LISBON
SCHOOL OF
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UNIVERSIDADE DE LISBOA

MASTER IN
FINANCE

MASTER'S FINAL WORK
DISSERTATION

TESTING THE RANDOM WALK HYPOTHESIS IN THE PORTUGUESE STOCK MARKET

GONÇALO FILIPE RODRIGUES ALVES

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By Gonçalo Alves

Abstract

This paper investigates the efficiency of the eighteen stocks that constitute the main Portuguese stock index, the PSI-20 of the Lisbon Stock Exchange. Tools used for the investigation were daily and monthly data from January 1999 to May of 2015, using the Augmented Dickey-Fuller (ADF) test, the automatic variance ratio by Choi and the individual and multiple variance ratios, by Lo and Mackinlay and Chow-Denning, which test the efficiency of the eighteen stocks and PSI-20 index. The Augmented Dickey-Fuller (ADF) tests the null hypothesis that the series has a unit root, while the variance ratio tests the random walk hypothesis. Based on these tests, the results provide mixed evidence against the random walk hypothesis. The results for the unit root tests do not reject the efficient market hypothesis for the entire sample, while the results from the variance ratio tests do, but tend to decrease in monthly data.

Keywords: Random walk hypothesis; Stock market efficiency; Variance ratio tests; Unit root test; Euronext Lisbon.

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1. Introduction

The theory of efficiency markets is extremely important in finance since the hypothesis of securities markets represent the origin of most of the research that is made in this field.

The efficient market hypothesis (EMH) in its weak form, (Fama, 1970) is a market in which the price of any financial asset fully reflects all the available information.

If a market is efficient, asset prices exhibit unpredictable movements. In such a market, the use of all the available information does not allow any investor to consistently obtain an unusual return. However, if a market is not efficient, asset prices indicate to have long memory, therefore those prices are not completely random. In this type of market the technical analysis has shown that it is possible to predict and identify trends to achieve abnormal returns.

The random walk hypothesis has strong implications in financial theories and investment strategies. For example, the trading strategies will be different when returns are characterized by random walks or by positive autocorrelations (or persistence) over short horizons and negative autocorrelations (or mean reversion) over long horizons.

In this work, we analyze the efficiency of the Portuguese market, by performing tests of the random walk hypothesis (RWH), including the Augmented Dickey-Fuller (ADF), the automatic variance ratio, the individual variance ratio test by Lo and Mackinlay and the multiple variance ratio tests by Chow and Denning. The use of variance ratios (VR), tests can be very important in order to test the alternative hypothesis of random walk hypothesis (RWH), namely the hypothesis of mean reversion.

Unlike earlier works concerning Portuguese markets, this study is about the eighteen stocks that make up the PSI-20 index, which differ from others studies about the Portuguese Index. One of the most complete works done in Portugal was performed by Dias et al. (2002), with daily data of the PSI-20 index from January 1993 to September 2001. They find strong evidence in support of the random walk hypothesis given by an Augmented Dickey-Fuller (ADF) test.

There is a publication that tests the random walk hypothesis (RWH) of the PSI-20's stocks (Gonzaga and Sebastião, 2012). However, changes in this index have occurred since then, such as the bankruptcy of Banco Espirito de Santo in 2014, and the fact that other companies were delisted. In addition, some companies joined the index as a result of privatizations that Portuguese government implemented (CTT or Banif, are an example).

The remainder of this work is organized as follows: Section 2 reviews the literature of market efficiency; Section 3 describes the theoretical framework; Section 4 explains the methodology; Section 5 describes the data and reports the empirical results; Section 6 summarizes the results of this study.

2. Literature review

The notion of efficient market hypothesis (EMH*)* was developed by Paul A. Samuelson (1965) and Eugene Fama (1965). Both authors argue that in a completely informational efficient market, the prices should be unpredictable, since this market incorporates all available information and expectations of all market participants. Fama (1970) wrote a paper that focuses on three forms of EMH, the weak, semi-strong and strong forms of efficiency. Each one, in different ways, test the predictability of security returns on the basis of past price changes.

The interest on EMH still continues nowadays by academics, searching for a better understanding of the price generation processes, and investors seeking investments that yield higher returns. In the last decade, some authors have argued that the EMH clearly no longer enjoy the level of strong support that was received during the golden era of 1970s. Nascimento (2007) says that until the end of 1970, most of the empirical results were consistent with EMH. In contrast, in the decade of the 80s there were many studies about the rejection of the random walk hypothesis (RWH).

