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MASTER

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MASTER FINAL WORK

PROJECT

INVESTMENT STRATEGIES OF A NON-LIFE INSURANCE COMPANY UNDER SOLVENCY II

MARIANA DA COSTA FERREIRA

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Abstract

On this study we develop a portfolio investment optimization process for a non-life insurance company, where capital requirement is calculated using the standard formula defined by Solvency II. The optimization aims to find the minimum solvency capital requirements for market risk and, simultaneously, maximize portfolio returns. The optimal investment strategy set is obtained using a multi-objective optimization process. To analyse the performance of the portfolio and the capital at risk, we compute the return on risk adjusted capital (RoRAC), that is the expected profit over the Solvency II market capital charge. Results show that is possible to define a set of investment strategies under Solvency II regime that accomplish the objectives on return and capital requirements.

Keywords: Portfolio optimization, Solvency II, market risk, performance measure, RoRAC.

Resumo

Neste trabalho é feita a otimização da carteira de uma empresa de seguros não vida, que utiliza a fórmula *standard* definida no regime de Solvência II para calcular os requisitos de capital, com o objetivo de encontrar a alocação dos ativos financeiros que minimizam o risco de mercado e, simultaneamente, maximizam o retorno da carteira. A solução é obtida a partir de um processo de otimização multi-objetivo. Para analisar o desempenho da carteira e o risco do capital investido, calculamos a rentabilidade ajustada ao risco (RoRAC), que é o rácio entre o retorno esperado e o valor de Solvência II relativo ao risco de mercado. Os resultados mostram que é possível definir uma estratégia de investimento no regime de Solvência II que permita atingir os objetivos em retorno e requisitos de capital.

Palavras-chave: Otimização de carteira , Solvência II , risco de mercado , medida de desempenho carteira , RoRAC .

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List of Abbreviations

BOF	Basic Own Funds
CIC	Complementary Identification Code
CVaR	Conditional Value-at-Risk
EEA	European Economic Area
EIOPA	European Insurance and Occupational Pensions Authority
GA	Genetic Algorithm
ID	Identity
MOP	Multi-objective problem
NSGA-II	Non-dominated Sorting Genetic Algorithm-II
OECD	Organisation for Economic Cooperation and Development
PF	Pareto Front
PS	Pareto Set
RoRAC	Return on Risk-adjusted Capital
SCR	Solvency Capital Requirements
SAM	Symmetric adjustment mechanism
VaR	Value-at-risk

List of Notation

$\mathbf{x}$	Decision vector
$\mathcal{X}$	Decision space
$\mathcal{Y}$	Objective space
$\mathcal{X}_E$	Set of all efficient solutions (Pareto set)
$\mathcal{Y}_E$	Set of all nondominated points (Pareto front)
P_t	Parents population
Q_t	Descendants population
r	Rank of a pareto front
F_l	Front l
n	Number of assets in the portfolio
x_i	Weight of each asset i
$E[P]$	Expected value of the portfolio's profit
SCR_{Market}	Solvency capital requirements for the market risk
w_i	Weight invested in each sub-asset class i
r_i	Return of each sub-asset class i
ΔBOF	Difference of the basic own funds
Interest^j	Solvency capital requirements for interest rate risk for shock j
s_t^j	Interest rate shock j for maturity t
r_t	Basic risk-free rate for maturity t
Equity	Solvency capital requirements for equity risk

Equity_i	Solvency capital requirements for equity risk with respect to category i
Property	Solvency capital requirements for property risk
Spread	Solvency capital requirements for spread risk
Spread^i	Solvency capital requirements for spread risk of i
MV_i	Market value of asset i
$duration$	Associated duration of asset i
$F^{up}(rating_i)$	Function of the credit quality step of asset i
Currency^i	Solvency capital requirements for currency risk after an i scenario
Concentration	Solvency capital requirements for concentration risk
$Conc_i$	Risk charge for exposures to counterparty i
XS_i	Excess exposure to counterparty i
E_i	Exposure at default to counterparty i
CT_i	Relative concentration threshold applicable to counterparty i
g_i	Reduction factor for counterparty i
SCR_{market}	Solvency capital requirements for the market risk

Contents

1 Introduction

The 2007-2008 financial crisis highlighted the need to review the European supervisory model and to develop a project that would deeply and comprehensively review the regulatory and supervisory framework of the European insurance sector - Solvency II. Since 1^{st} January 2016, insurance companies are required to hold eligible own funds for their solvency capital requirements. Solvency II framework is made with three pillars. The first pillar, covers the quantitative requirements and ensures that over a one year period the probability of ruin is bellow 0.5% by the computation of solvency capital requirements with a standard formula provided by the regulator, considering a VaR of 99.5%. The valuation of assets and liabilities under the Solvency II regime is set on a mark-to-market basis. The requirements for insurer's governance and risk management system are determined on pillar two and third pillar is about transparency and disclosure. The standard formula defines capital requirements based on various risk modules and respective sub-modules, market risk being a major contribution to this value (Ratings (2011)). The market risk module is calculated by applying stress factors to the different sub-modules and the different capital requirements, linked to the respective shocks, are then aggregated by a correlation matrix.

The most common way to find the optimal portfolio is using the mean-variance theory (Markowitz (1952)). However, to insurance companies that have to match the difference between assets and liabilities, with regulatory capital, there is a need to go further.

On the literature there are studies that analyse the impact of the implementation of Solvency II to asset managers (Heckel et al. (2012), Ratings (2011)). More specific papers study the impact of the market risk standard formula on investment strategies, as in Fischer and Schlütter (2012), that analyse the impact that standard formula equity risk calibration has on the allocation of assets to equity class and on an investment strategy of a shareholder-value-maximizing insurance company. Braun, Schmeiser, and Schreiber (2015b) investigate an optimization of the asset allocation, in the context of portfolio theory, for an insurance company that comply with the market risk capital requirements of Solvency II. These authors (Braun, Schmeiser, and Schreiber (2015a)) also analyse the standard formula for the market risk and propose to find the optimal portfolio, under Solvency II regime, by maximizing the RoRAC. Bruneau and Mankai (2012) do not consider the standard formula of Solvency II, but instead an internal model to find, simultaneously, the optimal investment portfolio for a non-life insurance company, by maximizing the RoRAC, and also by subjecting it to a global zero-conditional value-at-risk (CVaR) constraint. Concerning multi-objective portfolio optimisation, we consider the works of Duan (2007) and Steuer, Qi, and Hirschberger (2005), where they maximize returns and minimize risks. More specifically to non-life insurance companies, Jarraya and Bouri (2013) developed a model that integrates simulation approach with a multi-objective particle swarm optimization algorithm to find the optimal asset allocation which maximizes the shareholders expected utility and technical efficiency. Furthermore, Kaucic and Daris (2015) introduce a multi-objective stochastic optimization program for chance-constrained portfolio selection problems. At the best of our knowledge we use a new assessment approach on the combination of SCR and expected returns by performing a multi-objective process to find the optimal portfolio selection.

Since the SCR core is risk based and it does not take into account returns, which is the main point for portfolio optimisation targets, this study focus on an investment strategy that considers both objectives. The optimization of the investments of an insurance company, while considering the computation of solvency capital requirements, is of extreme relevance, because there is a direct link with portfolio composition in terms of asset classes, maturity, ratings and concentration. Unlike Solvency I, where capital requirements were a fixed percentage of the liabilities and independent of the insurer's asset allocation, now the regulatory capital must cover the difference between assets and liabilities. Consequently, SCR is very sensitive to asset related risks. We create a bi-objective problem, where our objective functions are the expected profit of the portfolio and the solvency capital requirements for market risk. We find the pareto curve between the two objectives, by changing the amount invested in each asset, and the respective pareto optimal solution to define the more advantageous investment strategy according to an initial allocation of asset classes.

