a diversified portfolio to minimize the impact of idiosyncratic risk on their investment outcomes. However, individual investors easily *fail to diversify*[9], preferring local and familiar stocks, avoiding investments in foreign stocks, which arguably provide strong diversification. Kumar (2009) shows that individuals prefer stocks with high idiosyncratic volatility, high idiosyncratic skewness, or low stocks prices. The results indicate that, unlike institutional investors, individual investors prefer stocks with lottery-type features. This lack of diversification appears to be the result of investor choices, rather than institutional constraints.

Social background, education, lack of skills and experience also play a major role in individual behavior and consequently in their performance. Anderson (2013) finds that lower income, poorer, younger, and less well-educated investors tend to invest a significant amount of their wealth in individual stocks, hold more highly concentrated portfolios, trade more, and have poorer trading performance. Nicolosi et al. (2009) point out that more experience and skills can lead to better trading performance. Similar conclusions are achieved by Seru et al. (2010). The authors conclude that performance improves as investors become more experienced. They also present a very interesting conclusion: others stop trading after realizing that their ability for trading is poor.

It is impossible to refute the deep and complex influence human emotions have in investment decisions and performance results[10]. Any feature that could help mitigate or preferable eradicate this handicap must be taken into serious consideration. The type of momentum investment strategy we present in this work, highly quantitative and computerized, can help minimizing emotional derivative risk. This goes in favor of Coval et al. (2005) findings, since they state that skillful individual investors exploit market inefficiencies to earn higher returns.

[9]Solnik and Zuo (2012) state that home bias remains a strong phenomenon around the globe.

[10]Feng and Seasholes (2005) and Glaser and Weber (2007) summarize that investors are hardly able to give a correct estimate of their own past realized stock portfolio performance. People overrate themselves. On average, investors think that they are better than others.

2.2 Momentum - The Premier Market Anomaly

Momentum is the tendency of investments to persist in their performance. The first influential paper on the subject was publish by Levy (1967), where he declares the possibility of higher returns being achievable by investing in securities that historically have been relatively strong in price movement. The *momentum effect* as one of the strongest and most pervasive financial phenomena, was declared by Jegadeesh and Titman (1993)[11] in their work. They demonstrate that strategies which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past, generate a significant positive return over 3- to 12-month holding periods.

After observing that the abnormal returns associated with momentum are prevalent, Fama and French (2008) called the momentum effect the *premier market anomaly*. Schwert (2003) explores many of the well known market anomalies, such as size effect, the value effect, the weekend effect, the dividend yield effect, and the momentum effect. They all seem to get weaker, disappear or arbitraged away, after the papers that highlighted them were published. All of them except momentum, which appears to be persistent, surviving since it has been published[12].

This possibility clashes with the EMH, as the perseverance of a market anomaly is a direct conflict with the market prices fully reflecting all available information. Momentum could be called a market inefficiency. Many attacks have recently been done to the efficient markets theory, having the most enduring critiques coming from psychologists and behavioral economists.

According to Lo (2007) the EMH is based on counter-factual assumptions regarding human behavior, that is, rationality. Investors do not always properly react in proportion to new information, arguing that markets are not rational, and are driven

[11] They present two alternative theories as to why momentum works: (1) Transactions by investors who buy past winners and sell past losers move the prices away from their long-run values temporarily and thereby causes prices to overreach. (2) Market under-reacts to information about the short-term prospects of firms but over-reacts to information about their long-term prospects.

[12] Jegadeesh and Titman (2001) confirm this hypothesis, showing evidences that momentum profits have continued in the 1990s, since their work was published.

by fear and greed instead[13].

Momentum seems to be persistent through all types of financial assets. As Asness et al. (2013) declare in their research, momentum is everywhere, becoming central to the market efficiency debate and asset pricing studies. Asness (1994) reinforces the presence of momentum prevalence on U.S. stocks, having the same occurrence being found on other equity markets, including foreign stock markets, as shown by Griffin et al. (2003), Bhojraj and Swaminathan (2006) and Chui et al. (2010). Similar effects have been found in currencies, Menkhoff et al. (2012), and in commodity markets, Miffre and Rallis (2007). Momentum strategies are also highly profitable among global government bonds and corporate bonds, Asness et al. (2013) and Jostova et al. (2013). Residential real estate is no exception in demonstrating evidences of momentum persistence as shown by Beracha and Skiba (2011).

Despite the abundance of momentum research, concept acceptance and Jegadeesh and Titman (1993) attempt to explain why it exists[14], no one is really sure why it works. Two major approaches to explain momentum can be classified as (i) risk-based and characteristics-based explanations, and (ii) explanations invoking cognitive biases or informational issues.

Risk-based explanations present the rational of momentum profits incurring risk premium because winners are riskier than losers, Johnson (2002) and Ahn et al. (2003). The most common explanations have to do with behavior factors, such as anchoring, overconfidence, herding, and the disposition effect. Behavior biases are unlikely to disappear, which may justify why momentum positive returns have persevered, and may continue to persevere, as a strong anomaly, as noted by Barberis et al. (1998), Daniel et al. (1998), Hong and Stein (1999) and Frazzini (2006).

The investment strategy we present in this work tries to take advantage of this market anomaly existence. We use as inspiration Foltice and Langer (2015) work,

[13]Lo (2004) states that, although cognitive neurosciences suggests that behavioral economics and EMH are two perspectives on opposite sides of the same coin, reconciling market efficiency with behavioral alternatives is possible by applying the principles of evolution/competition, adaptation, and natural selection to financial interactions.

[14]See foonote 11.

where the authors show that momentum trading profits can be attained by retail investors, even after factoring transaction costs and other pertinent market frictions.

2.3 The Risk Parity Weighting Process

This section concerns to the stage where proper asset allocation is calibrated in the investment portfolio, while minimizing risk.

In his groundbreaking work, Markowitz (1952)[15], contributes to market analysis by incorporate multiple assets that form a portfolio and by develop the mean-variance strategy. He determines the concept of an efficient portfolio and shows that there exists multiple efficient portfolios that form the efficient frontier. The Markowitz strategy requires two input parameters, namely the expected return, variance matrix of asset returns. The precise estimation of these parameters is often difficult and subjected to significant errors. Moreover, mean-variance portfolios have come under great criticism based on the poor performance experienced by asset managers during the global financial crisis, see Lee (2011) and Carvalho et al. (2016).

More recent research focuses on risk-based portfolio asset allocations, such as minimum variance, maximum diversification and risk parity, to protect investments against significant losses, with diversification controlling the investment decision. The main advantage of these strategies is that they diminish the input parameters when comparing to the traditional mean-variance strategy, specifically the estimation of expected returns. Institutional investment reports show that risk-based portfolio allocations were the only strategies that performed exceptionally during the late 2008-2009 crisis, see Podkaminer (2013), Romahi and Santiago (2012).

Minimum variance portfolios, as noted by Chow et al. (2011) are a special case of mean-variance efficient portfolios. The goal is the minimization of ex-ante portfolio risk so that the minimum variance portfolio lies on the left-most tip of the efficient frontier. Maximum diversification portfolios are based on a more recently introduced objective function by Choueifaty (2008) that maximizes the ratio of

[15] Diversification allows the ability to enhance portfolio returns while suppressing volatility.

weighted-average asset volatilities to portfolio volatility, an objective that is similar to maximizing the Sharpe ratio[16] but with asset volatilities replacing asset expected returns.

The concept of risk parity has evolved over time from the original concept embedded in research by Bridgewater in the 1990's. A more complete definition is introduced by Qian (2005). This phenomenon can be described as a strategy that allocates the weight of portfolio components through their risk contributions to the risk of the portfolio. In other words, in a stock portfolio, assets that have exhibited higher volatility will have a lower weight and those that have exhibited a lower volatility will have a higher weight[17]. Despite the quantitative underlying, risk parity is a heuristic[18] allocation approach. It is an intuitive approach and not theoretical like minimum variance or maximum diversification.

In their work, Clarke et al. (2013) claim the superiority of minimum variance portfolios in terms of minimizing risk, although risk parity equity portfolios reported in this study reveals to be promising in terms of a high Sharpe ratio. Maillard et al. (2008) achieve a similar conclusion, suggesting that risk parity portfolios appear to be an attractive alternative to minimum variance and other types of weighting portfolios, and might be considered a good trade-off between those approaches in terms of absolute level of risk, risk budgeting and diversification.

As declared by Schachter and Thiagarajan (2011), risk parity strongly appeals to the average individual investor intuition that risk diversification is the central goal in portfolio decision making, and equalizing estimated risk contributions is probably a good way to try to approach that goal. Risk parity is our strategy portfolio asset allocation method.

[16]It was developed by Nobel laureate William F. Sharpe. The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk.

[17]Risk Parity allocation can be used in portfolios consisting in different types of asset classes, e.g., stocks, bonds, commodities and others.

[18]Heuristic technique is any approach to problem solving, learning, or discovery that employs a practical method not guaranteed to be optimal or perfect, but sufficient for the immediate goals.

Chapter 3

The trading strategy

A well established strategy supported by a solid set of rules, allow us to backtest the concept, giving us reasonable expectations of performance, both during bull and bear markets. Hence, we start this chapter by exhibit a detailed summary of VAMS trading rules, which are picked and gathered by Clenow (2015). Then, we present a flowchart of how to implement the trading rules process.

In the following sections, we seek to explain in detail VAMS principal particularities, as well as the rational behind the major concepts of the investment strategy. Those will be discussed in the following subsections: (i) the rational behind the momentum ranking process, (ii) how the risk parity portfolio is created and rebalanced.

3.1 The strategy rules

VAMS trading rules, as shared by Clenow (2015), are as follows;

- **Only go long in stocks**.

There are several reasons justifying why we only allow long positions and forbid going short. First of all, stocks are prone to rapid volatility expansions in bear markets. This effect is confirmed by Schwert (1989), where the author states that stock volatility increase during recession periods. Schwert (1990) confirms that stock volatility increase during crashes, inducing a more erratic behavior in stock prices performance. Secondly, as Woolridge and Dickinson (1994) point out, short sellers do not earn abnormal profits. This inability makes the use of shorts in our strategy undesirable, as one of our goals is to outperform the market. Furthermore, we have to take into consideration the borrowing costs resulting for being short, and the limited availability of stock that could be borrowed.

- **Trade once per week. We chose Wednesdays**.

Avoiding acting too fast is a major feature of VAMS. In order to reduce workload and trading frequency, our investment strategy only looks to buy and sell signals, once per week. This frequency is enough to keep the feeling of actively managing the portfolio. This does not mean that we use weekly data in our analysis. Calculations are done by using daily data, but we chose to trade only once per week.
The selected weekday to realize trades are Wednesdays. This is due to Berument and Kiymaz (2001) work, where is declared that Wednesday is the less volatile day of the week.

