Lisbon School of Economics & Management
Universidade de Lisboa

Master

Mathematical Finance

Master's Final Work

Dissertation

Changes in Expectations About Monetary Policy Decisions for Short-term Interest Rates

MIGUEL SALGADO ANTUNES DE SOUSA

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SUPERVISION:
JORGE BARROS LUIS

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Resumo

Através de derivados financeiros podemos extrair informação valiosa quando estamos a analisar as expectativas dos investidores e as suas reações a choques já experenciados, bem como a potenciais, nos mercados financeiros. Para realizar esta análise, é necessário estimar a função de densidade de probabilidade neutra ao risco (DNR) implícita nos preços das opções.

O presente estudo examina as reações e expectativas sobre as decisões de política monetária das taxas de juro de curto prazo para o euro e o dólar americano, durante e depois da pandemia do COVID-19. Durante a pandemia, os principais bancos centrais como o BCE e o FED desempenharam um papel importante na economia e, para mitigar o efeito desta crise, baixaram as taxas de juro e a inflação começou a subir, situando-se hoje em níveis elevados.

Este processo pressionou tanto o mercado accionista como o mercado imobiliário, dado que estes estão em baixo, as obrigações de alto rendimento estão a deflacionar e a economia está num caminho que conduz à recessão. Enquanto os principais bancos centrais continuarem a aumentar as taxas de juro, os mercados descem cada vez mais.

Abstract

Through financial derivatives we can extract valuable information when we are analyzing the investor's expectations and their reactions to already experienced shocks, as well as the potential ones in the financial markets. To achieve this analysis, it's needed to estimate the risk-neutral probability density function (DNR) implied in option prices.

The present study examines the reactions and expectations about monetary policy decisions of the short-term interest rates for the Euro and US dollar, during the COVID-19 pandemic and after that. During the pandemic, the major central banks such as the ECB and the FED played an important role in the economy, and to mitigate the effect of this crisis, they lowered the interest rates and inflation began to rise, standing today at high levels.

This process put pressure on the stock market as well as the real estate market, given that those are down, high-yield bonds are deflating, and the economy is on a path leading to recession. As long as the major central banks continue to raise interest rates, the markets go lower and lower.

Abbreviations

RND - Risk-Neutral Density Function

PDF - Probability Density Function

ECB - European Central Bank

FED - United States Federal Reserve

CH - Clearing House

SDE - Stochastic Differential Equation

CAPM - Capital Asset Pricing Model

FOMC - Federal Open Committee

QE - Quantitative Easing

TIPS - US Treasury Inflation Protected Securities

PEPP - Pandemic Emergency Purchase Programm

TLTRO - Targeted Longer-Term Refinancing Operations

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Contents

1 Introduction

Financial derivatives are financial instruments whose value or price is derived from an underlying asset being the purpose of these instruments mainly to hedge risk. The main derivatives are forward, futures and options, and over time they have undergone major developments with increasingly complex and numerous contracts, reaching an increasing volume of transactions, that we have never seen.

The price of these financial contracts reflects the expectations of economic agents about the future evolution of the price of the underlying asset at any given moment, and in turn the price of the underlying asset reflects the expectations of market participants regarding the future evolution of its economic determinants.

With time, these contracts have been receiving a special attention from investors and central banks, as they contain valuable information for a variety of purposes. Of these, stand out the estimation of effectiveness of the monetary policies and the study of the expectations of the market agents about future developments in different financial variables, such as interest rates. For example, Rupert & Joseph (2012), from Bank of England, analyzed the options contract from FTSE 100 and short-selling, presenting potential for enhanced predictive power and future extensions incorporating economic variables post-June 2007 and Antoni & M.Magdalena (2017) studied the RND for three US indexes from 1996 to 2005 encompassing three major crisis.

From the options prices, we can extract the risk neutral density functions and then we can analyze the trader's attitudes and reactions to crisis or shocks already happened in the markets, assuming a risk-neutral world.

If we look at options to buy in the future, and view the different strikes that are traded, then the price of these contracts tells us the investors expectation of the probability that the price of the underlying asset will be above of the different strike prices. For the valuation of an option, many financial institutions use the Black-Scholes model.