The structure of this study is as follows. On section 2 we make an introduction to multi-objective optimization and the evolutionary algorithm used to solve the problem - Non-dominated Sorting Genetic Algorithm-II (NSGA-II). On section 3 we present the data and the methodology applied to find a more profitable investment strategy to a non-life insurance company. Section 4 analysis several allocation per asset sub-classes strategies following the works of Heckel et al. (2012), Ratings (2011) and Haslip (2011) and discusses the results of the multi-objective problem (MOP), that is reformulate as five distinct models, to overcome some restraints of NSGA-II. Lastly, section 5, states the conclusions and some limitations, as well as the new areas of future research.

2 Multi-objective Optimization

Multi-objective optimization is a process that involves at least two objective functions, usually conflicting in nature, at the same time. In mathematical terms, a multi-objective problem (MOP) can be formulated as:

$$\begin{aligned} &\text{minimize} \quad f(x) = (f_1(x), ..., f_m(x))^T, m \geq 2 \\ &\text{subject to} \quad x \in \mathcal{X} \end{aligned} \tag{2.1}$$

where $x = (x_1, ..., x_n)^T \in \mathbb{R}^n$ represents the decision, or design, vector and $\mathcal{X}$ is the feasible decision space, that is defined by some constraint functions. The objective, or criterion space is defined as the image of $\mathcal{X}$ as the set:

$$\mathcal{Y} = \{f(x) \in \mathbb{R}^m \mid \mathbf{x} \in \mathcal{X}\} \tag{2.2}$$

If some objective function is to be maximized it is equivalent to minimize its negative.

For a MOP there is not, in general, one global solution that simultaneously satisfies all the objectives, so it is necessary to define an optimal point, able to consider the tradeoffs among the different objectives.

Definition 1 *A point $x^* \in \mathcal{X}$ is called Pareto optimal to a multiple-objective problem if and only if does not exist another point $x \in \mathcal{X}$ such that:*

$$\forall i \in \{1, ..., m\} \;:\; f_i(x) \leq f_i(x^*) \;\; \textit{and} \;\; \exists j \in \{1, ..., m\} \;:\; f_j(x) < f_j(x^*). \tag{2.3}$$

The image of x^* *in* $\mathcal{Y}$*,i.e,* $f(x^*) = (f_1(x^*), ..., f_m(x^*))^T$*, is called a Pareto optimal (objective) vector and* x^* *is a non dominated point.*

Definition 2 *If* $x^1, x^2 \in \mathcal{X}$ *and* $f(x^1) \leq f(x^2)$*, we say* x^1 *dominates* x^2 *and* $f(x^1)$ *dominates* $f(x^2)$*.*

The set of Pareto optimal objective vectors forms the so-called Pareto, or efficient, front (PF) and the set with all Pareto points is the Pareto set (PS). Since PS represents optimal solutions to the MOP, it means that this solutions cannot be improved in any of the objective functions without deteriorating at least one of the other objectives, and PF results in being a subset of the boundary of the objective set.

Definition 3 *The set of all efficient solutions* $x^* \in \mathcal{X}$ *is denoted* $\mathcal{X}_E$ *and is called the Pareto set. The set of all nondominated points* $y^* = f(x^*) \in \mathcal{Y}$ *is denoted* $\mathcal{Y}_N$ *and is called the Pareto front.*

The range of values which can be attained by nondominated points are given by ideal and nadir points, defined as the lower and upper bounds of the PF.

Definition 4 *1. The point* $y^I = (y_1^I, ..., y_m^I)$ *is called the ideal point of the multicriteria optimization problem and is given by*

$$y_k^I := \min_{\forall x \in \mathcal{X}_E} f_k(x) = \min_{\forall y \in \mathcal{Y}_N} y_k \tag{2.4}$$

2. The point $y^N = (y_1^N, ..., y_m^N)$ *is called the nadir point of the multicriteria optimization problem and is given by*

$$y_k^N := \max_{\forall x \in \mathcal{X}_E} f_k(x) = \max_{\forall y \in \mathcal{Y}_N} y_k \tag{2.5}$$

2.1 Non-dominated Sorting Genetic Algorithm II

Non-dominated Sorting Genetic Algorithm II (NSGA-II), introduced by Deb et al. (2002), is an evolutionary algorithm that produces a set of the Pareto optimal solutions in one run. Has three principal components: elitist principle, emphasizes non-dominated solutions and explicit diversity preserving mechanism.

The NSGA-II procedure starts by creating a random population P_t, of size N, and for each solution a in this population a value of the number of solutions that dominates it is created and is computed a set with all solutions dominated by a. For each solution is assigned a rank. Crossover and mutation operators are applied to this parent population

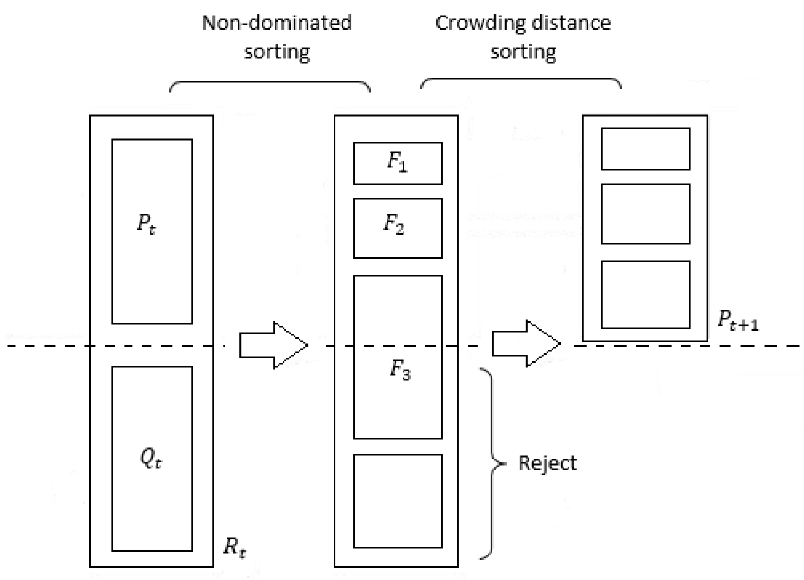


Figure 1: NSGA-II algorithm

to create a new pool of descendants Q_t, of size N, which are mixed with the parents population. This enlarged pool R_t, with size $2N$, is sorted into non-dominated fronts by descent order, where Pareto front has rank 0 and the individuals that are dominated, only by the ones with rank r have rank $r+1$. Since the count of the non-dominated solutions from the fronts $F_1, F_2, F_3, ..., F_l$ exceeds N, some of the lower ranked non-dominated solutions from the last front have to be reject, then is applied the elitism method by adding a crowding distance to each solution to generate the next population P_{t+1}. This distance, consider as being the density of individuals neighbouring a particular individual a, is computed as the perimeter of the hypercube where the vertices are the nearby individuals to a. The crowding distance sorting assures diversity in the population and explores the fitness landscape. The new population P_{t+1} suffers crossover and mutation to create a new descend population Q_{t+1}, of size N and the NSGA-II algorithm is repeated until the number of generations is reached.