Lo and Mackinlay (1988) developed the variance test and conclude that, despite individual stocks appearing to follow a random walk, the RWH is rejected by weekly data of US indices from 1962 to 1985. Particularly, Lo and Mackinlay (1988) find that variances grow faster than linearly as the holding period increases, implying positive serial correlations in weekly returns.

Chow and Denning (1993) proposed a multiple test alternatively to variance ratio (VR) individual tests, in order to avoid a double rejection of the null hypothesis, since it is

customary to calculate the VR test for various time periods and simply reject the hypothesis for a single period to reject the RWH.

Since the seminal work of Lo and Mackinlay (1988) the variance ratio test has emerged as the primary tool for testing whether stock return series are serially uncorrelated. The variance ratio test is based on the statistical property that if the stock price follows a random walk, then the variance of the k-period return is equal to k times the variance of the one-period return. The variance ratio should be equal to one for any k-period, under the null hypothesis of serially uncorrelated stock returns.

This literature has continued to grow over the last years albeit at a slower pace, with greater emphasis on emerging stock markets. A few examples of those studies are made by Squally (2006) and Moustafa (2004) for the United Arab Emirates stock market. Other studies were performed in South America (Charles and Darné, 2009) in Europe (Smith, 2003; M. Borges, 2010) and in Africa (Al-Khazali et al. 2007). The results from VR tests consistently show that most of the emerging markets are inefficient on the weak-form.

Some studies about the random walk hypothesis in the Portuguese stock market were made, possibly when it was very "immature". Gama (1998) and Smith and Ryoo (2003) used VR test to conclude that the Portuguese stock market was inefficient on the weak form until 1998. Borges (2010) used daily, weekly and monthly data of the PSI-20 index, for the period from January 1 of 1993 to 31 of December of 2006, to find strong correlation on the daily returns. This correlation might decrease on the weekly and monthly data. Dias et al. (2002) arrived at the same conclusion for the period between January 1993 and September 2001 using serial correlation tests and VR tests. Although they find strong evidence in support of the random walk by an Augmented Dickey-

Fuller (ADF) test. Another work that involved the PSI20 index and the twenty stocks, that at the time incorporated this index, was made by Gonzaga and Sebastião (2012). They used daily and weekly data, from 1987 to June of 2010 that was analyzed with serial correlations tests, variance ratio tests and also run tests. The main conclusion was that only during the last three years of their observation (2008-2010) the tests regarding the random walk does not reject the hypothesis of efficiency for the majority of the stocks and also for the PSI-20 index.

3. Theoretical framework

The market efficiency has been under debate for a long time. The main reason of EMH focus on the market capacity to reflect all available information and how that information is replicated in stocks prices. Under the EMH, the information is equal to all the participants, indicating that we cannot use historical prices to estimate future returns. Therefore, investors cannot achieve abnormal returns by buying undervalued stocks or selling inflated stocks.

Fama (1970) classify three forms of efficiency: weak-form, semi-strong and strong-form. He formulates three conditions for market efficiency when the current price of a security reflects all available information:

- There are no transactions cost
- All available information is costless
- All agree on the implications of current information for the current price and the distributions of future prices of each security

The weak-form of efficiency states that stock prices already reflect all available information contained in the historical prices. The semi-strong form states that the stock prices already reflect all publicly available information. The strong-form states that stocks prices reflect all relevant information including insider information.

4. Methodology

In this section we describe the approach to be followed along with the tests to be used in our study.

4.1 Choice of the subject

The Portuguese stock market has a large volatility to all the Euro zone news; for example, the latest news regarding Greek economical issues, penalize more strongly the Portuguese stock market than other European stock market. With higher volatility in the markets, traders can earn higher profits from their portfolios. Therefore we will not only to test the market efficiency for PSI-20 index, but also for the main stocks that make the Portuguese index.

4.2 Unit root test

The Augmented Dickey-Fuller (ADF) test is used to measure the existence of a unit root in a time series sample. It is an augmented version of the Dickey-Fuller test for a larger and more complicated set of time series models. The augmented Dickey–Fuller (ADF) statistic, used in the test, is a negative number. The more negative it is, the stronger is the rejection of the hypothesis of having a unit root at some level of confidence.

The ADF test estimates the following equation through Ordinary Least Squares regression (OLS):

$$\Delta P_t = \alpha_0 + \alpha_1 t + \rho_0 P_{t-1} + \sum_{i=1}^{q} \rho_i \Delta P_{it-I} + \varepsilon_{it} \quad (1)$$

Where P_t is the price at time t and $\Delta P_t = P_t - P_{t-1,}$ p_i are coefficients to be estimated, q is the lag order of the autoregressive process, t is the trend term, α_i is the estimated coefficient for the trend, α_0 is the constant and ε is the white noise.