Breeden & Litzenberger (1978) were the first authors to identify a connection between option prices and the RND, showing the RND can be obtained by taking the second derivative of the option price with respect to the option strike. After this discovery, Ross & Cox (1976) introduced the concept of risk-neutral valuation in financial theory, assuming investors are indifferent to systematic risk and focus solely on expected returns. It's used to calculate asset prices, especially options, using discount rates adjusted for market scenario probabilities. Based on this result, enumerous studies emerged and a special attention was taken by the central banks due to the potential of information that can be extracted from options.

Indeed, the first decade of the 21st century was characterized by an acceleration of expansion in the financial markets and a strong instability, leading to the financial asset prices to an uncontrolled volatility. We have assisted a several crisis, such as "Nasdaq Bubble" (2000-2011), the

”Subprime Crisis” (2007-2008) and the Sovereign debt crisis (2010-2011). These crisis had a disastrous effect on the financial markets and to add that, a pandemic appeared, COVID-19 (2019-Present) causing a giant turmoil.

About the History we are writing, let’s look at the most recent event, the pandemic. Started in the end of 2019, the COVID-19 pandemic has caused lockdowns, travel restrictions and other serious measures taken by the countries to stop the dissemination of the virus. Moreover, it provoked a turmoil in the markets and in the economy.

In the first half of 2020, a few studies emerged dealing with the effects and reactions in the financial markets, standing out the work of Michael et al (2020). They extracted the Risk-neutral densities from option prices on the major indices of the world and showed the reaction of the investors was late to the COVID-19, arguing this may could have happened because the investors or financial institutions did not take seriously the lockdown imposed in China when the virus appeared, but when Italy decided to put Lombardy on lockdown then the market’s participants start to get worried, and then the Risk-neutral densities functions start to diverge.

This pandemic has led to high levels of volatility in all exchanges around the world and has changed investors expectations about the future. The main central banks, ECB and the FED, had to take measures to mitigate the effects on the economy, which in turn were harmful, adopting strong monetary policies. This strong response was mainly lowering interest rates to support economic activity, however these action can also have side effects, one of which is inflation. When interest rates are lowered, borrowing becomes cheaper, and people are encouraged to spend more. This increased spending can lead to higher demand for goods and services, which can then drive-up prices.

2 Literature Review

2.1 Options

Options are financial instruments, whose value depends on the underlying asset fluctuations and this type of financial derivatives will constitute the most important part of this dissertation, since its through options we can extract information that will lead to the risk-neutral density functions.

To start, options are a contract between two parties that gives its buyer the right (but not the obligation) to buy or sell a particular asset (the underlying) to a pre-defined price (the exercise price) at a future date (maturity or expiry date), which is generally subject to the payment of an initial premium (price of option).

When we are talking about these options it's important to distinguish two styles of options: the European and the American ones. For the European options the holder can exercise the option only at the expiration date, while for the American option the holder can exercise at any time between the execution date and the expiration date, being for that reason American options more expensive than the European options.

The payoff of the call options at the maturity is given by: max $[S_T > K]$. When $[S_T > K]$ the option is exercised and when $[S_T < K]$ the option isn't exercised. On the other hand, the payoff for the put options is given by: max $[K - S_T]$, where $[S_T]$ represents the price of the underlying asset in the maturity and [K] the price of the option.

Moreover call options can classify into: In the money, meaning that the underlying asset price is above the call strike price, $[S_T > K]$; Out the money, meaning that the underlying price is below the strike price, $[S_T < K]$ and At the money, meaning that the underlying price and the strike price are the same. $[S_T = K]$

An investor can buy a call in these conditions, and he will pay a larger premium for an option that is in the money, since it already has intrinsic value. On the other hand, put options can be classified into: In the money, meaning that the underlying asset price is below the strike price, $[S_T < K]$; Out of the money, meaning that the underlying price is above the strike price, $[S_T > K]$ and At the money, meaning that the underlying price and the strike price are the same. $[S_T = K]$

The buyer of a put options will pay a larger premium when the option is in the money, for the same reason. The premium is calculated based on the price of exercises, the spot price, the volatility, and the time remaining until the option is in maturity.

Nowadays, options are one of the most used financial instruments with a variety of uses. For example, if an investor who has a stock portfolio thinks that the price of his portfolio may lose value, he can buy put options and if the stock price drops, he can become a beneficiary through put options. In times of crisis these strategies are widely used.

2.2 Futures contract

Futures contract is an agreement between the buyer and the seller, to stipulate the purchase or sale of an asset at a future date at a fixed price. They are essentially forward contract, modeled to reduce the risk of default and enhance liquidity. These contracts are traded on organized markets by hedgers and speculators, where there are regulations and supervision.