The set of solutions are in the first front of the final population and the optimal solution is the one with a lower rank, or having the same rank, the best solution is the one that is located in a less crowded region. The smallest and largest function values are the boundary solutions and are attributed an infinite distance.

3 Data and Methodology

In this section, we discuss the data collection as well as the procedure followed to optimize the investments of an insurance company. As a starting point we define the base scenario with an investment portfolio at the end of year 2015 and assess the solvency market capital requirements and the portfolio returns. The optimization process takes into account the maximization of the portfolio return to a minimization on the capital requirements.

3.1 Data Description

We consider the already built portfolio for a Non-life Insurance Company, that is composed with eight asset classes with a taxonomy set by the complementary identification code (CIC): cash and deposits, collateralised securities, corporate bonds, equity, government bonds, investment funds, mortgages and loans and property.

Considering the detailed information on investment funds, it is possible to apply the look-through approach to disaggregate all stocks and apply granular risk calculations. Consistent with the Solvency II regime, the valuation of assets is at market value and although there are no constraints on investments, due to the prudent person rule, all investments should be limited to assets and instruments whose risks can be properly identified, measured, monitored, managed, controlled and reported.

Table 1: Asset classes of the portfolio

Asset Classes			
Cash and Deposits	Corporate Bonds	Collateralised Securities	Equity
Government Bonds	Investment Funds	Mortgage and Loans	Property

We have a total of 3535 assets and a total amount of investment of €339.26 millions. Our initial allocation is 7.85% for cash and deposits, 0.73% for collateralised securities, 20.76% for corporate bonds, 13.39% for equity, 29.38% for government bonds, 1.56% for investment funds, 1.32% for mortgage and loans and 25.28% for property.

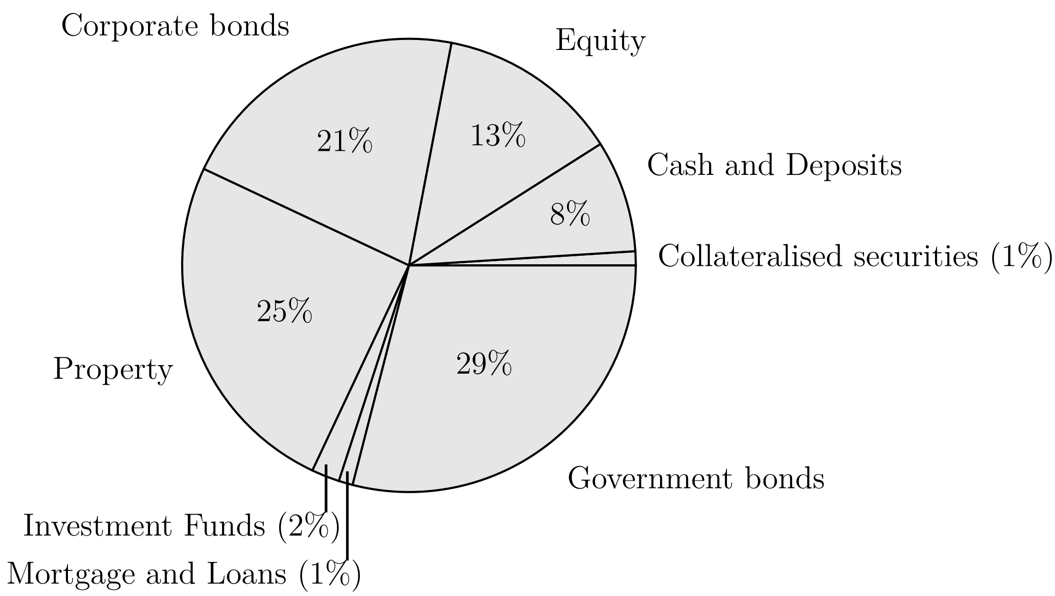


Figure 2: Alocation of the amount invested in each asset class

The data set for each asset includes an ID code, a description, the issuer group, the asset class, the total amount in Euro, the currency of investment, settlement date,maturity date, coupon rate, coupon frequency, CIC, basis and the rating. Liabilities are fixed and we consider the risk-free interest rate term structures provided by EIOPA (EIOPA 2016).

Considering the *Delegated Acts of October* 10^{th} (Parliament and Union (2009)), on the calculation of solvency capital requirements, equities have been separated in two types: *type* 1 are the equities listed in regulated markets in the countries which are members of the European Economic Area (EEA) or the Organisation for Economic Co-operation and Development (OECD) and *type* 2 are the equities listed in stock exchanges in countries which are not member of the EEA or the OECD, equities which are not listed, commodities and other alternative investments. The asset class of property has been divided into land and buildings (includes properties, plants and equipments held for own use and others than for own use) and real estate investment funds.

The assumption on expected returns of asset classes have been given by a consulting company and to map the classes of Solvency II (when different) we split it into sub-classes. On government bonds we consider the bonds issued by less developed countries as emerging market debt[1] and the others we divide by maturity: index-linked euro government (<5 years), fixed interest euro government (<5 years) and fixed interest euro government (<10 years). Collateralised securities, mortgage and loans are considered as euro corporate bonds (>10 years) in our study. The equities that are not listed in a stock exchange and have CIC XL and XT are classified as private equities, the eurozone equities are issued by the countries from the Eurozone and all others are classified as global equities. Investment funds are split into hedge funds, commodities and short duration global funds (remaining funds). At last, corporate bonds are divided into euro corporate bonds (>10 years), that have maturity greater than ten years and are traded in Euro, and absolute return bonds that includes all others.

[1]Bonds issued by South Africa, Brazil, Czech Republic, Greece, Mexico, Argentina, Hungary, Bulgaria and Indonesia

Asset Classes	Sub-asset Classes	Returns
Cash and Deposits	Cash and Deposits	0.86%
Collateralised securities	Euro corporate bonds (>10 years)	1.41%
Corporate bonds	Euro corporate bonds (>10 years)	1.41%
	Absolute return bonds	3.00%
Equity	Eurozone equities	6.60%
	Global equities	6.60%
	Private equities	8.90%
Government bonds	Emerging market debt	6.45%
	Index-linked euro government	0.86%
	Fixed interest euro government (>5 years)	0.86%
	Fixed interest euro government (>10 years)	0.86%
Investment funds	Hedge funds	4.86%
	Short duration global bond funds	2.36%
	Commodities	1.86%
Mortgages and loans	Euro corporate bonds (>10 years)	1.41%
Property	Property	4.13%

Table 2: Expected returns of the asset classes (over 1 year horizon)

The allocations, according to the mapping between asset classes and sub-classes, are now 18.47% on absolute return bonds, 7.58% on cash, 0.15% on emerging market debt, 4.34% on euro corporate bonds (>10years), 0.91% on eurozone equities, 0.25% on fixed interest euro government bonds (>10years), 0.37% fixed interest euro government bonds (>5years), 5.69% on global equities, 1.06% on hedge funds, 28.61% on index-linked euro government, 6.79% on private equity, 25.28% on property, 0.42% on short duration global bonds and 0.07% on commodities.