- **Do not buy stocks in bear markets. Check if S&P500 index is trading above or below its 200 days simple moving average (SMA(200))[1].**

The reasoning behind not buying stocks during bear markets is due to the correlations quickly approach 1 in this market condition. In this condition, does not matter which stocks we own, since all of them are going down. This effect is referred by Woodward and Anderson (2009), where the authors exhibit that down market betas were significantly higher than bull market betas[2]. In VAMS we will deliberately take beta risk during bull markets but not holding beta risk when markets are going down.
A very simple and practical way to measure if the current market regime is bull or bear is using moving averages. We declare the long term filter SMA(200) as the frontier between bull and bear market. See Han et al. (2015) for further details on portfolio management industry practices[3].
We draw attention to the following strategy feature: the index and its moving average will only tell us if we can buy more stocks, in case we have money at our

[1] A simple moving average (SMA) is the unweighted mean of the previous n days of data. Therefore, SMA(200) stands for 200 days simple moving average.

[2] The majority of stocks will always have a high correlation to the overall equity index market, whether bull or bear market taking place. This is why diversification in equity portfolios only helps decreasing risk to a certain extent, not being able to eliminate all nonsystematic risk like a fully diversified portfolio, comprehending different asset classes, would achieve. The difference lays in how much that correlation is, and is definitely higher during down markets.

[3] It is commom practice to use to use moving averages (MA) as trends representatives. Usually it is used the 50 days simple moving average (SMA(50)) for short trend signals, the 100 days simple moving average (SMA(100)) for medium trend signals and SMA(200) for long trend signals.

disposal. We do not sell only because the index moved down below the moving average. However we are not allowed to open new positions if the index is below its long term moving average. See Figure 9 in Appendix A to consult examples.

- **Check stock trading below its SMA(100) or gap in excess of 15%.**

These filters increase the robustness of VAMS approach, which is based in buying stocks that are moving up. Firstly, a stock must be trading above its SMA(100) to be considered as a candidate to our portfolio. This moving average is considered to be a medium trend signal, and in normal market conditions, any well ranked stock in terms of momentum should be trading above its SMA(100). This rule prevent us from buying stocks that are not moving up.
Secondly, any stock with a gap move, up or down, larger than 15% in the past 90 days is disqualified. Not excluding these gap situations could make us investing in stocks that are not really generating momentum. We are looking for smooth and consistently upward movements, not aggressive, sudden and erratic drives.
See Figure 10 and Figure 11 in Appendix A to consult some examples.

- **Calculate risk parity position sizes, based on a risk factor of 10 b.p.**

Volatility among stocks differs considerably, being some stocks more volatile than others. As a consequence, if we allocate the same amount of cash to each portfolio constituent, the portfolio is going to be dominated by the more volatile stocks. We can surpass this issue by using the risk based portfolio asset allocation, risk parity allocation methodology. This means that portfolio position sizes will be calculated to approximate risk parity, allocating the same risk to each position.
Setting positions sizes based on a risk factor of 10 basis points[4], VAMS acquires an average portfolio constituents of 28 stocks. As Statman (1987) states in his work, an well diversified portfolio should include around 30 stocks. More detail about risk parity asset allocation is given further on this chapter.

- **Rank all stocks based on volatility adjusted momentum.**

[4] This allows targeting a daily impact in the portfolio by each stock of 0.1%.

Capturing the momentum effect is the core and most important feature of VAMS approach. As denoted by Asness et al. (2013), the underlying concept is very simple, "a stock that has been moving up strongly, most likely will continue do to so a bit longer". Therefore, is our goal capturing stocks that show significant gains over time, but moving as tender as possible.

Table I: Ranked constituents S&P500 at 07/01/2015

Filter: S&500 index > MA(200) → OK

Order	Ticker	Adj Slope	MA100	Gap15%	ATR20	Price	Shares	RiskParity
1	LB	1,599	1	1	1,88	83,2	141	4,41%
2	EW	1,450	1	1	1,68	65,0	158	3,86%
3	SPLS	1,443	1	1	0,59	17,2	447	2,90%
4	MAC	1,382	1	1	1,62	80,6	164	4,97%
5	WFM	1,335	1	1	1,20	49,5	221	4,12%
6	LOW	1,239	1	1	1,49	67,2	179	4,52%
7	DLTR	1,198	1	1	1,65	70,8	160	4,26%
8	WHR	1,192	1	1	5,05	188,2	52	3,68%
9	EA	1,151	1	1	1,27	46,8	209	3,68%
10	ORLY	1,119	1	1	3,67	189,8	72	5,14%
11	IRM	1,103	1	1	0,99	37,2	267	3,73%
12	NAVI	1,100	1	1	0,53	20,0	503	3,79%
13	YHOO	1,089	1	1	1,39	48,6	191	3,49%
14	LUV	1,060	1	1	1,47	40,5	180	2,74%
15	ROST	1,042	1	1	1,05	47,4	252	4,49%
16	V	1,039	1	1	1,25	64,8	213	5,19%
17	TSCO	1,024	1	0	1,95	77,1		
18	KR	1,011	1	1	0,59	32,1	453	5,47%
19	LEG	0,962	1	1	0,96	41,5	277	4,33%
20	PNW	0,959	1	1	1,52	67,4	174	4,41%

VAMS captures momentum through an exponential regression method provided by Clenow (2015). This momentum measurement gives each stock a certain slope dimension. The higher the slope, the higher momentum the stocks exhibits. In order to minimize the inherent stock risk, we perform a volatility adjustment in the slope using the exponential regression coefficient of determination R^2. A detailed explanation of this momentum methodology is provided later in this chapter.

As an example, we present in Table I the top ranked 20 stocks of the S&P500 constituents, for the date 07/01/2015. These are sorted by momentum adjusted slope. In this table we provide for each constituent their others main characteristics discussed so far: risk parity position sizes, checking if stock is trading below its SMA(100)[5], disclosing if there is a gap in excess of 15%, and checking if S&P500

[5] 0 means that condition is validated, while 1 means the opposite.

index is trading below its SMA(200).

- **Construct portfolio using ranking list.**

Relying in the information compiled in tables such as the one presented in table I, we can begin constructing the portfolio. We start from the top of our ranking list until running out of cash. This aggregates, for the analyzed date t, the portfolio with the highest adjusted momentum effect.

- **Rebalance portfolio every week. In 20% strongest stocks, above SMA(100) and no gap in excess of 15%.**

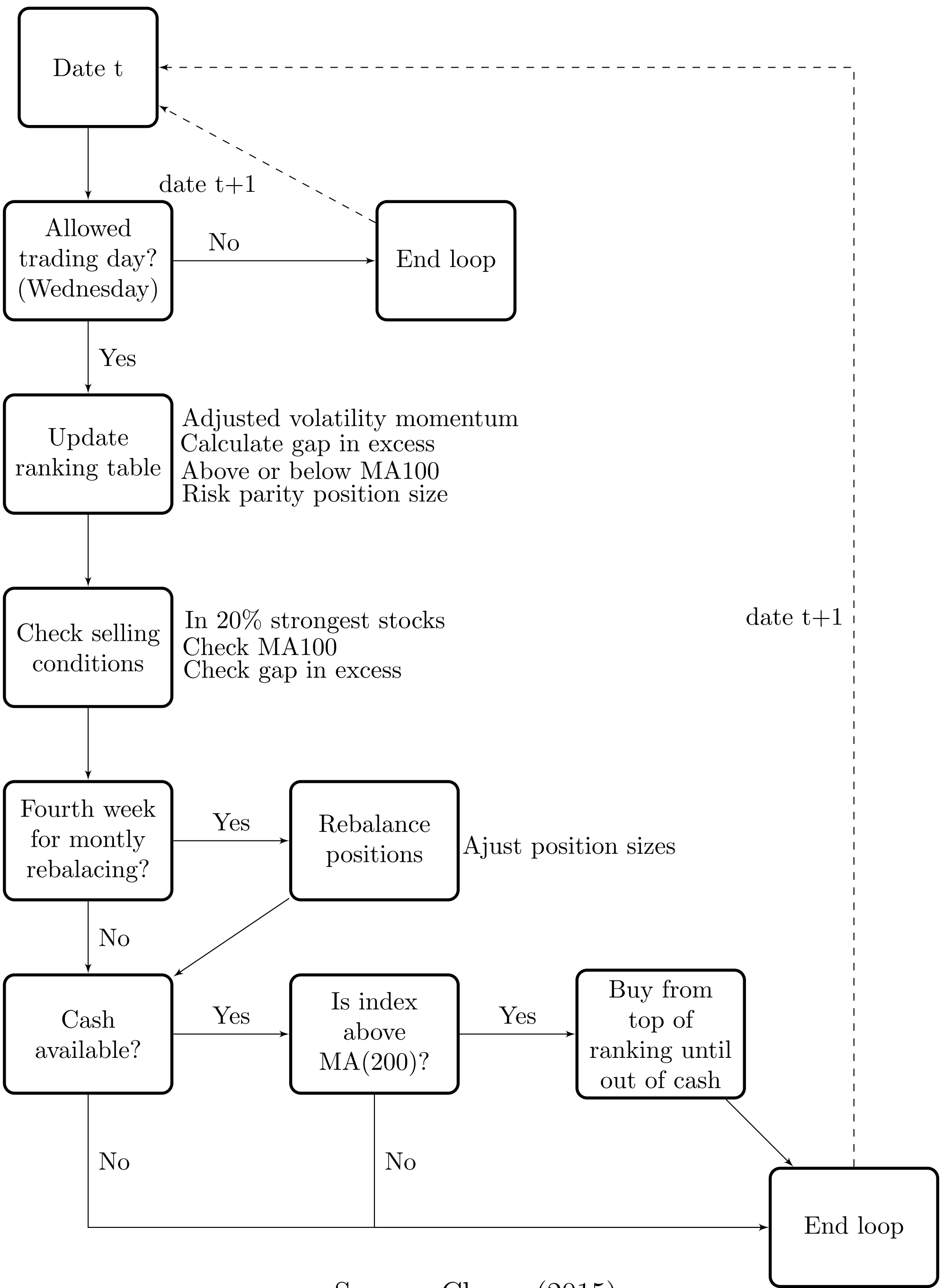
The rank presented in table I will fluctuate every day, reflecting the price oscillations of the S&P500 constituents acquire over time. Therefore, we need to check if our portfolio stocks are still fulfilling the criteria needed for them to stay. This criteria is keep being a stock with high adjusted volatility momentum.
After we enter a position, we need to give it a little space to move, avoiding too much trading activity and preventing selling good performance stocks too early. Thus, to perform a portfolio rebalance, each stock in the portfolio must be in the 20% top adjusted volatility momentum S&P500 constituents, otherwise we sell it. This means that we keep a stock while it remains one of strongest.
We recall that is only allowed to buy new positions if the index trades above is SMA(200), but we will let stay in the portfolio the already opened positions, as long as they keep showing high momentum performance. This helps avoiding selling perfectly good performance stocks just because the index is trading lower. However, to prevent continuing in the portfolio stocks moving down, we need to set a filter. We use the moving average filter SMA(100). If a stock starts trading below its 100 day moving average, SMA(100) we deploy a selling order. The same applies if a gap move, up or down, larger than 15% occurs.