One of the distinctive features of futures contracts is the ability for investors to cancel or terminate the contract before their maturity date, avoiding its full execution. This situation is quite common and occurs, for instance, when an investor anticipates price fluctuations that contradicts their initial position in the futures market. Investors often find themselves in a scenario where they have taken a long position in a futures contract, expecting the price of the underlying asset to increase, however, if the market takes a different turn and the asset's price starts to decline, the investor may opt not to hold onto that position until the contract matures. To mitigate potential losses, they can assume an opposing position in the same underlying asset with the same maturity date. This mechanism of early position closure is facilitated by the existence of a CH. So, futures contracts are not exercised between two parties, but between them and the clearing house (third party).

The CH is responsible for the standardization of contracts and for assuming the risk of default. In that way, all investors are required to create an account, and no one have to worry about the credit of default. In the CH, one party deposits an initial amount defined by them which serves as a guarantee for possible future losses and then the CH debits or credits this account, every day, according to the daily negative or positive variations. This daily tracking of price is named "Marking-to-market", which allows for the regular adjustment of contract values based on changes in market prices. Through marking-to-market, investors can monitor real-time fluctuations in their contract values. Margin requirements imposed by the clearing house provide a safeguard against potential losses, ensuring investors to have sufficient capital to cover adverse price variations.

These types of contracts represent an alternative investment as they are not correlated with the stock market as long as the underlying asset is not a stock and present very low transaction costs but on the other hand it presents a higher volatility. The flexibility to cancel futures contracts before maturity is one of the factors that make these instruments appealing to investors, granting them the ability to adjust their positions based on evolving market expectations and helping them to minimize potential losses or maximize gains.

In conclusion, by taking a long position in these contracts, the buyer bets on an increase in prices, which, if realized, will result in a gain of

$S_t - F$. If this doesn't happen, they will incur a negative gain $F - S_t$. On the other hand, the seller bets on a decrease in prices, and consequently, the value of their gain will be equal to the buyer's loss ($F - S_t$), and the value of their loss will be equal to the buyer's gain ($S_t - F$). This reflects the nature of futures contracts where the payoff is influenced by the movement of prices relative to the agreed-upon price (strike price). Here, *St* represents the market price of the underlying asset at the contract's expiration date, and F represents the strike price (also known as the futures price) agreed upon in the contract for the underlying asset.

2.3 Black-Scholes Model

The Black-Scholes-Merton model, also known as the Black-Scholes model (BSM) is a mathematical model to price financial instruments and calculates the theorical value of derivatives or other financial instruments, and its a fundamental strategy to hedging, or mitigate risks associated with volatility. Moreover, this model was created for European options since it doesn't take in account that American options could be exercised before the expiration date and for assets without the dividends, but later the formula was adapted to incorporate the dividends.

The mathematical model assumes the asset price follows a geometric Brownian motion, where Wt is a standard Brownian motion. The SDE is represented by:

$$dS_t = uS_t dt + \sigma dW_t \quad (1)$$

Where u is the drift part/expected rate of return and σ is the volatility. This SDE represents the evolution through time of the asset. From this equation (1) we can argue the fact that one part is deterministic and the other part is a stochastic part that depends on the Brownian motion, and since the volatility is constant this process follows a normal distribution.

This equation (1) can be also represented as an itô process. By itô lemma we know that:

$$F(t, S_t) = (\frac{dC}{dS} uS_t + \frac{dC}{dt} + \frac{1}{2}\frac{d^2C}{dS^2}\sigma^2 S^2)dt + \frac{dC}{dS}\sigma S_t dW_t \quad (2)$$

and applying the itô formula to F(t, S_t) = ln(S_t) we arrive to (3) which is the same as (1).

$$dS_t = (u - \frac{\sigma^2}{2})dt + \sigma dW_t \quad (3)$$

Considering a self-financing trading approach, where at each time t, we maintain x_t units of cash and y_t units of stock, the value P_t of this strategy at time t is determined by the cash holding x_t multiplied by the risk-free asset price B_t, plus the stock units y_t multiplied by the stock price S_t. The following equation (4) illustrates how the strategy's value is composed:

$$P_t = x_t B_t + y_t S_t \quad (4)$$

Choosing x_t and y_t so the strategy replicates the value of an option, the self financing it becomes:

$$dP_t = x_t B_t + y_t S_t \tag{5}$$

$$dP_t = r x_t B_t + y_t (u S_t dt + \sigma S_t dW_t) \tag{6}$$

$$dP_t = (r x_t B_t + y_t u S_t) dt + y_t \sigma S_t dW_t \tag{7}$$

Equating terms of (2) with the terms of (7) we obtain:

$$y_t = \frac{dC}{dS} \tag{8}$$

$$r x_t B_t = \frac{dC}{dt} + \frac{1}{2}\sigma^2 S^2 \frac{d^2C}{dS^2} \tag{9}$$