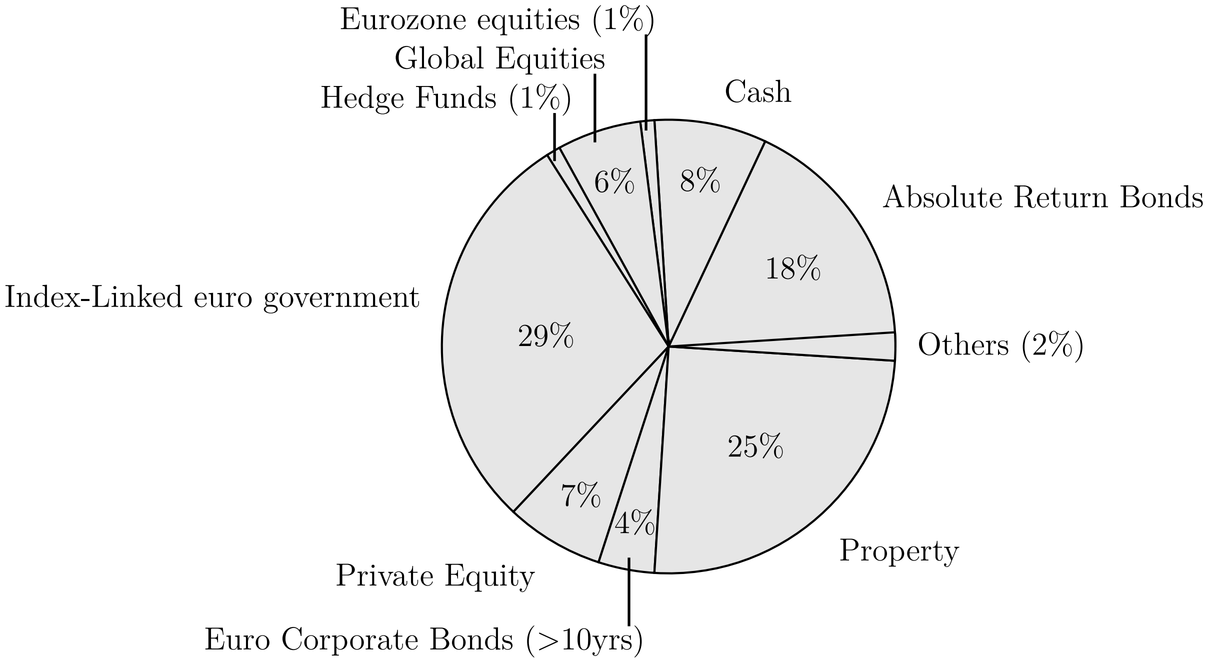


Figure 3: Alocation of the amount invested in each sub-asset class

We can then divide the total assets of the portfolio per sub-asset class.

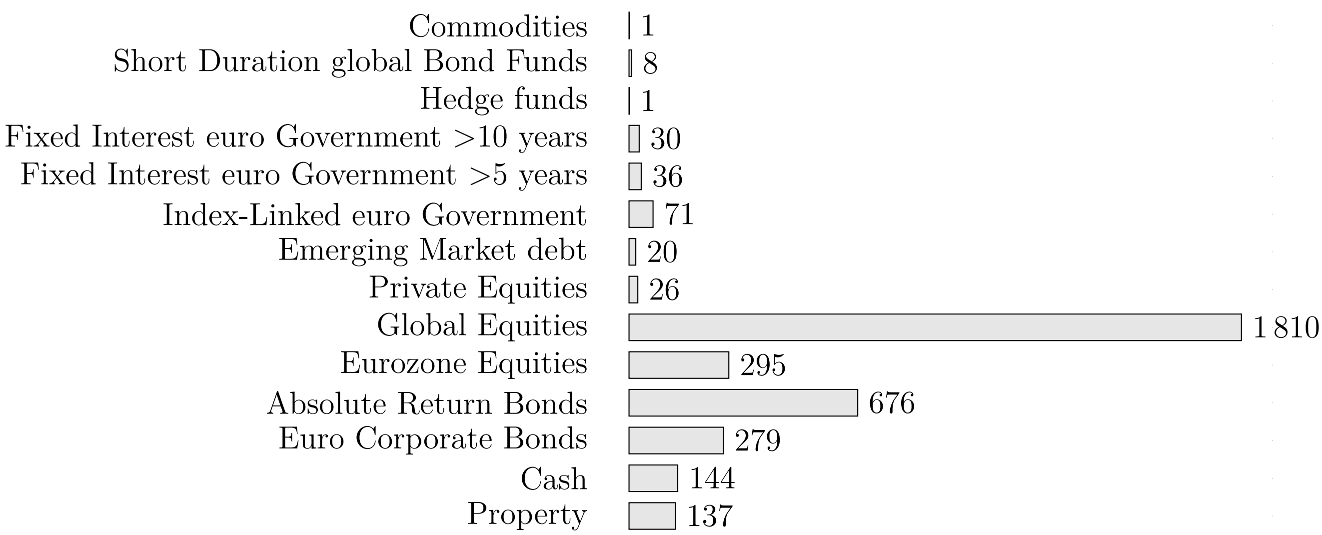


Figure 4: Number of assets per asset sub-class

3.2 Methodology

3.2.1 Optimization Problem

Based on the assets chosen in each asset class shown on Table 1, we have a portfolio with a certain amount invested in each asset:

$$\mathbf{x} = (x_1, x_2, ..., x_n), \tag{3.1}$$

with vector $\mathbf{x}$ being the weight of each asset and n the number of assets in the portfolio. The total amount of investment should be divided amongst the n assets as

$$\sum_{i=1}^{n} x_i = 1 \tag{3.2}$$

We can add other constraints by excluding short sales due to difficulties involved in shorting most of the asset types as well as to simplify the calculus:

$$x_i \geq 0 \tag{3.3}$$

Then, considering a bi-objective problem, that takes into account, simultaneously, the capital level for a non-life insurance company and the optimal investment portfolio on returns, we can state our objective function as:

$$\begin{cases} \text{maximize } E[P] \\ \text{minimize } SCR_{Market} \end{cases} \tag{3.4}$$

3.2.2 Expected Profit

Having the weight invested in each sub-asset class, w_i, and the respective returns, r_i, we calculate the expected profit as follow:

$$E[P] = \sum_i w_i.r_i \tag{3.5}$$

3.2.3 Market Risk Standard Formula

With the introduction of Solvency II, insurance companies were provided with a standard formula for different risk types to calculate their solvency capital requirements (SCR), which is calibrated using the Value-at-Risk (Var) of the basic own funds (BOF) subject to a confidence level of 99.5 percent over a one year period (see Parliament and Union 2015).This study focus on the market risk module, which is a very relevant risk category in the insurance industry. European insurers are a major investor in Europe's financial markets and market risk represents a relevant percentage of their solvency capital requirements (Ratings 2011).

Market risk module reflects the risk arising from the level or volatility of market prices of financial instruments which have an impact upon the level of the BOF of the undertaking. BOF is defined, mainly, as the difference between assets and liabilities on the economic balance sheet.

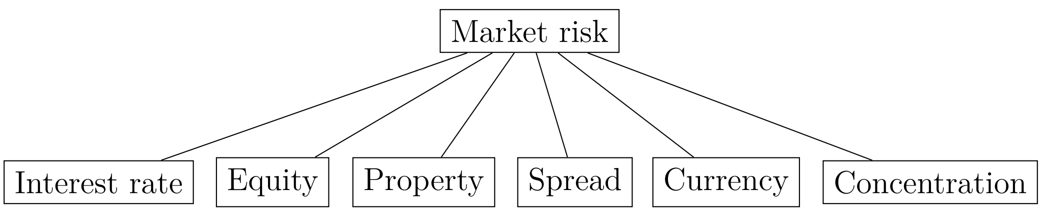


Figure 5: Market risk module and respective sub-modules.