The null hypothesis on test is the existence of a unit root, meaning that the time series is not stationary. The alternative hypothesis is that the time series does not have a unit root, and consequently is stationary. If we do not reject the null hypothesis, we also do not reject that the time series has random walk properties.

H_0: $p_0 = 0$ (nonstationary time series)

H_1: $p_0 < 0$ (stationary time series)

To see if the time series are non stationary, we have estimated the correlation coefficient to check if $p_0 = 0$, based on τ (tau) statistic and in critical values estimated by Dickey-Fuller. If the absolute value of τ is higher than the critical values, it means that the null hypothesis is rejected.

4.3 Individual Variance Ratio Test by Lo and Mackinlay (1988)

Variance ratio tests have been widely used and are particularly useful for examining the behavior of asset prices. These tests are based on the variance of returns and have good size and power properties against interesting alternative hypotheses and, in this respect, are superior to many other tests (Campbell *et at*., 1997).

The variance of the k-th difference scaled by k to the variance of first difference tends to equal one, that is, the variance of the k-difference increases linearly in the observation interval,

$$VR(k) = \frac{\hat{\sigma}^2(k)}{\hat{\sigma}^2(1)} \quad (2)$$

where σ^2 (k) is 1/k the variance of the k-differences and σ^2 (1) is the variance of the first differences that can be defined,

$$\hat{\sigma}^2(1) = (n-1)^{-1} \sum_{t-1}^{n} (r_t - \hat{\mu})^2 = (n-1)^{-1} \sum_{t-1}^{n} (P_t - P_{t-1} - \hat{\mu})^2 \quad (3)$$

Lo and MacKinlay (1988), proposed for a sample size of $nq + 1$ observations$(P0, P1, \dots, Pnq)$:

$$\hat{\sigma}^2(k) = m^{-1} \sum_{t=k}^{n} (r_t + r_{t-1} + \cdots + r_{t-k+1} - k\hat{\mu})^2 \quad (4)$$

with $m = k(n - k + 1)(1 - kn^{-1})$. Thus, the value of m is chosen such that $\sigma^2(k)$ to be a unbiased estimator of the variance of the return of k-th period and that σ^2_t is constant over the time

Lo and MacKinlay (1988) derive the asymptotic distribution of the estimated variance ratios and suggest two test statistics, M_1(k) and M_2(k), under the null hypothesis of homoskedastic increments random walk and heteroskedastic increments random walk, respectively. If the null hypothesis is true, the associated test statistic has an asymptotic standard normal distribution. Assuming homoskedastic increments, we have

$$M_1(k) = \frac{VR(k) - 1}{\phi(k)^{1/2}}, \ N(0,1) \quad (5)$$

Where $\phi(k) = \frac{2(2k-1)(k-1)}{3kn}$. Besides the $M_1(k)$ they also proposed a test assuming heteroskedastic increments, the test statistic is:

$$M_2(k) = \frac{VR(k)-1}{\phi^*(k)^{1/2}} \text{ , N(0,1)} \quad (6)$$

Where under the null hypothesis, V(k)=1,

$$\phi^*(k) = \sum_{j=1}^{k-1} \left[\frac{2(k-j)}{k}\right]^2 \delta(j) \quad (7)$$

and

$$\delta(j) = \frac{\sum_{t=j+1}^{n}(r_t-\hat{\mu})2}{[\sum_{t=1}^{n}(r_t-\hat{\mu})^2]} \quad (8)$$

The procedure proposed by Lo and MacKinlay (1988) was developed to test individual variance ratio tests for a specific k-difference, but under the random walk hypothesis, we must have VR(k)=1 for all k.

If the returns are positively (negatively) autocorrelated, VR(k) must be higher (lower) by one. The series has mean reversion (level) if VR(k) is significantly less than one and aversion to the average if VR(k) is significantly greater than one. The RWH returns must be tested for various time intervals (lags), for k periods, as rejection for a given value of k, the rejection means of the RWH returns.

4.4 Multiple variance ratio test by Chow and Denning (1993)

Chow and Denning (1993) had proposed a multiple variance ratio test to control the test size. They extended Lo and Mackinlay (1988) conventional variance ratio test methodology and form a simple multiple variance ratio test, which uses Lo and Mackinlay test statistic.