By plugging in the expressions from equations (8) and (9) into equation (4), emerges the Black-Scholes differential equation (10) which establishes the condition to which the call option price must respect under the assumptions of neutrality to the investors risk.

$$\frac{dC}{dt} + \frac{1}{2}\sigma^2 S^2 \frac{d^2C}{dS^2} + rS\frac{dC}{dS} - rC = 0 \tag{10}$$

Here C is a function of the variables S ∈]0, +∞[and t ∈]0, T[, r is the risk-free interest, t is the time to maturity, K the strike price, S the current price and sigma is the volatility.

The Black-Scholes closed formulas for an European call and put, solutions of the (10), will be respectively:

$$C = S_t N(d_1) - Ke^{-rt} N(d_2) \tag{11}$$

$$C + Ke^{-r(T-t)} = P + S \tag{12}$$

Where:

$$d_1 = \frac{ln(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}} \tag{13}$$

$$d_2 = d_1 - \sigma\sqrt{t} \tag{14}$$

This formula (11) was later adapted to incorporate the dividends, so if the stock pays a continuous dividends yield q, the formula is

$$C(S_t, t) = e^{-q(T-t)} S_t N(d_1) - e^{-r(T-t)} KN(d_2) \tag{15}$$

Where

$$d_1 = \frac{ln(\frac{S_t}{K}) + (r - q + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{16}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{17}$$

Despite BSM is considered one of the best methods in a finance industry to price financial instruments, it present certain assumptions that are rejected by the data.

In fact, it appears, the volatility is assumed to be constant for all the prices of the assets. This is a wrong assumption since for the same expiration date and underlying asset, the volatility is a convex function, called volatility smile. Allan (2014) estimated the RND based on the volatility smile, using a robust technique avoiding violation of options no-arbitrage restrictions. The volatility smile suggests that investors are willing to pay a higher premium for out-of-the-money options, possibly due to concerns about impactful events that could affect asset prices. Here, stochastic volatility models such as ARCH and GARCH assume a key role since they can be used to forecast the volatility. The ARCH model assumes the current volatility is a linear function of past squared residuals (errors). According to Nicolau (2011), this implies that periods of high volatility tend to be followed by other high-volatility periods and vice versa (Volatility Clustering), while the GARCH model is an extension of the ARCH model that takes into account both past residuals and past volatilities to forecast future volatility. This allows the model to better capture the persistence dynamics and rapid dissipation of volatility.

It assumes there are no arbitrage in the markets, meaning the put-call parity always hold, which is not true, and finally, it assumes asset prices follow a lognormal distribution, based on the principle that asset prices cannot take a negative value. However, the returns verified in the market tend to present heavy tail distributions (Kurtosis greater than 3).

2.4 Hypothesis of risk neutral valuation

The risk neutral assumption is an important concept in financial markets. Thus, it is understood that an investor is risk neutral when he only analyzes the potential return of a particular investment and does not look at the risk of it.

There are three types of investors in the markets: risk averse, who choose capital preservation and tend to avoid risk; risk-lovers, who love risk and choose to invest according to the highest risk, accepting a decrease in expected return as risk increases and risk-neutrals who make decisions based on return, being indifferent to risk.

In terms of PDF, the risk-neutral hypothesis points that prices of financial assets are determined using an adjusted PDF. The central idea is that asset prices should be adjusted according to investors risk preferences, such that the pricing of an asset is carried out as if investors were indifferent to risk. To price these assets, many agents use the Black-Scholes model, since the resulting value of the model is completely independent of investors risk preferences. Moreover, the risk-neutral PDF is a probability density function which prices are discounted by the risk-free discount factor, implying investors expected returns are equal to the risk-free rate.

The value of a European call option on an asset that doesn't pay dividends can be thought of as the anticipated future gain at maturity, discounted at the risk-free interest rate "r."

$$C(S,t) = e^{-r(T-t)} \hat{E}[max(S_T - E, 0)] \tag{18}$$

Where, Ê represents the expected value of a random variable in a risk-neutral world, S is the deterministic value of the asset under consideration at time t and S_T is the random variable describing the behavior of the asset at the option's maturity.