- **Rebalance positions, considering changes in volatility every month.**

Rebalancing our position size through time is very important. This need results from the fact that risk changes over time, not being a static factor. Hence, we have to rebalance our portfolio, otherwise we will be unbalanced in terms of risk. We proceed this task on a monthly basis to avoid incurring high trading costs.

The flowchart presented in Figure 1 aggregates all trading rule information, and how to logically group them. Please see Clenow (2015)(pp.103). The structure displayed is used as rational to create an automatized process.

Figure 1: Trading rules loop chart



Source: Clenow(2015)

3.2 Volatility adjusted momentum process

The core scope of VAMS investment is to capture the momentum effect prevailing in S&P500 stocks. However, momentum by itself only measures the stock recent performance. It does not take into account the stock volatility, which measures the returns degree of dispersion as stated by Brownlees and Engle (2010). Therefore, for any given point in time, we will take into consideration both the momentum and the volatility when creating our portfolio constituents.

First, we have to find a way of measuring momentum in a stock historical price series. Fabozzi et al. (2010) declare that a possible method to measure momentum is using the mathematical concept of linear regression, as denoted in the following expression;

$$y_i = \alpha + \beta x_i + \varepsilon_i \quad , \tag{3.1}$$

where y_i represents the stock price, x_i denotes time expressed in days, α is the intercept value, β is the slope and ε is the error. Linear regression is a method of fitting a line over a series of prices, where the resulting line will be the best linear fit to the price data, minimizing the least squared errors. The slope of that line give us the direction of the stock price. The linear regression slope on a daily price series is the same as calculating the average incline or decline per day over the same time period.

However, this linear slope will be expressed in currency units, and consequently can't be used to compare the momentum between stocks. A slope of a stock quoting at \$100 has a complete different magnitude of slope from a stock quoting at \$10. This is why Clenow (2015) suggests using exponential regression as measurement of momentum, instead of linear regression. The exponential slope give us the average percentage move a stock is expected to acquire per trading day. This allow us to compare the slopes, i.e, momentum, among different stocks. The greater the exponential slope presented by a stock, the stronger the momentum is.

Momentum can be captured by performing a logarithmic transformation in equation 3.1. Considering the following exponential model;

$$y_i = \alpha e^{\beta x_i} \quad , \tag{3.2}$$

taking the natural log in both sides of the equation, we have the following equivalent equation in the form of a linear regression model, commonly named the *log-linear model*, which is formally represent as follows;

$$\ln y_i = \delta + \beta x_i + \varepsilon_i \quad , \tag{3.3}$$

where $\alpha = e^{\delta}$. The literal interpretation of the estimated coefficient β is that a one-unit increase in x_i will produce an expected increase in $\ln y_i$ of β. In terms of y_i itself, this means that the expected value of y_i is multiplied by e^{β}.[6]

Slope can then be easily calculate by a linear regression line (considering the *log-linear model*) through a set of given points using the following expression:

$$\beta = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(\ln y_i - \ln \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad , \tag{3.4}$$

Where the values of $\bar{x}$ and $\ln \bar{y}$ are the sample means (the averages) of the known $x's$ and the known $\ln y's$.

In this work, we consider the past 90 trading days in Equation 3.4, for slope calculations. This 90 days term represents a medium term period of 4.5 months for past stocks return. Clenow (2015) shows the robustness of the exponential regression model in capturing momentum for this period, achieving good results. This is within the range 3- to 12 month holding period referred by Jegadeesh and Titman (1993) as showing momentum prevalence. Since regression slopes are often very small numbers, for numerical practical reasons, we annualize the stock slope as follows:

$$\beta_{annualized} = e^{250.\beta_{daily}} - 1 \quad , \tag{3.5}$$

Now that we have presented how to measure the momentum effect in our investment

[6] For small values of β, we can approximately assume $e^{\beta} \approx 1 + \beta$. This means that $100.\beta$ is the expected percentage change in y_i for a unit increase in x_i. For instance, if $\beta = .06$, $e^{.06} \approx 1.06$, then a 1-unit change in x_i corresponds to (approximately) an expected increase in y_i of 6%. This is the approach we need to be able to compare momentum performance among stocks.

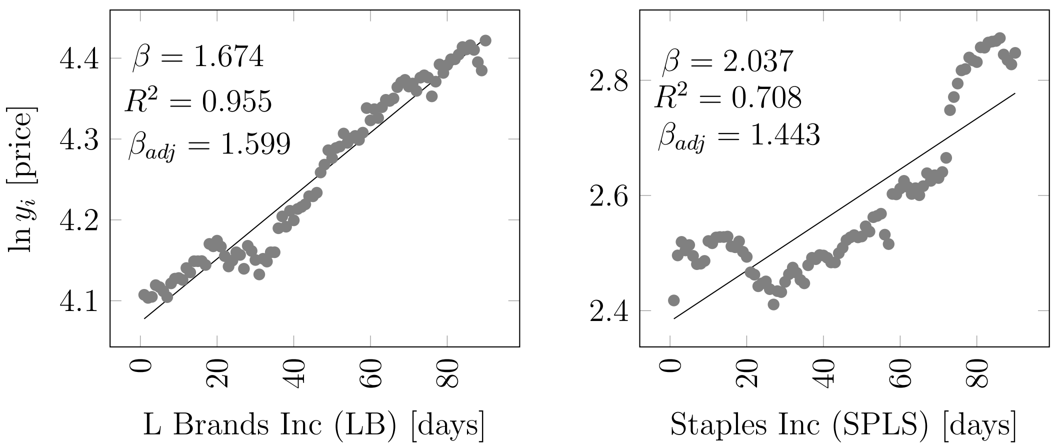
strategy, we need to adjust it for volatility. This adjustment is done multiplying the annualized exponential regression slope, β, by the coefficient of determination, R^2.[7] The higher the volatility, the worse the punishment through slope adjustment. The result of this product, known as volatility adjusted momentum, β_{adj}, is used to rank the S&P500 constituents.

As previously showed in Table I, those higher ranked, i.e, displaying higher β_{adj} are the first in line to be invested.

The graphs presented in Figure 2 show two examples of the volatility adjusted momentum process, for a certain date t.

SPLS presents the strongest momentum, since its β achieves the higher number ($\beta^{SPLS} = 2.037$ vs $\beta^{LB} = 1.674$). However, it also exhibits the highest volatility as consequence of the coefficient of determination, R^2, being the lowest among both stocks ($R^2_{SPLS} = 0.708$ vs $R^2_{LB} = 0.955$). This means that the ranking, represented by the volatility adjusted momentum beta, β_{adj}, is more punished in SPLS than in LB ($\beta^{SPLS}_{adj} = 1.443$ vs $\beta^{LB}_{adj} = 1.559$). Prices are less erratic in LB being the uptrend smoother than the one presented by SPLS.

Figure 2: Volatility adjusted momentum examples



[7]This coefficient will take values between 0 and 1. An R^2 of 1 indicates that the regression line perfectly fits the data, and the lower R^2 is, the worse the fit is. Thus, a stock exhibiting lower volatility will present a higher R^2, while a more volatile stock will get a lower R^2 value.

3.3 Risk parity positions and rebalancing process

3.3.1 Position size

The portfolio asset allocation method is very important, because when determining position size, we are actually allocating risk. If all stocks have identical volatility, an equally weighted positioning size method would be satisfactory. However, volatility among stocks differs and we use a risk parity asset allocation process. This means we distribute the same risk per portfolio position. Thus, for a certain date t, the amount of shares to be invested and guarantee risk parity for each stock, is given by the following equation;

$$Shares_t^i = \frac{AccountValue_t * RiskFactor}{ATR_t^i} \quad , \tag{3.6}$$

where *AccountValue* is the amount of money we have in our disposal at time t, *Riskfactor* represents the daily impact for the stock in the overall portfolio and ATR states for average true range (ATR), which is an indicator that measures volatility. For example, if we set *riskfactor* equal to 0.001, then we are targeting a daily impact in the portfolio of 0.1%, or 10 basis points. The lower this number is, the higher the number of stocks we will have in our portfolio. Therefore, diversification will increase as we lower the *riskfactor*.

ATR gives us an *n-day* smoothed moving average (SMMA) of the true range values, being the true range the maximum of day's high to low or move from previous day. True range can be obtained by using the following expression;

$$TR = max[(high - low), abs(high - close_{prev}), abs(low - close_{prev})] \quad , \tag{3.7}$$

The ATR at the moment of time t can be generically calculated as follows;

$$ATR_t = \frac{ATR_{t-1} * (n-1) + TR_t}{n} \quad , \tag{3.8}$$

We take $n = 20$, as it represents the average number of working days in a month. The first ATR value to be considered is calculated using the arithmetic mean formula;

$$ATR = \frac{1}{n}\sum_{i=1}^{n}(TR_i) \quad , \tag{3.9}$$

Considering this risk based asset allocation heuristic approach we can aim for a risk parity portfolio, where every stock as an equal theoretical chance to impact our strategy overall portfolio.

3.3.2 Position rebalancing

Since the ATR indicator can easily capture the volatility dynamics, we will use the ATR based formula presented in equation 3.6 to rebalance our position every month ($\delta = 20$).

$$Shares^i_{t+\delta} = \frac{AccountValue_{t+\delta} * RiskFactor}{ATR^i_{t+\delta}} \quad , \tag{3.10}$$

This means that in a monthly basis, we change the number of shares of each stock, in order to obtain a steady risk allocation over time. Please pay attention to the fact that the *AccountValue* changes through time, impacted by the performance of all portfolio position over time.

In order to prevent doing many small trades when rebalancing, we set a minimum value of 10%[8] in shares variation between months.

[8]In our simulations, this is the optimal rebalance value in order to achieve a constant risk allocation in our portfolio.