Considering a set of variance ratio estimates,$\{ VR(k_i)|I = 1,2, \dots, m\}$, where m corresponds to a set of pre-defined number of lag and $M_1(k_i)$ and $M_2(k_i)$ the statistical tests Lo and Mackinlay (1988). Under the random walk hypothesis, we test a set of sub-hypotheses:

$$H_{0i} : VR(k_i) = 1, \text{ for } i = 1,2, \dots, m$$

$$H_{1i:} VR(k_i) \neq 1, \text{ for } i = 1,2, \dots, m$$

Since any rejection of H_{0i} will directly reject the RWH, it allows the maximum absolute value of the test statistic be

$$MV_1 \max \mid \text{M1 (ki)}$$

$$1 \leq i \leq m$$

$$MV_2 = \max \mid \text{M2 (ki)}$$

$$1 \leq i \leq m$$

where $M_1(k_i)$ and $M_2(k_i)$ are defined in equations (5) and (6), respectively.

The rejection of the null hypothesis can be based on the maximum absolute value of individual variance ratio test statistic. The test follows a Sudentized Maximum Modulos distribution with M and T (the sample size) degrees of freedom. When T is large, the null hypothesis is rejected at α level of significance if MV_1 [or MV_2] is greater than the $[1 - \left(\frac{\alpha^*}{2}\right)]$ where $\alpha^* = 1 - (1 - \alpha)^{1/m}$.

Chow and Denning (1993) manage the size of the multiple variance ratio test by comparing the calculated values of the standardized test statistics, also $M_1(k_i)$ or $M_2(k_i)$ with the SMM critical values. Every time that the maximum absolute value of $M_1(k_i)$ or $M_2(k_i)$ are greater than the critical value at a predetermined significance level, the RWH is rejected.

4.5 Automatic VR by Choi (1999)

When implementing the VR tests, we need to choose what our holding period (k) is. To overcome this issue, Choi (1999) proposed a data-dependent procedure to determinate the optimal value of k automatically using an optimal data dependent method. Choi (1999) suggested a VR based in frequencies. The VR estimator is defined as

$$VR(k) = 1+2 \sum_{i=1}^{T-1} h\left(\frac{i}{k}\right) \hat{p}(i), \quad (9)$$

Where $\hat{p}(i)$ is the autocorrelation function, and *h(x)* is the Quadratic Spectral (QS) window defined as

$$h(x) = \frac{25}{12\pi^2\ x^2}\left[\frac{\sin\left(\frac{6\pi x}{5}\right)}{6\pi x/5} - \cos\left(\frac{6\pi x}{5}\right)\right] \quad (10)$$

The standardized statistic is

$$VR_f = \frac{VR(k) - 1}{(2)^{1/2}\left(\frac{T}{k}\right)^{-1/2}} \quad (11)$$

The null hypothesis, of no serial correlation, of this test statistic follows the standard normal distribution.

5. Empirical analysis and Results

5.1 Data

Our data is daily adjusted closing prices of the eighteen Portuguese stocks that currently form the PSI-20 index: Altri, BCP, BPI, CTT, EDP, EDP-Renewable, Galp, Impresa, Jerónimo Martins, Mota-Engil, NOS, PT, Portucel, Ren, Semapa, Sonae and Teixeira Duarte, and also of the PSI-20 index, which is the Portuguese benchmark index, that reflect the evolution of the prices of the twenty largest and most liquid stocks selected from the universe of companies listed on the Portuguese main market. The source of all data is Datastream. The data includes different number of observations for each of the stocks, because they initiate in different periods and finish on the 6th of May of 2015. The oldest available date is for the PSI-20 index and reports to the 4th of January of 1999. Considering all together, it gives 16 years of data for some of the stocks.

We apply the tests to the daily closing prices and also for the monthly closing prices. For the monthly price series, we use the observation of day 15 of each month. In case of a missing observation on day 15, we use the day before (14), and if the day before is also not available we use the next day (16). The returns are computed as the logarithmic difference between two consecutive prices in a series.

Table I shows the descriptive statistic for the daily returns of the eighteen stocks and also for the PSI-20 index. The mean return for the daily sample are between -0.49% for the Banif and 0.54% for the Semapa. For the PSI-20 the mean return is -0.02%. The mean standard deviation for the eighteen stocks is 0.0236 while for the PSI-20 index is lower, with a value of 0.0119. The returns are in most stocks positive skewed, with exception of Banif, CTT, EDP, Jerónimo Martins, Sonae and also the PSI-20 index. The

kurtosis level is highest in the whole sample indicating that the distribution of returns is leptokurtic. The Jarque-Bera statistic rejects the hypothesis of a normal distribution for entire stocks returns at a significance level of 1%

Table I- Descriptive statistic for the eighteen Portuguese stocks and also for PSI-20 index.