Assets and liabilities are interest rate sensitive and upward and downward shocks of the interest rate term structure may have a detrimental influence on the BOF, creating a loss to the undertaking. The capital requirement for interest rate risk is calculated as (see Parliament and Union 2015):

$$\text{Interest}^{up} = \Delta BOF|_{up} \quad and \quad \text{Interest}^{down} = \Delta BOF|_{down} \tag{3.6}$$

The stress factors are applied to the basic risk-free interest rates as follows:

$$\Delta r_t^{up} = r_t(1 + s_t^{up}) - r_t \quad and \quad \Delta r_t^{down} = r_t(1 + s_t^{down}) - r_t \tag{3.7}$$

with s_t^{up} and s_t^{down} being the interest rate shocks for up and down scenario and r_t being the basic risk-free rate for maturity t.

Equity risk arises from the risk of changes in the market prices of equities. All assets and liabilities whose value is sensitive to modifications in equities prices is exposed, but this sub-module only covers a downward stress scenario. The computation of this risk was based on the "standard" approach, with a symmetric adjustment mechanism (SAM), as (see Parliament and Union 2015):

$$\begin{aligned} \text{Equity} = \sqrt{\text{Equity}_1^2 + \text{Equity}_2^2 + 2 \times 75\% \times \text{Equity}_1^2 \text{Equity}_2^2}, \\ \text{Equity}_i = \max\left(\Delta\ BOF \mid \text{equity shock}_i;\ 0\right). \end{aligned} \tag{3.8}$$

with $i \in \{1, 2\}$ and equity shock$_i$ being an instantaneous increase in the value of equities classified as *type* 1 by 39% plus a SAM and being an instantaneous decrease in the value of equities classified as *type* 2 by 49% plus a SAM. This symmetric adjustment mechanism allows the equity shock to move within an interval of 10% on either side of the underlying standard equity stress.

The third sub-module, property risk, is the result of sensitiveness of assets, liabilities and financial investments to the level or volatility of market prices of properties (see Parliament and Union 2015):

$$\text{Property} = \max\left(\ \Delta BOF \mid \text{property shock};\ 0\right), \tag{3.9}$$

where property shock represents a decrease in the value of properties by 25%.

Changes in the credit worthiness of the issuers of the securities held in the insurer's investment portfolio, that will be reflected in changes on the underlying credit spread, creates the spread risk (see Parliament and Union 2015):

$$\text{Spread} = \text{Spread}^{bonds} + \text{Spread}^{securisation} + \text{Spread}^{derivatives}. \tag{3.10}$$

We do not consider the capital requirements for securisations and derivatives because of the available data.

$$\begin{aligned} \text{Spread}^{bonds} &= \max\ (\ \Delta BOF \mid \text{spread shock on bonds};\ 0), \\ \text{spread shock on bonds} &= \sum_i MV_i duration_i F^{up}(rating_i), \end{aligned} \tag{3.11}$$

where MV is the market value of asset $i \in \{1, ..., n\}$, $F^{up}(rating_i)$ is a function of the credit quality step of asset i and *duration* is the associated duration of asset i.

When investors are exposed to assets denominated in foreign currencies, face the risk of an adverse movement in the exchange rate of the denominate currency in relation to the base currency, known as currency risk (see Parliament and Union 2015):

$$\begin{aligned} \text{Currency}^{up} &= \max\ (\ \Delta BOF \mid \text{currency upward schock};\ 0), \\ \text{Currency}^{down} &= \max\ (\ \Delta BOF \mid \text{currency downward schock};\ 0), \end{aligned} \tag{3.12}$$

with currency upward and downward shock being an instantaneous increase and an instantaneous decrease, respectively, of 25% of the value of the currency invested against the local currency.

Concentration risk is originated by an increased exposure to specific counterparties and extents to assets considered in the equity, spread and property risk and exclude assets covered by the counterparty default risk (see Parliament and Union 2015):

$$\text{Concentration} = \sqrt{\sum_i Conc_i^2}, \tag{3.13}$$

where $Conc_i$ is the risk charge for exposures to counterparty i and is defined as:

$$\begin{aligned} Conc_i &= \Delta BOF \mid \text{concentration downward shock}, \\ \text{concentration downward shock} &= XS_i \times g_i, \\ XS_i &= \max\left(0; E_i - CT_i \times Assets\right), \end{aligned} \tag{3.14}$$

with XS_i being the excess exposure to counterparty i, E_i denoting the exposure at default to counterparty i, CT_i denoting the relative concentration threshold applicable to counterparty i and g_i being a reduction factor.

Then the total capital requirement for market risk is a combination of all the above sub-risks using a correlation matrix (see Parliament and Union 2015):

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} SCR_i SCR_j}, \tag{3.15}$$

where $i, j \in \{interest,\ equity,\ property,\ spread,\ currency,\ concentration\}$ and $Corr_{i,j}$ denotes the correlation matrix entry for the pair of risks (i, j).

3.2.4 Return on Risk-adjusted Capital

An insurance company is subject to different investment risks and with the introduction of Solvency II the losses derived from these risks shall be absorbed by a certain amount of capital. The capital charges are calculated with the standard formula provided by the regulator. Although there are a lot of performance measures, the focus of this study is the Return on Risk-adjusted Capital, because the aim is to evaluate investments based on the capital at risk and also due to the fact that RoRAC is adequate to compare portfolios with different levels of risk or different risk profiles.

In RoRAC the capital is adjusted for risk and yields a financial analysis from the relationship between the expected profit and the risk capital necessary to achieve this profit (Matten and Warburg (1996)):

$$RoRAC = \frac{E[P]}{SCR_{market}}, \tag{3.16}$$

the SCR_{market} in the denominator causes an implicit risk-adjustment of the profit.

4 Results

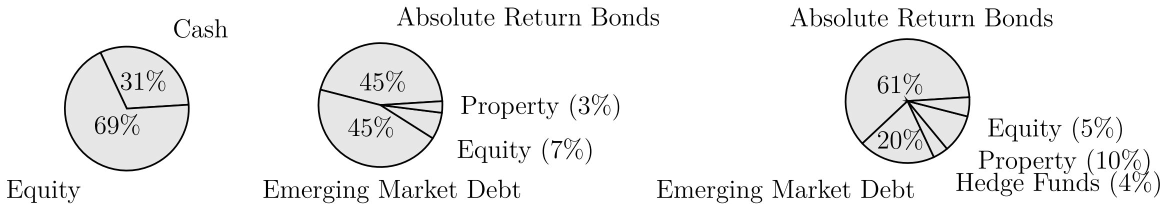
Considering the initial composition of a portfolio of a stylized Non-Life Insurance company we compute the values of the SCR_{market}, $E[P]$ and $RoRAC$.

Table 3: Solvency capital requirements for the market risk and expected profit in millions, and return on risk-adjusted capital of the initial portfolio

Risk i	SCR_i
Market	€48.36
Interest rate	€7.08
Equity	€14.22
Property	€19.96
Spread	€6.24
Currency	€1.34
Concentration	€30.51
E[P]	€1.72
RoRAC	0.036

We start by testing three investment strategies, changing the percentage in each asset class, to define a set of qualitative possibilities to improve the designing of an investment policy and without loss of generality, we allocate on sub-asset classes with higher returns.

Following the ideia of Heckel et al. (2012), we start by changing the allocation to 31% on cash and 69% on equity. The next case, based on Ratings (2011), we set 45% on absolute return bonds, 45% on emerging market debt, 7% on equity and 3% on property. At last, as in Haslip (2011), we allocate our portfolio 61% on absolute return bonds, 5% on equity, 20% on emerging market debt, 4% on hedge funds and 10% on property.