Chapter 4

Data and methodology

In this chapter we describe the data and the methodology we used for constructing the volatility adjusted momentum strategy (VAMS) previously defined.

4.1 Data

We draw your attention to the fact that our strategy comprises only stocks. Other financial assets such as bonds, commodities or derivatives are not taken into consideration. In order to significantly reduce the amount of data to be analyzed, we consider the assumption of not existing trading costs[1]. Our main source for stock data is Yahoo Finance[2]. We employ their free database website to download and compile historical prices for all the index components. This website has the massive advantage of providing historical price time-series accounting to cash dividends and splits. This is highly important because to get a correct picture of the actual financial development of a share price, the series need to be adjusted back in time, and Yahoo Finance allows it.

The index we consider as benchmark and as source of picking our momentum strategy stocks is the S&P500[3]. We collect the S&P500 historical index components membership and index delisted stocks by using a Bloomberg[4] terminal.

The data-set spans a eleven years period, between January 2005 to December 2015,

[1]In real world trading costs can't be ignored. Later in our project we conduct a stress test to VAMS with the propose of measuring the impact of these in our portfolio.

[2]http://finance.yahoo.com/

[3]The S&P500 index is itself a momentum index, therefore by definition this index is essentially a very long term investment momentum strategy. A stock to be considered for inclusion in the SP&500, market capitalization must be over 5.3 billion dollars. This means that a certain stock is part of the index because it had had a strong price development in the past. When a stock leaves the index, it's usually because it had poor price performance and dropped below the market cap requirement.

[4]http://www.bloomberg.com/europe

comprising a total of 574 weeks. This translates into constructing a portfolio in January 2005 and manage it according to VAMS until December 2015. We also study performance for other holding periods. These consist in similarly constructing a portfolio in the beginning of all the subsequent years until Dec 2015. For example, Jan 2006 to Dec 2015, Jan 2007 to Dec 2015, etc. Thus, a total of 11 backtests are performed to VAMS.

In order to perform and backtest our strategy we collect historical daily stock prices on a weekly basis. We analyzed over 280000 stock time-series and performed over 15000 trades distributed through the 11 main backtests we performed. An average of 4.6 trades per week.

We provide further confirmation regarding VAMS effectiveness by carrying out a out-of-sample test, from January 2016 to July 2016. This comprehends a total of 28 weeks.

4.2 Methodology

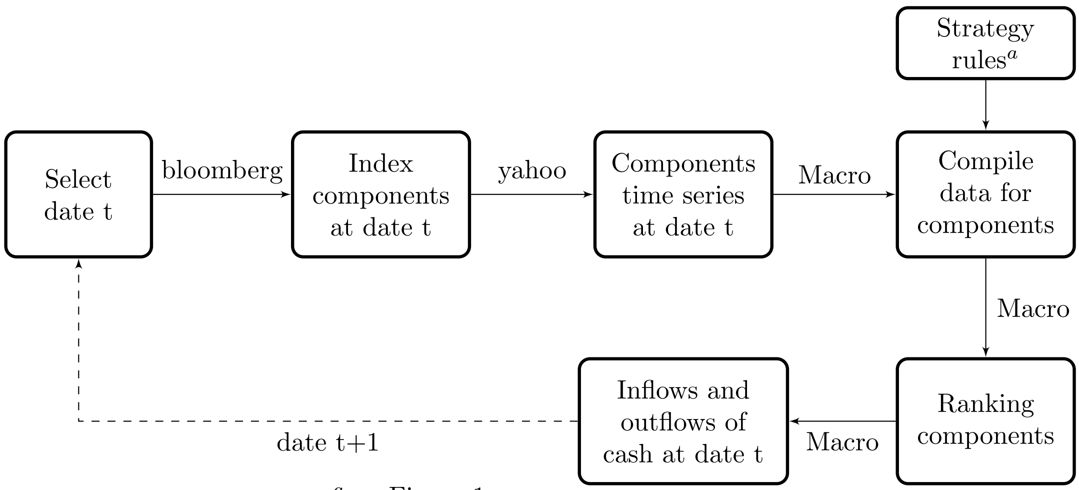
Our main objective is to capture the momentum effect of a large set of stocks, and allocate our investment in the stocks showing the strongest momentum performance. Therefore, we need to find a proper way to rank the S&P500 stock constituents, considering all of our strategy features and particularities. The flowchart presented in Figure 3 illustrates the overall implementation process step by step. As VAMS is fully automatized this rational is not time-depended, i.e. it can be used for any given time interval as long as Yahoo and Bloomberg databases are accessible.

The steps to proceed are as follows, being also illustrated in Figure 3;

- The first step comprehends choosing a certain date t for which we want to set up our portfolio.
- Secondly, we consider the index components, for the previously selected date t, as potential stocks to be included in our portfolio.
- Next, after knowing which index stocks should be considered, we access to their historical price time series.

- According to the VAMS rules described in section 3.1, compile all the information for every index components.
- Considering all the compiled information we rank all the index components by momentum.
- Create, delete or rebalance positions according to the new inflows and outflows of cash necessities.

Figure 3: VAMS implementation flowchart



[a] see Figure 1

Chapter 5

Results

In this chapter we present the overall results achieved by applying VAMS to the S&P500 index. We compare the strategy' performance with the index itself, in terms of absolute returns, annualized returns, risk-adjusted returns (Sharpe ratio) and value-at-risk (VaR), as well as descriptive statistics. We also compare drawdowns performance[1].

We study performance for 11 different holding periods, as previously described, being the longest from Jan 2005 to Dec 2015.

Later, we realize a stress test to VAMS. This test consists in accounting trading costs in VAMS, and see how performance is affected. Multiple possibilities, per stock trade, are considered.

Finally, in order to check VAMS robustness, we perform a out-of-sample process. The period selected goes from Jan 2016 to Jul 2016. Even though a longer out-of-sample period would have been preferable this will be helpful on guiding us on how well the system might perform with new data.

5.1 VAMS vs benchmark performance

We start by presenting, in Table II, the overall results VAMS achieved for the 11 years investment period, and compare it to the outcomes obtained by the benchmark. The VAMS approach outperforms the index, with absolute returns 52% higher (179.5% vs 118.0%). In annualized terms the values are 9.8% and 7.4%, for the VAMS and the index respectively. In terms of risk-adjusted returns, VAMS performs also better with a Sharpe ratio of 0.64 vs 0.42 of the S&P500. The huge difference

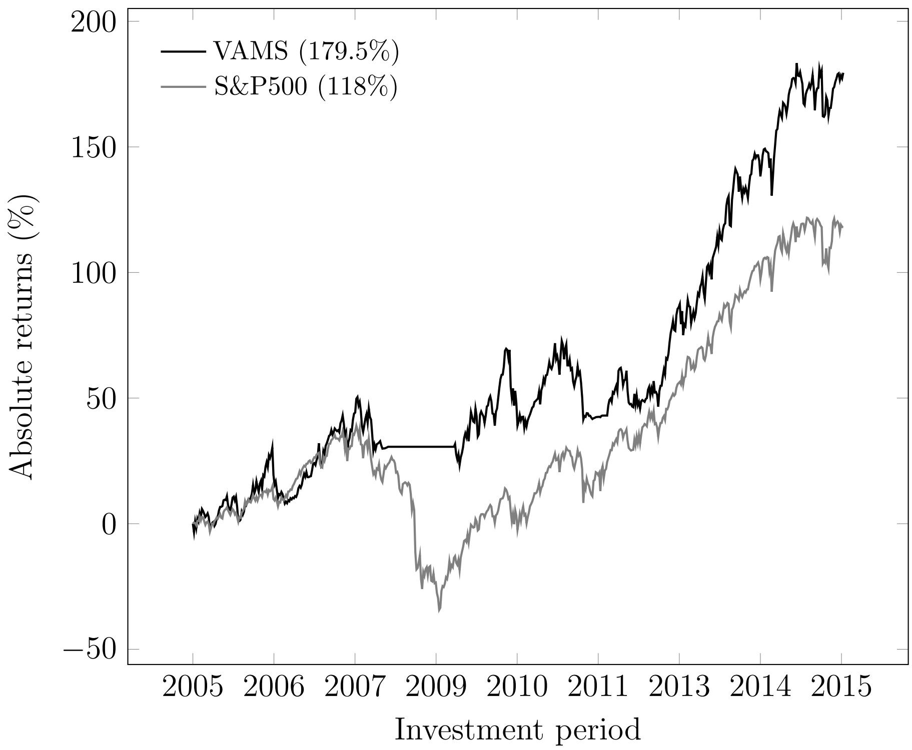
[1] A drawdown is the peak-to-trough decline during a specific recorded period of an investment. In our work the time period is 1 year. The drawdown is quoted as the percentage between the peak and the subsequent trough.

Table II: 11 years VAMS performance (Jan 2005 - Dec 2015)

	2005-15 Period	
	VAMS	S&P500
Absolute Returns	179,5%	118,0%
Annualized Returns	9,8%	7,4%
Sharpe Ratio	0,63	0,42
Max. 1Y Drawdown	-18,6%	-48,1%
Historical VaR 95%	-0,033	-0,036
Gaussian VaR 95%	-0,032	-0,037

between 1 year maximum drawdowns, and inferior VaR[2] values, corroborate that VAMS allow for a better risk control than passively investing in the S&P500. Figure 4 shows how absolute returns progressed during the eleven years investment period, while Figure 5 illustrates the development of annualized returns.