	Start	End	Observations	Minimum	Mean	Maximum	Std. Deviation	skewness	kurtosis	Jarque-bera
Altri	01-03-2005	06-05-2015	2649	-0.1849	0.0013	0.7221	0.0271	7.2600	194.8210	408456**
Banif	21-12-2012	06-05-2015	620	-1.2112	-0.0049	0.3118	0.0622	-11.5851	231.7230	136532**
BCP	03-01-2000	06-05-2015	3996	-0.1709	-0.0007	0.2384	0.0241	0.2836	11.9767	13487.16**
BPI	03-01-2000	06-05-2015	3996	-0.1398	-0.0002	0.2393	0.0218	0.8218	14.1839	21275.55**
CTT	06-12-2013	06-05-2015	371	-0.0655	0.0017	0.0722	0.0176	-0.1673	5.2291	81.08**
EDP	03-01-2000	06-05-2015	3996	-0.1649	0.0002	0.1315	0.0160	-0.0181	11.7802	12836.01**
EDP-R	04-06-2008	06-05-2015	1768	-0.0121	0.0001	0.1358	0.0206	0.0620	7.0139	1210.21**
Galp	24-10-2006	06-05-2015	2205	-0.1308	0.0006	0.2471	0.0224	0.5144	13.1829	9641.49**
Impresa	06-06-2000	06-05-2015	3837	-0.1228	-0.0005	0.3244	0.0277	1.3918	15.3226	25515.07**
Jeronimo Martins	03-01-2000	06-05-2015	3996	-0.1626	0.0005	0.1212	0.0207	-0.4599	10.8771	10472.12**
Mota-Engil	25-01-2001	06-05-2015	3719	-0.1133	0.0003	0.1833	0.0208	0.5353	9.7640	7280.87**
Nos	03-01-2000	06-05-2015	3828	-0.2269	-0.0003	0.1441	0.0217	0.0128	12.3043	14266.11**
Portucel	03-01-2000	06-05-2015	3884	-0.0872	0.0005	0.1395	0.0157	0.2873	8.1871	4534.80**
PT	03-01-2000	06-05-2015	3993	-0.2167	-0.0006	0.2147	0.0215	0.0475	13.6457	18894.58**
Ren	10-07-2007	06-05-2015	2031	-0.1325	0.0001	0.1227	0.0140	0.0727	15.2025	12602.49**
Semapa	03-01-2000	06-05-2015	3871	-0.1265	0.0054	0.1109	0.0165	0.2460	7.3518	3174.35**
Sonae	03-01-2000	06-05-2015	3926	-0.2683	0.0001	0.2136	0.0219	-0.1788	13.7856	19050.46**
Teixeira Duarte	16-08-2010	06-05-2015	1231	-0.1286	-0.0003	0.1823	0.0335	0.6696	6.4672	708.57**
PSI-20	04-01-1999	06-05-2015	4151	-0.1038	-0.0002	0.1019	0.0119	-0.1821	9.0998	6456**

Notes: The Jarque-Bera is a normality test, which is asymptotically distributed as X^2 (2), based on the sample kurtosis and skewness, under the null hypothesis of both the skewness and excess of kurtosis being zero. An ** indicates that null hypothesis rejection significance at 1% level.

5.2 Results

In this point, we are going to present the results for the four random walk tests utilized, such as, unit root test, the individual variance ratio test by Lo and Mackinlay, the multiple variance ratio test by Chow-Denning and also the automatic variance ratio

5.2.1 Unit Root tests

Table II reports the results of the ADF statistic to test the null hypothesis of a unit root in the eighteen stocks and also for the PSI-20 index. It is important to say that we only have done the ADF test for the daily series because in the monthly series we have, for example, 18 observations for the CTT and with this number of observations the ADF test has low power.

The optimal lag length for the test is chosen with Akaike info criteria, from a maximum of 15 lags allowed, by the number of observations. The test is conducted under the specification of an intercept and trend. In the eighteen stocks and also for the PSI-20 index, the test statistic of the ADF is less than the critical values at the 1%, 5%, and 10% significant levels.

The results support the random walk hypothesis, as the null hypothesis of the existence of a unit-root is not rejected. These results are consistent with other studies, by Dias et al. (2002) and Borges (2007).

Table II- ADF test for the eighteen Portuguese stocks and also for PSI-20 index for daily prices.