(a) 1^{st} case (b) 2^{nd} case (c) 3^{rd} case

Figure 6: Alocation of the amount invested in each sub-asset class for the three discrete cases

The results of this three allocations are as follow.

Table 4: Solvency capital requirements for the market risk and expected profit in millions, and return on risk-adjusted capital for the three discrete cases

	1^{st} case	2^{nd} case	3^{rd} case
Risk i	SCR_i	SCR_i	SCR_i
Market	€68.08	€60.13	€46.89
Interest rate	€9.23	€40.73	€12.60
Equity	€51.50	€5.55	€7.81
Property	€0	€2.37	€7.89
Spread	€3.27	€24.99	€20.38
Currency	€13.66	€1.39	€0.99
Concentration	€31.79	€30.89	€31.38
E[P]	€14.65	€6.66	€4.90
RoRAC	0.215	0.111	0.105

This discrete testing procedure indicates that there is room for implementing a different investment strategy. On the 3^{rd} case it becomes clear that by using a different asset allocation is possible to lower the capital at risk and to improve the returns of the company. Thus it is possible to set quantitative hypothesis to define a more profitable investment strategy.

Using SolveXL software (Savić, Bicik, and Morley (2011)) to implement the multi-objective optimization on the initial portfolio, described on section 3, we can obtain a set of optimal asset allocation strategies.

A genetic algorithm (GA) maintains a large population of candidate solutions and each population is generated from its predecessor. Given that a GA is a stochastic search method, is difficult for the solutions to satisfy equality constraints (Reid (1996)) and due to the complexity of the computations of the SCR_{market}, on all following approaches we do not impose a total investment restriction. Instead, we handle this restriction by reformulating the initial MOP, but we always maximize the expected profit of the portfolio and minimize the capital requirements for market risk.

We bound the investment in each asset by €0 and €10 millions. We subject it to be higher than 0, since we are not considering short sales due to difficulties associated with short selling most of the asset classes and the legal restrictions faced by insurance companies in the regulated markets. The upper bound is a mandatory input of SolveXL and although there are not a maximum limit value to be invest in each asset, we choose €10 millions to not expose too much the portfolio to the concentration risk and to insure an investment diversification among all 3535 assets. Our population of solutions is 50, which is kept to select, mutate and crossover. By applying NSGA-II, we have the Pareto front, set of non-dominated solutions with rank 0 - optimal solutions for the optimization process. Although the preferable solution, among the obtained set, is located in a less crowded region, we choose the solutions that fit best the interests of the insurance company - first, an investment amount on the interval of €339 millions and, second, the smaller value for the SRC_{market}.

First we consider the initial MOP without any constraint.

$$\begin{cases} \max\ E[P] \\ \min\ SCR_{market} \end{cases} \tag{4.1}$$

After having the set of optimal solutions, we normalize the weight of each asset, dividing by their total sum. This way we fulfil the initial restriction that the total amount available must be invested.

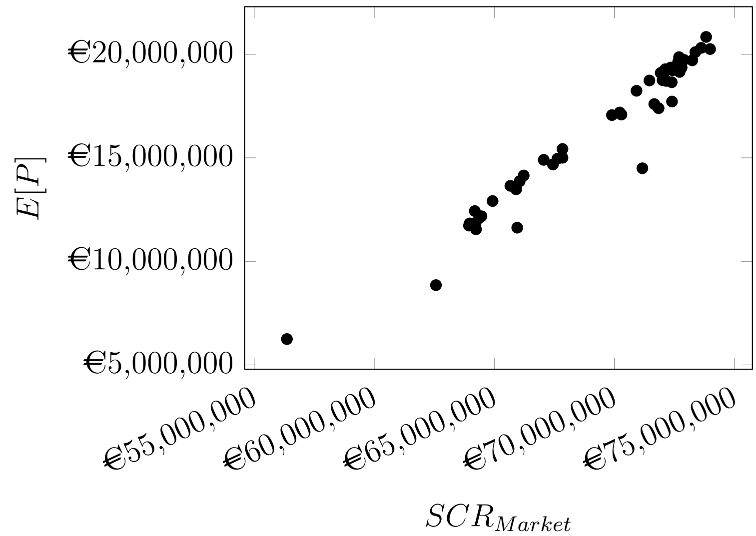


Figure 7: Pareto front for the 1^{st} run

On the above figure, we have the Pareto optimal solutions for problem (4.1), where weights are normalized and the investment amount is, for all points, equal to €339.26 millions. On Figure 7, the values for the SCR_{market} have had a significant increase compared with the initial value of €48.36 millions. Because we want to minimize the capital requirements for the market risk, the only solution to be consider is the strategy with the lower SCR_{market}.

Table 5: Result for the 1^{st} run - investment, SCR_{market} and profit in millions

Investment	SCR_{market}	Profit	RoRAC
€339.26	€56.36	€6.25	0.1109

From these procedure on, it would not be possible to normalize the weights of assets due to the nature of the optimization problem reformulations. Therefore, we have different investment amounts and we consider the expected profit and the investment amount as dependent values of the SRC_{market}.

Since the values obtained, on problem (4.1), for the SCR_{market} are higher than our initial values, we make some adjustment to the MOP by subjecting the SCR_{market} to a risk target - be less than €48 millions. This leads to low investment amounts, with the higher amount being just €195.55 millions. Consequently, we increase the inequality constraint to €60 millions, to get higher values of investment.

$$\begin{cases} \max\ E[P] \\ \min\ SCR_{market} \\ \text{s.t. } SCR_{market} < 60000000 \end{cases} \tag{4.2}$$

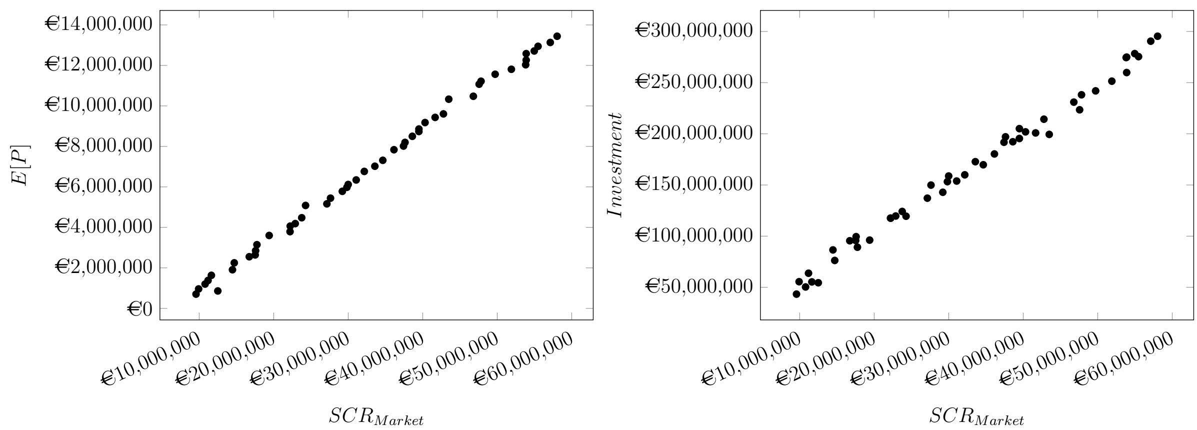


Figure 8: Pareto front for the 2^{nd} run

From the analysis of Figure 8, where the solutions for the optimization problem (4.2) are presented, and taking into consideration that our investment amount should be on the region of the initial value the company was investing, we just consider the strategy with the higher investment amount.