Figure 4: Absolute returns dynamic performance - Jan 2005 to Dec 2015

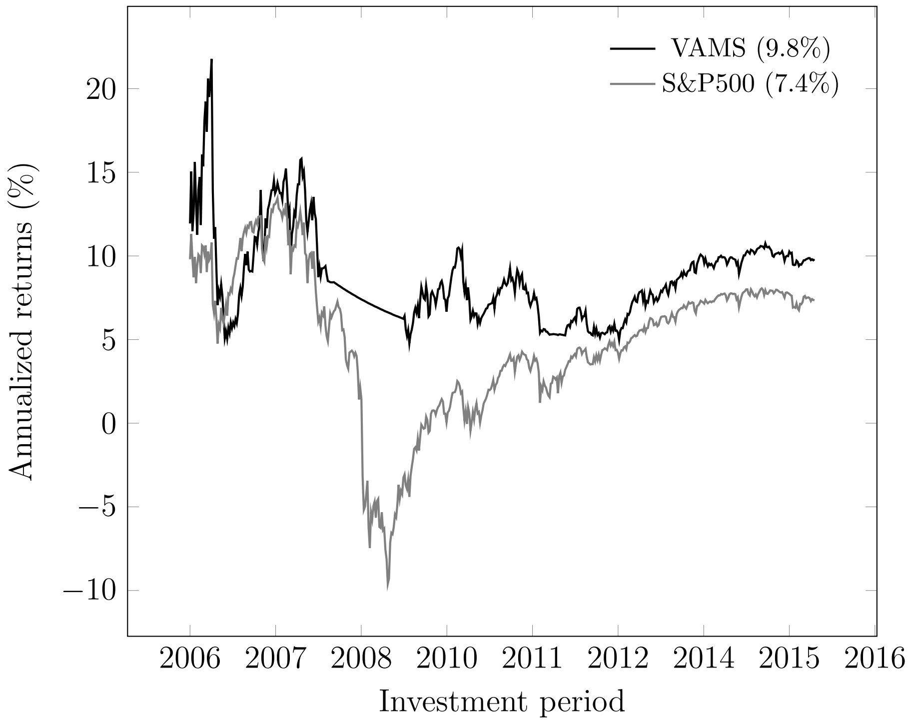


In terms of absolute returns, VAMS also outperforms the index. From Figure 4 it is clear that our approach exceeds the index almost the entire investment period, except for a small time period in the beginning years. It is also evident the capability

[2]VaR represents the maximum potential loss which can occur to a portfolio of an investor, for certain level of confidence and for a given time period. All VaR results in our work considers a 1 week time frame. If the confidence level is 95%, this means that 5% of the times the loss will be larger than what VaR predicted. VaR does not give any information about the severity of loss.

of VAMS to avoid major market turns, such as the market meltdown occurred in the 2008-09 financial crisis. For further information in terms of each year relative gains, see Table VIII in Appendix A. To consult VAMS constituents over time at the beginning of each year, see Table IX in Appendix A.
The annualized returns progression over time, exhibited in the Figure 5, helps supporting VAMS better performance. Before stabilizing near the 10% value, our approach produces an annualized return always positive and above 5%. The index exhibits most of the time lower and more volatile values.

Figure 5: Annualized returns dynamic performance - Jan 2005 to Dec 2015



More insights can be withdraw from Table III, specifically the apparent lower risk VAMS shows when compared to the index. The majority of the years show in terms of Sharpe ratio, maximum 1Y drawdown and VaR, better performance as exhibit higher values. VaR calculations consider two quantitative approaches, the historical VaR and gaussian (parametric) VaR, for 95% confidence levels[3]. VAMS also presents

[3]We recall the historical approach as assuming that the returns in the future will have the same distribution as they had in the past, while the gaussian approach assumes returns as being normally distributed. We performed the Shapiro–Wilk test in order to check for normality. The p-values obtained were $2.85e-12$ for our strategy returns and $2.2e-16$ for the S&P500 returns. The null hypothesis is rejected and there is evidence that the data tested are not normally distributed.

more coherent values through the years, while the index is more inconsistent.

Table III: Risk indicators evolution by year - Jan 2005 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawdown VAMS	Maximum 1Y Drawdown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2005	0,80	1,02	-9,0%	-5,7%	-0,034	-0,016	-0,029	-0,018
2006	0,58	1,22	-17,1%	-6,8%	-0,032	-0,016	-0,032	-0,017
2007	0,74	0,77	-11,1%	-9,3%	-0,033	-0,025	-0,034	-0,024
2008	0,47	-0,27	-14,2%	-44,6%	-0,032	-0,038	-0,031	-0,041
2009	0,56	0,07	-7,6%	-48,1%	-0,033	-0,044	-0,031	-0,043
2010	0,53	0,18	-18,6%	-14,4%	-0,034	-0,043	-0,033	-0,042
2011	0,34	0,16	-18,1%	-16,8%	-0,033	-0,044	-0,033	-0,043
2012	0,42	0,26	-14,9%	-7,0%	-0,033	-0,041	-0,033	-0,041
2013	0,58	0,39	-6,6%	-3,8%	-0,033	-0,039	-0,032	-0,039
2014	0,65	0,43	-7,5%	-6,7%	-0,033	-0,038	-0,032	-0,038
2015	0,64	0,42	-7,5%	-9,1%	-0,033	-0,036	-0,031	-0,037

Next, in Table IV we compile the main descriptive statistics, for both VAMS and the S&P500, while histograms in Figure 6 give some extra insights. The presented statistics concern not only the 11 years investment period covered so far but also 3 other important investment horizons. The 2007-15 period which comprehends the beginning of the financial crisis, the 2009-15 period that concerns the financial crisis ending and lately, the considerable 2012-15 bull market.

VAMS exhibits better performance in terms of annualized mean returns in all cases, excepting for the 2009-15 period. The standard deviation, as well as the absolute values of maximum and minimum weekly returns, are in almost situations higher for the S&P500. This might be an indication of higher volatility presence in the index. Both VAMS and the index present negative skewness. VAMS always presents a value of skewness higher than -1, which allows us to consider its returns as moderately skewed. On the other hand, the index returns half of times present a skewness lower then -1, making it highly skewed. This implies that negative returns are more common when investing in the index.

VAMS kurtosis are lower than 3. This indicates that our approach distribution produces fewer and less extreme outliers than the normal distribution. This situation helps investors avoiding extreme impacts in their overall balance. The index returns

Table IV: VAMS and S&P500 descriptive statistics

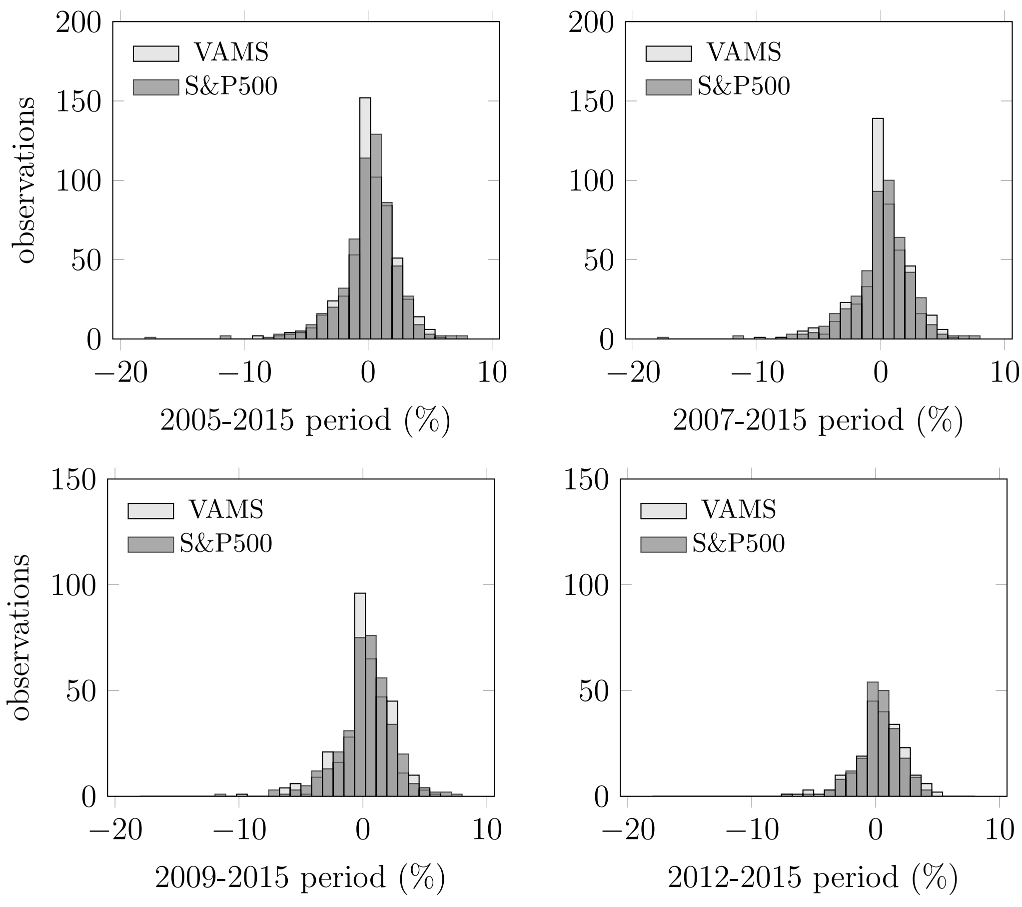
2005–2015	Ann. Mean	Ann. St.Dev	Skewness	Kurtosis	Minimum	Maximum
VAMS	9,33%	14,67%	-0,80	2,06	-8,99%	5,56%
S&P500	7,28%	16,73%	-1,32	8,44	-17,42%	9,56%
VAMS and S&P500 correlation = 0,6068						

2007–2015	Ann. Mean	Ann. St.Dev	Skewness	Kurtosis	Minimum	Maximum
VAMS	10,31%	15,01%	-0,78	2,14	-9,68%	5,47%
S&P500	6,24%	18,02%	-1,28	7,43	-17,42%	9,56%
VAMS and S&P500 correlation = 0,6048						

2009–2015	Ann. Mean	Ann. St.Dev	Skewness	Kurtosis	Minimum	Maximum
Strategy	11,35%	15,09%	-0,75	1,87	-9,72%	5,43%
S&P500	13,75%	16,24%	-0,59	3,40	-11,65%	9,56%
Strategy and S&P500 correlation = 0,7147						

2012–2015	Ann. Mean	Ann. St.Dev	Skewness	Kurtosis	Minimum	Maximum
Strategy	17,47%	14,20%	-0,63	1,28	-6,80%	5,26%
S&P500	13,99%	11,68%	-0,61	1,89	-6,88%	3,98%
Strategy and S&P500 correlation = 0,8402						

Figure 6: VAMS vs S&P500 weekly returns histograms - Multiperiods



distribution generates a very high kurtosis because it produces more outliers than the normal distribution. The "heavy tails" presence can easily be confirmed when checking the index histograms in figure 6.
It is important to notice that VAMS returns are moderately positively correlated (+0.6048 to +0.6068) to those presented by the index in the periods that cover the 2008-09 financial crisis. The correlation substantially increases in the years after the crisis (+0.7147 to +0.8402). A positive correlation is expected since our portfolio constituents are all selected from the S&P500. However, the discrepancy in correlation values, before and after the crisis, indicates the capability of VAMS outperform the index in market turmoils. Correlation decreases in market turn downs because VAMS stops investing while the index keeps moving.

Table V provides a complete overview of VAMS performance versus the one obtained by the S&P500. The table background is presented in two different colors. In case of being displayed in light gray, the S&P500 index performed better, otherwise it is VAMS that has outperformed. More detailed information can be consulted in Appendix A.
In 9 of the 11 backtests, VAMS seems to outperform the index, presenting only worst outcomes in the 2009-15 and 2010-15 tests. These results make us strongly believe that in the long run VAMS is expected to outperform the benchmark S&P500, specially in terms of reducing portfolio risk and avoiding exposure to major market turns.