Sample	Daily prices			
	ADF test statistic	**Included observations**	**Number of lags**	**p-value**
Altri	-13.434	2650	13	0.01
Banif	-7.963	614	8	0.01
BCP	-17.292	3995	15	0.01
BPI	-14.940	3997	15	0.01
CTT	-7.688	371	7	0.01
EDP	-16.181	3882	15	0.01
EDP-R	-12.319	1768	12	0.01
Galp	-12.452	2205	13	0.01
Impresa	-13.528	3838	15	0.01
Jeronimo Martins	-15.422	3884	15	0.01
Mota-Engil	-13.673	3720	15	0.01
Nos	-13.821	3828	15	0.01
Portucel	-14.242	3884	15	0.01
PT	-15.832	3995	15	0.01
Ren	-14.019	2032	12	0.01
Semapa	-15.375	3871	15	0.01
Sonae	-14.164	3927	15	0.01
Teixeira Duarte	-10.901	1232	10	0.01
PSI-20	-14.145	4151	16	0.01

Notes: ADF statistic test the hypothesis, H_0: unit root ; H_1: there is no unit root (stationary). The model includes constant and trend.

5.2.2 Individual Variance Ratio tests by Lo and Mackinlay (1988)

Table III reports the results of the individual variance ratios tests by Lo and Mackinlay (1988) for the eighteen main Portuguese stocks and also for the PSI-20 stock index. In order to make more easier the comparisons between other studies, we adopt the regular procedure of selecting lags 2, 5, 10 and 30 for daily prices and the lags 2, 4, 8, 16 for monthly prices. For each period we report the estimated variance ratio, VR (k), and also the heteroscedasticity robust statistic, $M_2(k)$. Since this is an individual test if we reject the null hypothesis for one of k periods, we reject the RWH for the entire sample.

Considering the daily prices all the stocks with exception of BCP, CTT, EDP, REN, Semapa and Teixeira Duarte, have at least one variance ratio larger than unity, which indicates that the variances grow more than proportionally with time, existing thus a positive autocorrelation of the data. On the other hand, the test statistic $M_2(k)$, reject the RWH for ten of the eighteen stocks, namely for Altri, BCP, BPI, Impresa, Jerónimo Martins, Mota-Engil, Nos, Pt, Ren and Teixeira Duarte, and also for the PSI-20 index.

Considering the monthly prices, the only variance ratio that is lower than the unity is for Galp, which indicates that it is the only stock where the variance increases less than proportionally with time, suggesting mean reversion for the monthly series. Analyzing now the test statistic $M_2(k)$, from ten stocks that we have rejected the RWH for the daily prices, now this number decreases for seven, namely for Altri, BCP, EDP, Impresa, Mota-Engil, Semapa and Sonae, and also for the PSI-20 index.

As an overview, we can conclude that in the monthly data the number of stocks that reject the RWH is fewer, suggesting that when we decrease the frequency of the observations, past prices becomes less capable to explain the future prices.

Table III- Individual Variance Ratio tests by Lo and Mackinlay for the eighteen Portuguese stocks and also for PSI-20 index.