Table 6: Result for the 2^{nd} run - investment, SCR_{market} and profit in millions

Investment	SCR_{market}	Profit	RoRAC
€295.34	€58.04	€13.44	0.2316

To improve our investment strategy, we set higher figures for the investment amount, and introduce a new objective function - maximize the investment. To avoid that the solutions surpassed too much the initial amount, we impose a cap to this function of €400 millions. Also, we add a restriction for the SCR_{market} to be less than €48 millions.

$$\begin{cases} \max\ E[P] \\ \min\ SCR_{market} \\ \max\ Investment \\ \text{s.t. } Investment < 400000000 \\ \qquad SCR_{market} < 48000000 \end{cases} \tag{4.3}$$

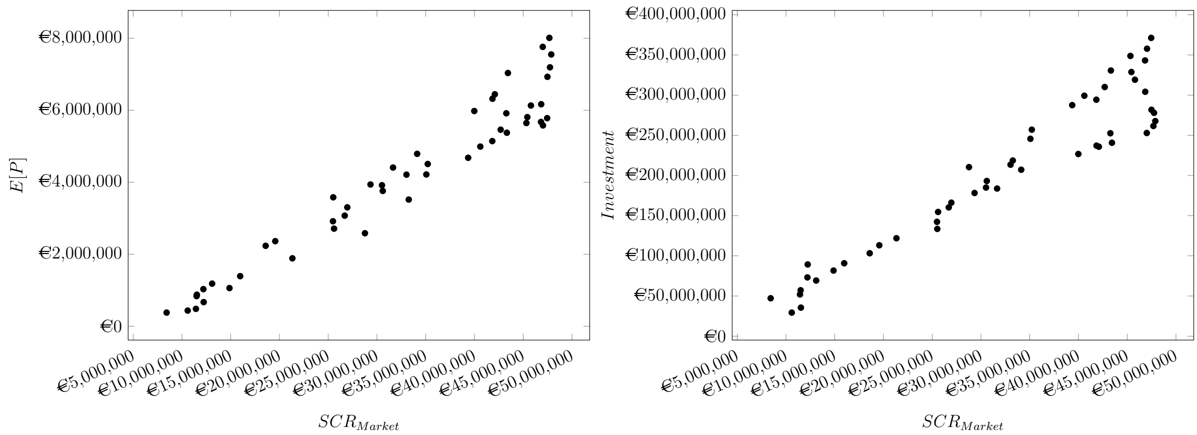


Figure 9: Pareto front for the 3^{rd} run

The solutions for the problem (4.3), on Figure 9, are selected from the strategies with a value of investment closer to the starting point.

Table 7: Results for the 3^{rd} run - investment, SCR_{market} and profit in millions

Investment	SCR_{market}	Profit	RoRAC
€330.65	€43.32	€5.38	0.1241
€343.18	€46.83	€5.67	0.1212

As can be seen on Figure 9, the risk target is limiting our solution space and the problem does not achieve the maximum value for the investment given the inequality constraint, therefore we increase the risk target to SCR_{market} less than €55 millions.

$$\begin{cases} \max E[P] \\ \min SCR_{market} \\ \max Investment \\ \text{s.t. } Investment < 400000000 \\ \qquad SCR_{market} < 55000000 \end{cases} \tag{4.4}$$

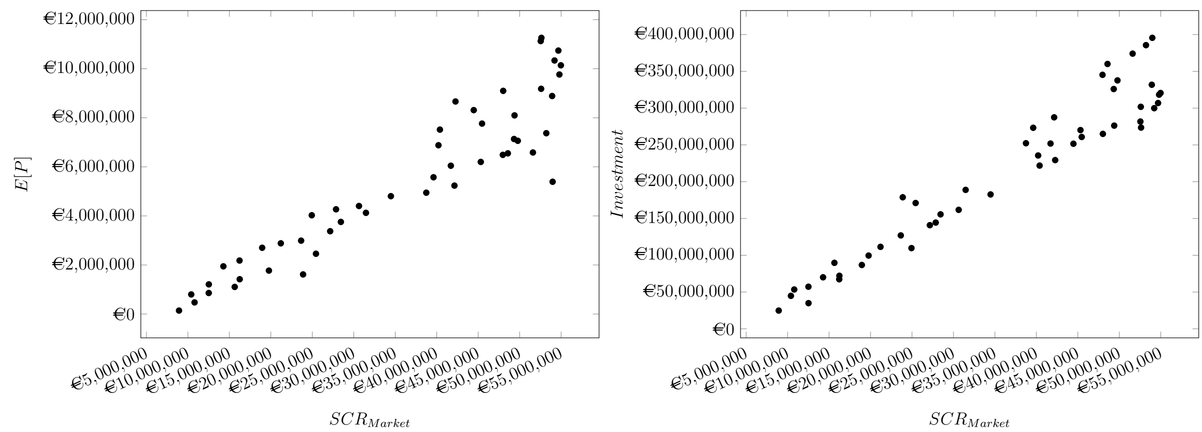


Figure 10: Pareto front for the 5^{th} run

Again, the solutions on Figure 10, are selected from the ones which investment amount is closer to €339.26 millions.

Table 8: Results for the 5^{th} run - investment, SCR_{market} and profit in millions

Investment	SCR_{market}	Profit	RoRAC
€337.69	€49.78	€7.06	0.1419
€345.26	€47.97	€6.49	0.1353

Figures 7-10 suggests that there is an increasing, almost linear, relation between the SCR_{market}, the expected profit and the investment value. As the amount of investment increases, so are the variables for return and capital at risk. By imposing different constraints and adding an objective function it is possible to have different investment strategies that meet our objectives and that represent an improvement for the company.

There are solutions where the SCR_{market} has really low values, as can be seen on Figure 8-10, but they are not worth to mention since it represents also low values of investment[1]. For an insurance company that has an amount available to invest in a portfolio, investing less would mean that the difference would be anyway invested in cash, what could increase others risks of the standard formula, for example the counterparty risk.

We can resume our solutions to a set with six investment strategies.

Table 9: Summary of the optimal solutions - investment, SCR_{market} and profit in millions

Investment Strategy Set

Strategy no.	Investment	SCR_{market}	Profit	RoRAC
1	€339.26	€56.36	€6.25	0.1109
2	€295.34	€58.04	€13.44	0.2316
3	€330.65	€43.32	€5.38	0.1241
4	€343.18	€46.83	€5.67	0.1212
5	€337.69	€49.78	€7.06	0.1419
6	€345.26	€47.97	€6.49	0.1353

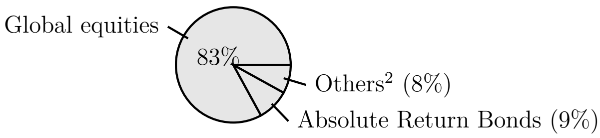
[1]Investments are funding the liabilities of an insurance company

A RoRAC greater than one implies that the expected profit exceeds the market risk capital. In other words, per Euro risk capital a profit above one has been obtained. On Table 9, the highest RoRAC equals to 0.2316, meaning that per Euro risk capital a profit of 0.2316€ is achieved. The strategy number 2 grants more expected return of the portfolio for a less value of capital at risk, but it increases the capital requirements for the market risk at almost €10 millions and the expected profit at almost €12 millions. Also, by implementing this strategy the company would not invest the whole amount what implies an increase on other risks of the SCR. The second highest RoRAC is 0.1419 for strategy number 5, where we have a slightly increase of the SCR_{market} and a substantial increase of the profits, compared to the initial values.