Table V: Strategy and benchmark main results

	2005-15 Period		2006-15 Period		2007-15 Period	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Absolute Returns	179,5%	118,0%	169,5%	98,6%	153,4%	75,6%
Annualized Returns	9,8%	7,4%	10,4%	7,1%	10,9%	6,5%
Sharpe Ratio	0,63	0,42	0,65	0,39	0,69	0,35
Max. 1Y Drawdown	-18,6%	-48,1%	-19,4%	-48,1%	-19,4%	-48,1%
Historical VaR 95%	-0,033	-0,036	-0,033	-0,038	-0,034	-0,040
Gaussian VaR 95%	-0,032	-0,037	-0,033	-0,038	-0,032	-0,040

	2008-15 Period		2009-15 Period		2010-15 Period	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Absolute Returns	121,8%	68,2%	121,8%	162,5%	90,1%	104,6%
Annualized Returns	10,4%	6,7%	12,1%	14,8%	11,3%	12,7%
Sharpe Ratio	0,70	0,35	0,75	0,85	0,70	0,81
Max. 1Y Drawdown	-19,7%	-48,1%	-19,7%	-20,9%	-19,7%	-16,8%
Historical VaR 95%	-0,033	-0,040	-0,033	-0,036	-0,034	-0,034
Gaussian VaR 95%	-0,030	-0,041	-0,032	-0,034	-0,033	-0,031

	2011-15 Period		2012-15 Period		2013-15 Period	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Absolute Returns	79,7%	78,7%	101,8%	75,0%	74,7%	49,7%
Annualized Returns	12,5%	12,4%	19,2%	15,1%	20,5%	14,4%
Sharpe Ratio	0,80	0,82	1,23	1,192	1,35	1,162
Max. 1Y Drawdown	-19,0%	-16,8%	-9,9%	-9,1%	-8,8%	-9,1%
Historical VaR 95%	-0,033	-0,027	-0,030	-0,025	-0,027	-0,024
Gaussian VaR 95%	-0,031	-0,031	-0,029	-0,024	-0,027	-0,024

	2014-15 Period		2015-15 Period		All Periods Average	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Absolute Returns	29,4%	17,1%	4,7%	3,9%	101,8%	77,4%
Annualized Returns	13,6%	8,4%	4,8%	4,0%	12,3%	9,9%
Sharpe Ratio	0,95	0,64	0,38	0,303	0,80	0,662
Max. 1Y Drawdown	-8,6%	-9,1%	-8,3%	-9,1%	-15,5%	-25,7%
Historical VaR 95%	-0,026	-0,026	-0,027	-0,022	-0,031	-0,032
Gaussian VaR 95%	-0,027	-0,027	-0,026	-0,027	-0,030	-0,032

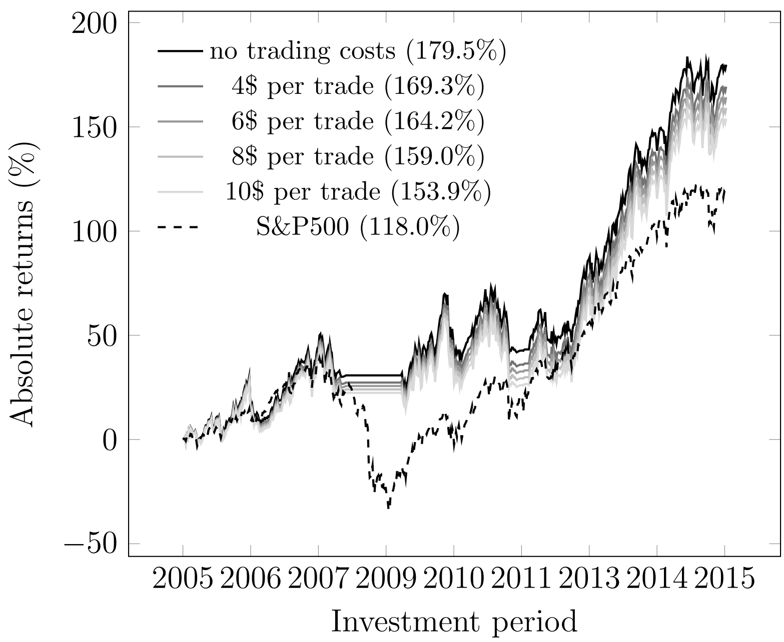
5.2 VAMS trading costs stress test

So far in our study, we have not considered the existence of trading costs in VAMS performance analysis. This was an assumption that reduced considerably the amount of data to be analyzed. However in the real world trading costs exist and cannot be ignored. We now present a stress test that measures the impact these costs would have done to our portfolio absolute returns, considering an initial \$100.000 account, in the 11 years backtest. From Jan 2005 to Dec 2015, which is the longest time period studied.

Trading costs are the brokerage commission payed to a brokerage firm for executing our trades. Each stocks buy or sell incurs on fee that must be deducted from our portfolio account. Since the fee value differs among brokers, we consider 4 distinct values: 4\$, 6\$, 8\$ and 10\$ per trade.

During the 11 years, around 2700 trades were triggered, and Figure 7 shows the degree of susceptibility VAMS has to these. In the worst scenario, 10\$ per trade, absolute returns are only 85% of those achieved by base scenario. In the best scenario returns present a higher value of 94%. Thus, returns can be considerably affected by trading costs, even though VAMS keeps outperforming the index in every scenario.

Figure 7: Trading costs stress test - Jan 2005 to Dec 2015



5.3 VAMS out-of-sample process

We proceed to VAMS an out-of-sample test, considering Jan to Jul 2016 the time period. The period length is too small to be conclusive but some important insights can be extracted. VAMS presents absolute and annualized returns very closed to the ones obtained by the index. However, in terms of risk VAMS clearly outperforms the index. Risk-adjusted returns are significantly higher, being drawdowns and VaR considerably lower. The out-of-sample process provides indications of VAMS capability of reducing portfolio risk and major market turns (see Figure 8), corroborating the findings previously showed in backtests.

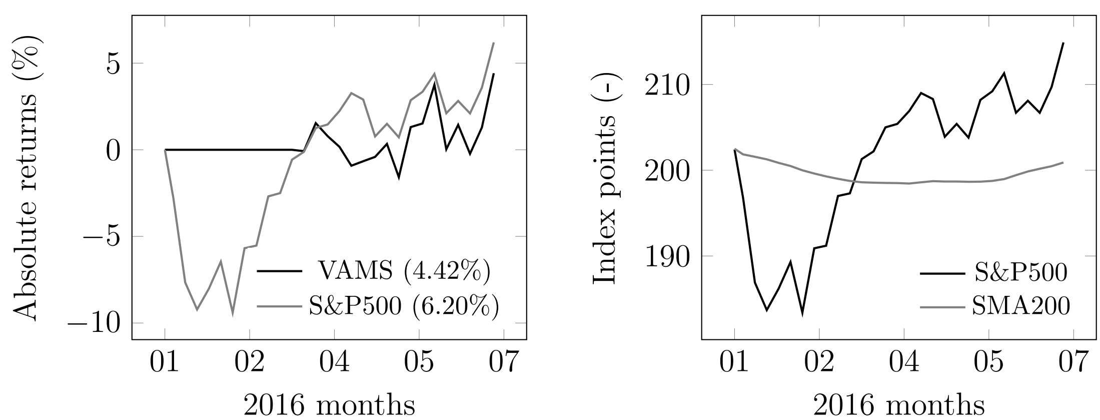
Table VI: 2016 Out-of-sample VAMS vs S&P500 performance

	VAMS	S&P500
Absolute Returns	4,4%	6,2%
Annualized Returns	8,5%	12,0%
Sharpe Ratio	0,80	0,34
Max. 1Y Drawdown	-3,8%	-9,4%
Historical VaR 95%	-0.008	-0,041
Gaussian VaR 95%	-0,015	-0,033

Table VII: 2016 Out-of-sample descriptive statistics

	Ann. Mean	Ann. St.Dev	Skewness	Kurtosis	Min	Max
VAMS	7,76%	9,76%	-0,18	1,88	-3,65%	3,05%
S&P500	10,79%	14,22%	-0,73	0,90	-5,16%	4,03%
VAMS and S&P500 correlation = 0,3138						

Figure 8: Absolute returns and index SMA200 evolution - Jan 16 to Jul 16



Chapter 6

Conclusion

We have proposed a fully automatized active investment strategy that is expected to outperform the passive investments. The so called volatility adjusted momentum strategy (VAMS) is immune to human emotions, which are documented to have a deep negative influence in investment decisions and performance.

VAMS exploits the well known market phenomena, declared by Jegadeesh and Titman (1993) as the *momentum effect*. The VAMS proposed in this study improves on the traditional momentum strategies by adjusting each securities momentum to volatility and by using a risk parity asset allocation process.

The results show that, at least for the S&P500, and for various investment horizons from 2005-2015, VAMS achieves higher returns with lower exposition to risk, and proves capability of stanching major market downturns. For 9 of 11 investment horizons considered, VAMS performs better with absolute and annualized returns, as well as in risk-adjusted returns. A substantial better performance is also achieved in terms of drawdowns and VaR. Good results are also accomplished by VAMS for the out-of-sample process, specially in terms of risk-adjusted returns, drawdowns and VaR.

The elaborated stress test shows that VAMS returns can be affected if tradings costs are high. Nevertheless, VAMS still outperforms the index in every scenario.

It is important to develop VAMS considering other benchmarks. Specifically, multiple currency markets. This will bring new challenges such as: hedging exposure, costs in foreign exchange spreads and higher capability of database management.