Sample	Daily prices								Monthly prices							
	VR(2)	M2(2)	VR(5)	M2(5)	VR(10)	M2(10)	VR(30)	M2(30)	VR(2)	M2(2)	VR(4)	M2(4)	VR(8)	M2(8)	VR(16)	M2(16)
Altri	1.059	1.095	1.105	1.112	1.154	1.301	1.346	2.016*	1.161	1.510	1.395	2.153*	1.848	3.169**	2.368	3.366**
Banif	1.020	0.444	1.128	1.235	1.155	1.025	1.459	1.161	1.137	1.680	1.246	1.892	0.863	-0.566	0.293	-1.903
BCP	0.826	-4.600**	0.637	-4.409**	0.565	-3.513**	0.538	-2.311*	1.209	2.325*	1.395	2.494*	1.729	2.944**	1.949	2.624**
BPI	1.041	1.580	1.091	1.687	1.163	2.057*	1.323	2.401*	1.141	1.663	1.181	1.023	1.443	1.630	1.502	1.312
CTT	0.918	-1.354	0.806	-1.544	0.726	-1.508	0.538	-1.456	1.054	0.389	0.756	-0.877	0.322	-1.399	0.059	-1.276
EDP	0.984	-0.597	0.957	-0.782	0.885	-1.332	0.724	-1.771	0.983	-0.230	1.052	0.394	1.365	1.693	1.653	2.054*
EDP-R	1.054	1.288	1.003	0.035	0.933	-0.514	0.917	-0.365	1.086	0.500	1.219	0.751	0.989	-0.026	0.709	-0.537
Galp	1.056	1.270	1.017	0.194	1.013	0.112	0.986	-0.070	0.944	-0.382	0.906	-0.352	0.907	-0.240	0.541	-0.857
Impresa	1.021	0.818	1.044	0.840	1.103	1.318	1.270	2.028*	0.944	-0.382	1.220	1.390	1.585	2.377*	1.585	1.638
Jerónimo Martins	1.067	2.535*	1.059	1.096	0.964	-0.448	0.928	-0.518	1.032	0.406	1.060	0.404	1.123	0.506	1.134	0.373
Mota-Engil	1.056	2.031*	1.096	1.712	1.142	1.740	1.330	2.421*	1.214	2.921**	1.308	2.149*	1.656	2.807**	1.809	2.316*
Nos	1.097	3.585**	1.156	2.495*	1.282	2.939**	1.455	2.760**	0.881	-0.943	1.077	0.355	1.422	1.359	1.724	1.773
Portucel	0.974	-1.155	0.909	-1.825	0.912	-1.206	1.000	-0.001	1.068	0.938	1.146	1.105	1.118	0.547	0.917	-0.255
PT	1.066	2.168*	1.045	0.726	1.064	0.716	1.044	0.286	0.881	-0.985	0.974	-0.120	1.158	0.492	1.084	0.193
Ren	0.985	-0.320	0.838	-1.677	0.716	-2.040*	0.651	-1.597	1.040	0.356	0.894	-0.533	0.811	-0.601	0.731	-0.579
Semapa	0.985	-0.655	0.934	-1.346	0.902	-1.380	0.908	-0.789	1.050	0.694	1.291	2.169*	1.483	2.306*	1.496	1.615
Sonae	0.981	-0.792	0.964	-0.743	0.966	-0.465	1.125	1.021	1.178	2.435*	1.388	2.831**	1.846	3.808**	1.971	2.893**
Teixeira Duarte	0.880	-3.190**	0.844	-2.022*	0.872	-1.114	0.934	-0.326	1.135	0.998	1.247	1.021	1.728	1.960	1.907	1.644
PSI-20	1.084	3.430**	1.127	2.384*	1.127	1.535	1.265	1.815	1.207	2.693**	1.354	2.565*	1.605	2.718**	1.530	1.622

Notes: VR(k)- variance ratio estimate, $M_2(2)$- test statistic for null hypothesis of heteroskedastic increments random walk; * null hypothesis rejection significant at the 5% level; ** null hypothesis rejection at the 1% level.

5.2.3 Multiple variance ratio tests by Chow and Denning (1993)

Table IV reports the results of the multiple VR tests by Chow-Denning. This test is robust to the presence of heteroscedasticity. Considering the daily series at the 5% level, we only reject the hypothesis of random walk for Jerónimo Martins. At the 1% level, we reject for BCP, Nos, Teixeira Duarte and also for the PSI-20 index. Furthermore for the monthly series we reject the hypothesis of random walk, at 5% significant level for BCP, Mota-Engil and also for the PSI-20 index. At the 1% level, we only reject for Altri and Sonae.

Table IV - Multiple variance ratio tests by Chow and Denning for the eighteen Portuguese stocks and also for PSI-20 index.

Sample	**Daily prices**	**Monthly prices**
	MV2	**MV2**
Altri	2.016	3.372**
Banif	1.235	1.904
BCP	4.600**	2.938*
BPI	2.401	1.660
CTT	1.544	1.403
EDP	1.771	2.049
EDP-R	1.288	0.750
Galp	1.270	0.862
Impresa	2.028	2.376
Jeronimo Martins	2.535*	0.510
Mota-Engil	2.421	2.921*
Nos	3.585**	1.783
Portucel	1.825	1.112
PT	2.168	0.994
Ren	2.040	0.601
Semapa	1.380	2.308
Sonae	1.021	3.812**
Teixeira Duarte	3.190**	1.960
PSI-20	3.430**	2.720*

Note: $M_2(k)$- test statistic for null hypothesis of heteroskedastic increments random walk; * null hypothesis rejection significant at the 5% level; ** null hypothesis rejection at the 1% level.

5.2.4 Automatic Variance Ratio tests by Choi (1999)

The results of the automatic variance ratio test (AVR) by Choi are reported in Table V for the daily and monthly data. Observing the values by daily data for the stocks: Altri, BCP, BPI, Jerónimo Martins, Mota-Engil, Nos, Teixeira Duarte and also for the PSI-20 index, with the critical values (two-sided) of normal distribution we find that the null hypothesis of no serial correlation is rejected for seven of the eighteen stocks and also for the PSI-20 index. This suggests that all the seven stocks and also the PSI-20 index are inefficient with daily data.