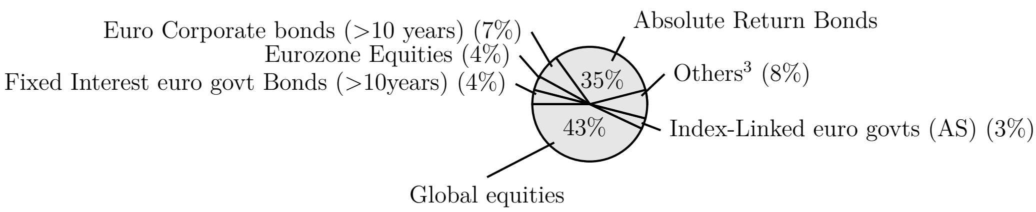
In view of improving the financial position of the company, studying an allocation to minimize the market risk and to maximize the return, we do not consider strategy number 1. Thence, it has the lowest RoRAC and as can be noted on Table 9, there are strategies with a lower value of the SCR_{market} and similar profit. Strategies number 3, 4 and 6 yield a result lower than the initial strategy for the SCR_{market} and an increase of the profit.

For the company to choose one of this investment strategies, should be taken into account the internal specifications of the company and its investment profile. First of all, must be considered that for all five strategies the investment value is different from the initial. When this difference is positive, it should be studied whether it is possible, or not, to transfer more money to be invested in the portfolio. When this difference is negative, the amount is anyway invested in cash, so it should be studied the impact this has on the remaining risks of the standard formula. To the strategies in which the SCR_{market} is greater than the initial, which implies an increase in the SCR , it is necessary to examine whether the company has the possibility of increasing the value of this fund that is intended to absorb future losses although it is possible to increase the net income from investment returns. In addition to that, with the introduction of Solvency II , insurance companies must monitor, at all time, their solvency ratio which should be greater than 100% , ie, to put into practice the specified above strategies is necessary that the available capital funds the solvency capital requirement. A good strategy is to have a higher level for the solvency value, for example 150% , since it demonstrates the healthy financial situation of the company and because it is seen as a better protection against adverse events. At last, it depends on the weight given to each variable by the insurer, if it is more critical to reduce the SRC_{market} and not as much important to have an higher expected profit or higher RoRAC.

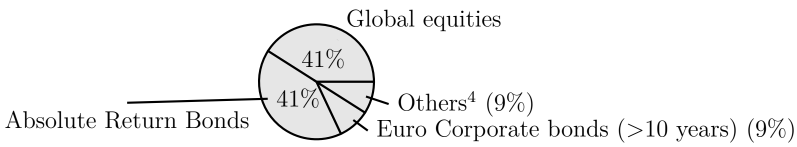
Figure 11 plots the allocations per sub-asset class of the five investment strategies considered. First, we notice that most portion of amount investment is allocated to Global Equities and Absolute Return Bonds. This can be explained by the fact that these sub-classes are composed by more assets and due to the reason that they have an higher rate of return comparing with the inital strategy. Investing a substantial amount in these two asset sub-classes allows a diversified investment policy, since they represent a large share of our portfolio (71%). On strategy number 2, there is a higher exposure to global equities, which justifies the amount for the SCR_{market}, because we are increasing the exposure to equity risk, and the wide boost in the expected profit, since this sub-class has an higher return compared to absolute return bonds. A comparison between the initial allocation of the portfolio, Figure 3, shows that an investment strategy goes by reducing the exposure to the sub-classes Property and Index-Linked Euro Government.



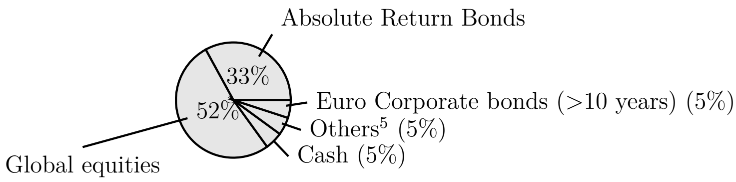
(a) Strategy no. 2



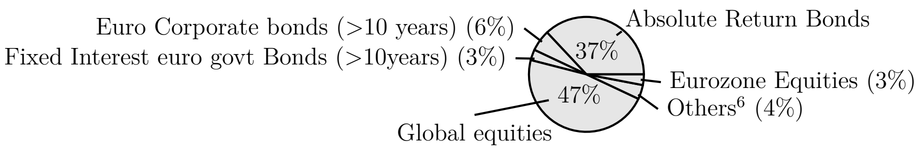
(b) Strategy no. 3



(c) Strategy no. 4



(d) Strategy no. 5



(e) Strategy no. 6

Figure 11: Alocation of the amount invested in each sub-asset class for the five strategies

[1] Euro Corporate Bonds (>10 years) (2%), Eurozone Equities (2%), Index-Linked Euro Government (1%), Fixed Interest Euro Government Bonds (>10 years) (1%), Cash (1%), Others (1%)

[2] Property (2%), Eurozone Equities (2%), Fixed Interest Euro Government Bonds (>5 years) (1%), Cash (1%), Emerging Market Debt (2%)

[3] Eurozone Equities (2%), Index-Linked Euro Government (2%), Fixed Interest Euro Government Bonds (>5 years) (1%), Fixed Interest Euro Government Bonds (>10 years) (2%), Cash (2%)

[4] Eurozone Equities (1%), Index-Linked Euro Government (1%), Fixed Interest Euro Government Bonds (>10 years) (1%), Property (3%)

[5] Index-Linked Euro Government (1%), Fixed Interest Euro Government Bonds (>5 years) (2%), Cash (1%)

5 Conclusions

In this paper we propose an approach to define an investment strategy for a stylized non-life insurance company. The procedure involves multi-objective optimization, to generate a set of investment strategies. We propose four problems to solve jointly for the optimal capital requirement and its optimal portfolio expected profit. Given equality constraints are hard to satisfy for a GA, we do not consider the restriction that the sum of the weighs invested must sum 1, but different reformulations of the initial problem. Each model is constructed based on the MOP to maximize the $E[P]$ and minimize the SCR_{market}. We begin by consider the initial optimization problem without any constraint and we normalize the weigh of each asset to comply with the restriction that all the available amount must be invested. Based on the results, we then consider the problem with a restriction for the SCR_{market} value. The nature of the new optimization impossible the use of normalization, the same for the next two models. To overcome the difficulties of the investment amount we add another objective function - maximize the investment amount. For this models, we also consider a risk target for the SRC_{market}, with different values, choose according to the results obtain.

The results show that it is possible to define a strategy considering the new regime for insurance companies, as it is obtained a set with five alternative investment strategies. We found that to improve the initial investment strategy, the exposure to the sub-classes of Propriety and Index-Linked Euro Government should be reduced, in return for a strategy with more focus on Absolute Return Bonds and Global Equities.

Under Solvency II, when splitting the assets by the CIC there are fifteen categories, and our study only contemplate eight classes. Therefore, future studies should include: Structured Notes, Futures, Call Options, Put Options, Swaps, Forwards and Credit Derivatives. Also, remaining risks of the standard formula use to compute the capital requirements of the company must be taken into account in setting the optimal SCR.

A limitation to our study is the use of NSGA-II, since the optimal solutions obtained with this algorithm may not be Pareto optimal. We can only guarantee that none of the solutions generated dominates the others.

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