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Appendix A

Examples, details and outcomes

Figure 9: S&P500 trend filter examples

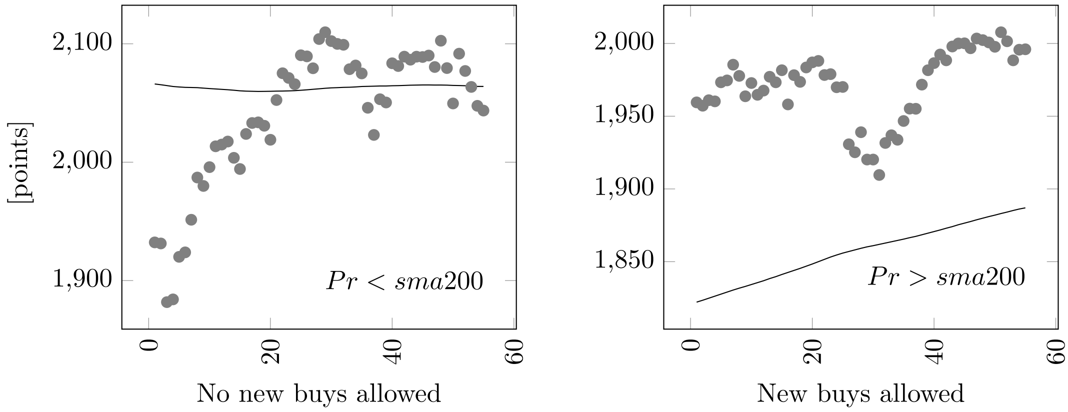

Figure 10: Stock buying filters examples - Up trend

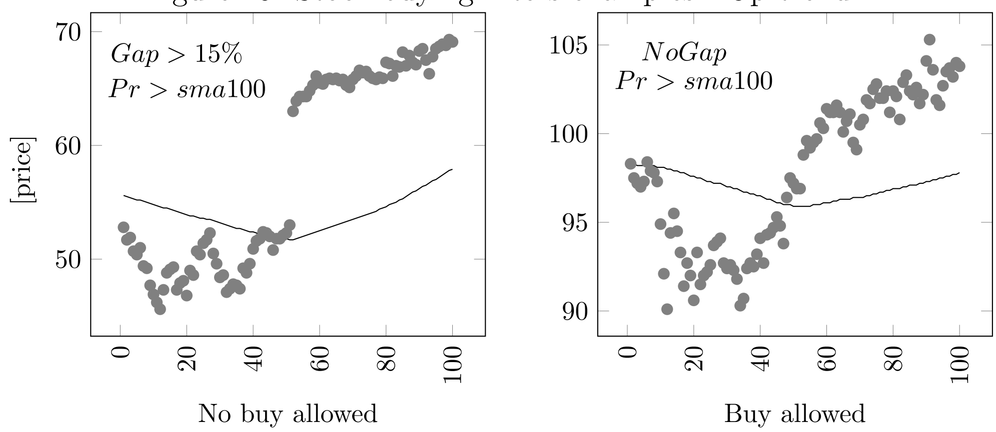

Figure 11: Stock buying filters examples - down trend

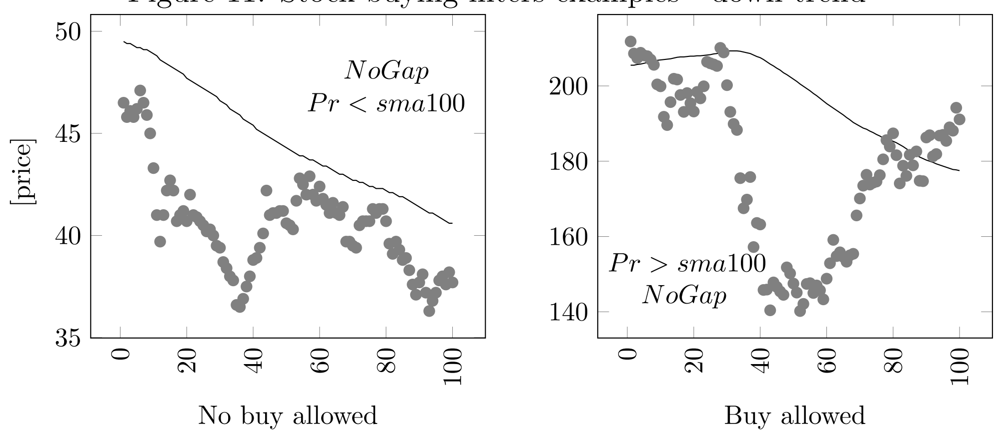

Table VIII: Yearly relative returns - 2005 to 2015

	2005	2006	2007	2008	2009	2010
VAMS	11,9%	6,3%	19,8%	-8,3%	14,6%	7,9%
S&P500	9,8%	13,1%	4,4%	-35,9%	28,3%	14,5%

	2011	2012	2013	2014	2015	Average
VAMS	-11,4%	16,0%	32,3%	21,2%	5,1%	10,5%
S&P500	2,1%	16,9%	27,8%	12,8%	3,9%	8,9%

Table IX: VAMS vs S&P500 constituents, 11 years investment (2005-2015)

	2005		2006		2007		2008	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Technology	31,0%	12,0%	25,9%	11,4%	22,2%	11,0%	3,2%	10,8%
Non-cyclical	24,1%	19,4%	18,5%	20,2%	7,4%	19,8%	19,4%	19,6%
Cyclical	10,3%	14,8%	11,1%	14,2%	29,6%	13,6%	6,5%	13,6%
Utilities	3,4%	6,6%	0,0%	6,4%	0,0%	6,4%	3,2%	6,2%
Financial	6,9%	16,4%	14,8%	16,8%	7,4%	17,4%	19,4%	18,4%
Industrial	6,9%	12,6%	11,1%	12,4%	3,7%	12,4%	12,9%	12,4%
Communic.	6,9%	7,8%	11,1%	8,0%	22,2%	8,6%	3,2%	7,2%
B.Materials	6,9%	5,0%	7,4%	4,6%	7,4%	4,4%	9,7%	4,4%
Energy	3,4%	5,4%	0,0%	5,8%	0,0%	6,4%	22,6%	7,2%

	2009		2010		2011		2012	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Technology	0,0%	10,8%	12,5%	11,2%	16,1%	10,6%	6,3%	10,0%
Non-cyclical	0,0%	20,6%	12,5%	21,0%	3,2%	20,4%	15,6%	20,4%
Cyclical	0,0%	12,6%	18,8%	12,2%	29,0%	12,4%	40,6%	12,8%
Utilities	0,0%	6,6%	3,1%	6,6%	0,0%	6,6%	0,0%	6,4%
Financial	0,0%	16,6%	12,5%	16,2%	0,0%	16,6%	6,3%	16,4%
Industrial	0,0%	13,0%	21,9%	12,2%	9,7%	12,0%	6,3%	12,6%
Communic.	0,0%	7,0%	3,1%	7,2%	9,7%	7,8%	9,4%	7,4%
B.Materials	0,0%	4,4%	9,4%	4,8%	6,5%	4,8%	3,1%	4,8%
Energy	0,0%	8,2%	6,3%	8,4%	25,8%	8,6%	12,5%	9,0%

	2013		2014		2015		Average	
	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
Technology	0,0%	10,0%	13,0%	9,0%	0,0%	9,0%	13,0%	10,5%
Non-cyclical	17,9%	20,4%	17,4%	20,2%	42,9%	20,0%	17,9%	20,2%
Cyclical	35,7%	13,4%	17,4%	13,8%	33,3%	14,2%	23,2%	13,4%
Utilities	0,0%	6,0%	0,0%	6,0%	0,0%	6,0%	1,0%	6,4%
Financial	14,3%	16,4%	8,7%	16,6%	14,3%	16,8%	10,4%	16,8%
Industrial	10,7%	12,6%	17,4%	13,0%	4,8%	12,8%	10,5%	12,6%
Communic.	10,7%	7,2%	17,4%	7,2%	4,8%	7,8%	9,9%	7,6%
B.Materials	3,6%	4,8%	4,3%	4,8%	0,0%	4,4%	5,8%	4,7%
Energy	7,1%	9,0%	4,3%	9,2%	0,0%	8,8%	8,2%	7,8%