Applying the same process, but now with the monthly data, we can compare the values of the AVR test with the critical values (two-sided) of normal distribution and we find that for the stocks of, Altri, BCP, Sonae and also for the PSI-20 index the null hypothesis is rejected, suggesting that this three stocks and also the index are inefficient with monthly data.

Table V - Automatic Variance Ratio tests for the eighteen Portuguese stocks and also for PSI-20 index, for daily and monthly prices.

Sample	Daily prices	Monthly prices
	AVR test statistic	AVR test statistic
Altri	2.101*	1.963*
Banif	0.335	0.899
BCP	-9.162**	2.621**
BPI	2.190*	0.954
CTT	-0.671	1.332
EDP	-0.400	0.338
EDP-R	1.079	0.501
Galp	0.964	-0.107
Impresa	0.962	0.123
Jeronimo Martins	2.185*	0.263
Mota-Engil	2.480	1.884
Nos	3.745**	0.055
Portucel	-1.132	0.811
PT	1.920	-0.912
Ren	0.043	0.254
Semapa	-0.442	0.568
Sonae	-0.767	1.983*
Teixeira Duarte	-2.972**	0.600
PSI-20	3.558**	2.224*

Note: * indicates significant value of the test statistic at 5% level of significance and ** indicates the same at 1% level of significance.

6. Conclusions

Table 6 summarizes the results of all the tests performed in this study. The empirical evidence from the unit root tests does not allow the rejection of the RWH for the eighteen shares and also for the PSI-20 index, while the other tests provide mixed conclusions. The automatic variance ratio test of Choi in daily data rejected the RWH for eight of the eighteen stocks including the PSI-20 index, but when we consider monthly data the numbers of stocks that reject this hypothesis decreased to four, including the PSI-20 index. When we use the individual and multiple variance ratios tests, the number of stocks that does not following a random walk decreases as well when we use monthly data. On the other hand, all tests with the exception of the ADF reject the RWH for the PSI-20 index in both, daily and monthly data.

The findings that we achieve with this study confirm with previous results on the Portuguese stock market, such as those made by Dias et al. (2002) and Borges (2007), who provide evidence in favor of the RWH when ADF tests are used. On the other hand, they find evidence against the RWH with variance ratio tests and correlation tests.

Therefore, even though there has been a crisis in Europe, it seems that the Portuguese stock market has become more efficient. Nevertheless, for some stocks there is still some space, for transaction strategies based on technical analysis that may increase investor's profits.

Table VI- Summary of test results for the random walk hypothesis. Daily and monthly data are presented.

	Daily prices				Monthly prices			
	ADF	Auto VR	Lo and Mac	Chow	ADF	Auto VR	Lo and Mac	Chow
Altri	No	Yes	Yes	No	N/A	Yes	Yes	Yes
Banif	No	No	No	No	N/A	No	No	No
BCP	No	Yes	Yes	Yes	N/A	Yes	Yes	Yes
BPI	No	Yes	Yes	No	N/A	No	No	No
CTT	No	No	No	No	N/A	No	No	No
EDP	No	No	No	No	N/A	No	Yes	No
EDP-R	No	No	No	No	N/A	No	No	No
Galp	No	No	No	No	N/A	No	No	No
Impresa	No	No	Yes	No	N/A	No	Yes	No
Jerónimo Martins	No	Yes	Yes	Yes	N/A	No	No	No
Mota-Engil	No	Yes	Yes	No	N/A	No	Yes	Yes
Nos	No	Yes	Yes	Yes	N/A	No	No	No
Portucel	No	No	No	No	N/A	No	No	No
PT	No	No	Yes	No	N/A	No	No	No
Ren	No	No	Yes	No	N/A	No	No	No
Semapa	No	No	No	No	N/A	No	Yes	No
Sonae	No	No	No	No	N/A	Yes	Yes	Yes
Teixeira Duarte	No	Yes	Yes	Yes	N/A	No	No	No
PSI-20	No	Yes	Yes	Yes	N/A	Yes	Yes	Yes

Notes: This table is a summary of the RWH tests. The columns "ADF", "AVR", "Lo and Mac" and "Chow" , represents the results for the Augmented Dickey-Fuller test, Automatic Variance Ratio test, the multiple and also the individual variance ratio tests, respectively. "N/A" means that the results are not available for this sample. "Yes" and "No" means that we reject or not reject the RWH with 5% of significance level.

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