Figure 12: Absolute returns performance - Years 2006 and 2007 to 2015

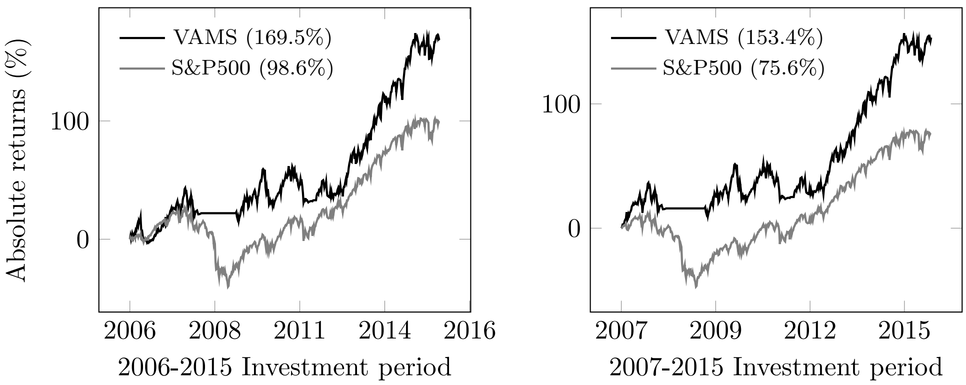

Figure 13: Absolute returns performance - Years 2008 and 2009 to 2015

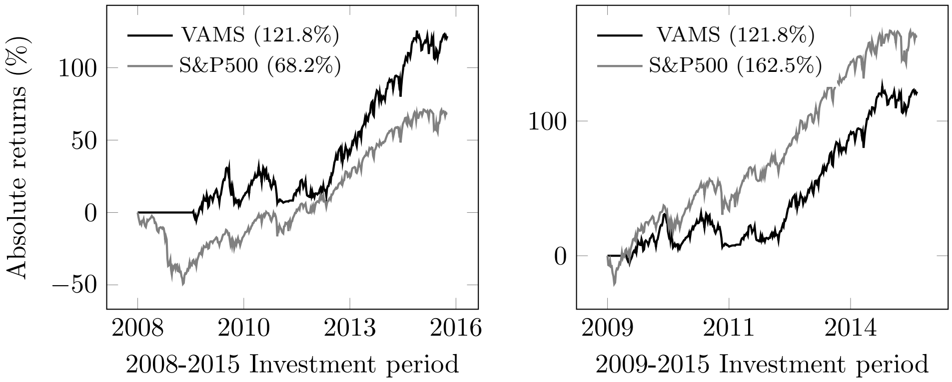

Figure 14: Absolute returns performance - Years 2010 and 2011 to 2015

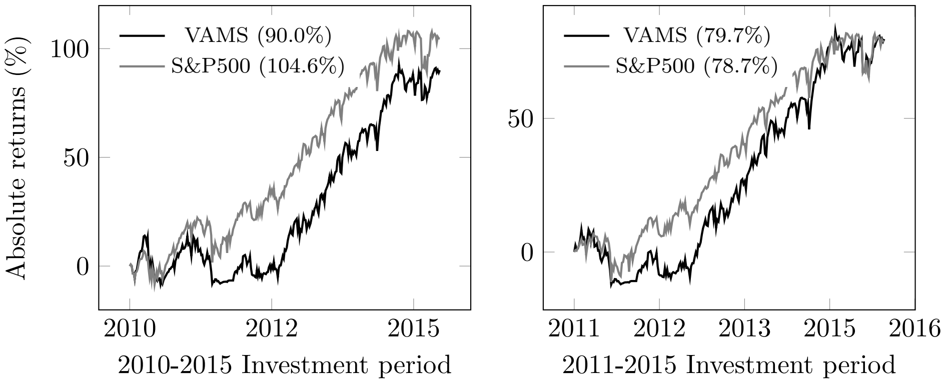

Figure 15: Absolute returns performance - Years 2012 and 2013 to 2015

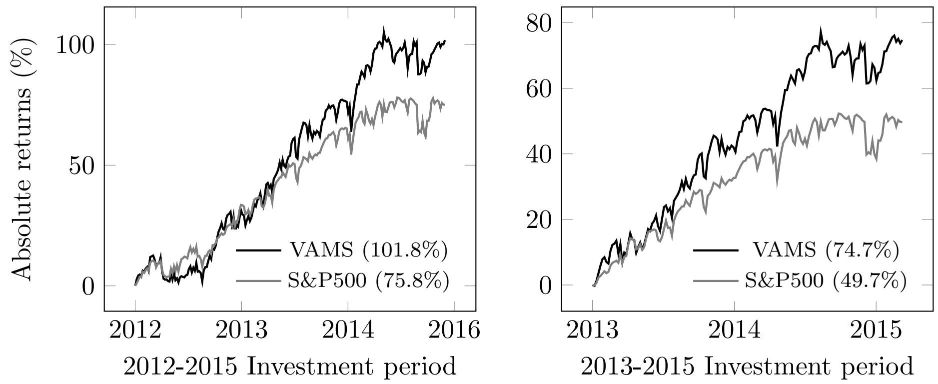

Figure 16: Absolute returns performance - Years 2014 and 2015 to 2015

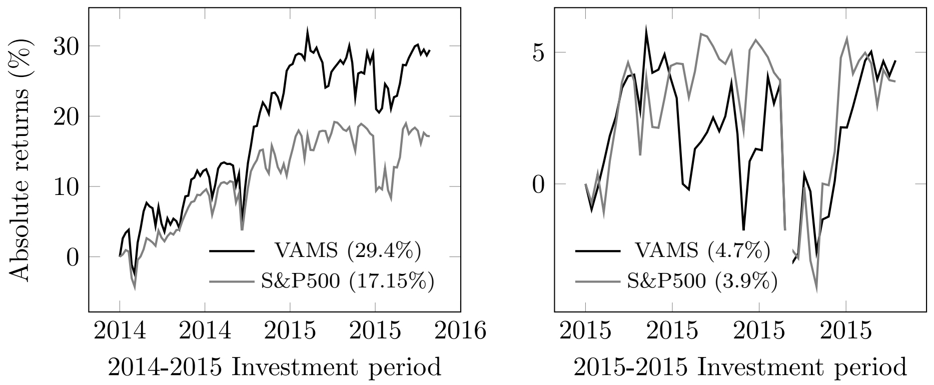

Table X: Risk indicators evolution by year - Jan 2006 to Dec 2015

End	Risk adjusted		Maximum 1Y		Historical VaR		Gaussian VaR	
	Sharpe Ratio		Drawndown		95%	95%	95%	95%
Year	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
2006	0,42	1,42	-18,2%	-6,8%	-0,033	-0,015	-0,035	-0,017
2007	0,90	0,68	-11,6%	-9,3%	-0,035	-0,034	-0,037	-0,026
2008	0,43	-0,48	-16,3%	-44,6%	-0,032	-0,044	-0,033	-0,046
2009	0,54	-0,04	-7,6%	-48,1%	-0,032	-0,047	-0,032	-0,047
2010	0,51	0,11	-19,4%	-14,4%	-0,033	-0,044	-0,034	-0,045
2011	0,30	0,10	-19,0%	-16,8%	-0,035	-0,046	-0,035	-0,046
2012	0,36	0,21	-15,5%	-7,0%	-0,034	-0,044	-0,035	-0,043
2013	0,57	0,36	-6,7%	-3,8%	-0,033	-0,041	-0,034	-0,041
2014	0,67	0,40	-7,5%	-6,7%	-0,033	-0,040	-0,033	-0,039
2015	0,65	0,40	-8,0%	-9,1%	-0,033	-0,038	-0,033	-0,038

Table XI: Risk indicators evolution by year - Jan 2007 to Dec 2015

End	Risk adjusted		Maximum 1Y		Historical VaR		Gaussian VaR	
	Sharpe Ratio		Drawndown		95%	95%	95%	95%
Year	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
2007	1,45	0,29	-11,7%	-9,3%	-0,040	-0,037	-0,037	-0,033
2008	0,50	-0,87	-16,3%	-44,6%	-0,028	-0,051	-0,032	-0,056
2009	0,63	-0,22	-7,6%	-48,1%	-0,029	-0,053	-0,031	-0,054
2010	0,55	-0,02	-19,4%	-14,4%	-0,034	-0,049	-0,034	-0,050
2011	0,29	0,00	-19,1%	-16,8%	-0,034	-0,048	-0,034	-0,050
2012	0,36	0,13	-15,5%	-7,0%	-0,034	-0,046	-0,034	-0,046
2013	0,60	0,30	-6,7%	-3,8%	-0,034	-0,042	-0,033	-0,043
2014	0,71	0,35	-7,5%	-6,7%	-0,034	-0,041	-0,033	-0,041
2015	0,68	0,35	-8,6%	-9,1%	-0,034	-0,040	-0,032	-0,040

Table XII: Risk indicators evolution by year - Jan 2008 to Dec 2015

End	Risk adjusted		Maximum 1Y		Historical VaR		Gaussian VaR	
	Sharpe Ratio		Drawndown		95%	95%	95%	95%
Year	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
2008	N/A	-1,55	N/A	-42,9%	N/A	-0,067	N/A	-0,072
2009	0,72	-0,37	-7,5%	-48,1%	-0,018	-0,058	-0,021	-0,062
2010	0,53	-0,08	-19,7%	-14,4%	-0,032	-0,053	-0,029	-0,055
2011	0,14	-0,04	-19,0%	-16,8%	-0,033	-0,051	-0,031	-0,053
2012	0,33	0,11	-15,4%	-7,0%	-0,033	-0,048	-0,032	-0,048
2013	0,61	0,30	-6,6%	-3,8%	-0,033	-0,043	-0,031	-0,044
2014	0,73	0,36	-7,4%	-6,7%	-0,033	-0,041	-0,031	-0,042
2015	0,70	0,35	-8,7%	-9,1%	-0,033	-0,040	-0,030	-0,041

Table XIII: Risk indicators evolution by year - Jan 2009 to Dec 2015

End	Risk adjusted		Maximum 1Y		Historical VaR		Gaussian VaR	
	Sharpe Ratio		Drawndown		95%	95%	95%	95%
Year	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500	VAMS	S&P500
2009	1,02	1,05	-7,5%	-20,9%	-0,028	-0,051	-0,028	-0,048
2010	0,65	0,93	-19,7%	-14,4%	-0,033	-0,048	-0,036	-0,043
2011	0,16	0,65	-19,0%	-16,8%	-0,036	-0,048	-0,036	-0,045
2012	0,37	0,74	-15,4%	-7,0%	-0,034	-0,041	-0,035	-0,041
2013	0,67	0,92	-6,6%	-3,8%	-0,034	-0,038	-0,034	-0,037
2014	0,79	0,92	-7,4%	-6,7%	-0,033	-0,037	-0,033	-0,035
2015	0,75	0,85	-8,7%	-9,1%	-0,033	-0,036	-0,032	-0,034

Table XIV: Risk indicators evolution by year - Jan 2010 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2010	0,40	0,78	-19,7%	-14,4%	-0,041	-0,041	-0,041	-0,036
2011	-0,20	0,40	-19,3%	-16,8%	-0,038	-0,041	-0,039	-0,043
2012	0,15	0,60	-15,5%	-7,0%	-0,035	-0,037	-0,037	-0,037
2013	0,56	0,88	-6,7%	-3,8%	-0,035	-0,035	-0,035	-0,033
2014	0,72	0,90	-7,4%	-6,7%	-0,034	-0,035	-0,034	-0,032
2015	0,70	0,81	-8,3%	-9,1%	-0,034	-0,034	-0,033	-0,031

Table XV: Risk indicators evolution by year - Jan 2011 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2011	-0,76	0,10	-19,0%	-16,8%	-0,035	-0,043	-0,036	-0,048
2012	0,09	0,51	-15,5%	-7,0%	-0,033	-0,035	-0,034	-0,038
2013	0,69	0,92	-6,6%	-3,8%	-0,033	-0,027	-0,033	-0,032
2014	0,88	0,93	-7,5%	-6,7%	-0,033	-0,027	-0,032	-0,031
2015	0,81	0,82	-8,4%	-9,1%	-0,033	-0,027	-0,031	-0,031

Table XVI: Risk indicators evolution by year - Jan 2012 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2012	0,93	1,27	-9,9%	-7,0%	-0,032	-0,026	-0,032	-0,025
2013	1,42	1,81	-6,9%	-3,8%	-0,032	-0,024	-0,030	-0,021
2014	1,44	1,52	-7,5%	-6,7%	-0,031	-0,025	-0,030	-0,022
2015	1,23	1,20	-8,5%	-9,1%	-0,030	-0,025	-0,029	-0,024

Table XVII: Risk indicators evolution by year - Jan 2013 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2013	2,04	2,50	-5,8%	-3,8%	-0,032	-0,022	-0,026	-0,017
2014	1,73	1,66	-7,5%	-6,7%	-0,027	-0,024	-0,026	-0,021
2015	1,36	1,17	-8,8%	-9,1%	-0,027	-0,024	-0,027	-0,024

Table XVIII: Risk indicators evolution by year - Jan 2014 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2014	1,37	0,99	-7,7%	-6,7%	-0,025	-0,025	-0,029	-0,025
2015	0,96	0,65	-8,6%	-9,1%	-0,026	-0,026	-0,027	-0,027

Table XIX: Risk indicators evolution by year - Jan 2015 to Dec 2015

End Year	Risk adjusted Sharpe Ratio VAMS	Risk adjusted Sharpe Ratio S&P500	Maximum 1Y Drawndown VAMS	Maximum 1Y Drawndown S&P500	Historical VaR 95% VAMS	Historical VaR 95% S&P500	Gaussian VaR 95% VAMS	Gaussian VaR 95% S&P500
2015	0,39	0,31	-8,3%	-9,1%	-0,027	-0,022	-0,026	-0,027