Mathematicians Cox & Ross (1976) formulated a significant result in asset pricing theory, establishing that under certain conditions, risk-neutral probabilities correspond to equivalent martingale measures, where a martingale measure is a probability measure reflecting the absence of arbitrage opportunities.

This implies when working with these risk-neutral probabilities, the prices of financial assets are computed in a manner that eliminates arbitrage opportunities and mirrors risk-free return expectations. This discovery is pivotal in the formulation and development of asset pricing models such as the CAPM and the Black-Scholes Model. With the assumption of no arbitrage emerges the work of Pascal et al (2022) in which they extrapolated implied volatilities of far out-of-the-money options by modelling the tails of the risk neutral distributions.

Thus, according to the Cornuejols & Tütüncü (2006), there are no arbitrage opportunities if the lime price function, C, satisfies the following conditions: C must be strictly decreasing, which means call prices must decrease as exercise prices increases, ensuring higher prices correspond to lower option prices; C must be strictly convex, meaning that call prices must be convex with respect to strike prices, or in other words, the rate of change of the option increases as the strike price increases; and C must be strictly positive in the considered interval, since the option should have some value based on the potential future gains. If the options satisfy the points described above, there are no arbitrage opportunities, which means the estimated probability distributions are risk-neutral distributions.

2.5 Monetary policies of ECB and FED

As an world power, the monetary policies adopted by the FED and ECB can affect directly and indirectly other countries, so it is extremely important to study their decisions and how they affect the expectations of the stakeholders of the financial markets.

The ECB has with mainly objective maintain the price stability. In other words, this can be achievable by maintaining the inflation low, stable, and predictable. They defined a 2% inflation over the medium term, and to ensure it they make monetary policy decisions every six weeks. For the FED, the objective is to enhance the economic growth in the US, ensuring price stability, keeping the unemployment rate at low levels and long-term interest rates at a moderate level. The FOMC is responsible for the definition of interest rates in the US.

Central banks have manipulated interest rates by directly fixing bank lending operations. With the crisis seen in recent years, central banks have gradually reduced interest rates, to stimulate the economy and make credit more accessible. However, there is no agreement by several people with this subject that this is the best way to resolve the matter. According to Correia et al (2011), when interest rates are low, central banks face difficulties in making further cuts, as rates cannot drop below zero (Called a zero-bound limit). This situation limits the central banks ability to stimulate the economy through interest rate reductions. In a scenario where the interest rate is zero, loans cannot pay interest, resulting in the full nominal value of the loan being paid at the maturity date. Consequently, these monetary policies may become less effective in boosting the economy, and can have negative side effects, such as the creation of bubbles in financial markets or the decrease in banks profit margins, which also hampers central banks ability to respond to future crisis.

During the most recent event, COVID-19 pandemic, the two most powerful central banks, ECB and FED had to take measures to mitigate the effects in the economy. This pandemic has had harmful effects on global economies, leading to successive bankruptcies of several companies and causing crisis in several countries. With this they focused on helping families and companies to continue to finance themselves through this crisis.

Although the pandemic started in the final of 2019, only in March the central banks started to take measures. On March 23, 2020, the FED launches its response to the pandemic. They announced a set of measures, as part of the QE program, among which it stood out in the purchase of government and corporate debt and the purchase of other securities. The purchase of debt involved acquiring Treasury securities issued by the United States government, resulting in assistance for funding public expenditures and maintaining controlled borrowing costs. They also bought securities issued by companies, helping to sustain the functioning of credit markets. They believe with these efforts it would help limit short-term negative impacts by supporting credit flows to households and businesses and then ease tension in financial markets. In many countries the credit flows through the bank system, but in the US the case is different, the US credit flows through the financial markets, so the FED had to work very hard to keep the financial markets stable and functional. Former Vice Chair of the FED, Don Kohn states that the US treasure market is the base for trading around the world, and if it's non-functional then every market will be affected.

Moreover, they lowered the US interest rates, close to 0, at the beginning of the pandemic, to stimulate consumption and potentiate the economy. Since demand was very strong, supply couldn't keep up, the prices rose, and inflation skyrocketed to 7% in 2021. Now, in 2023, the FED keeps raising the interest rates since the inflation reached high levels and stopped the purchase of assets, due to the part of the QE program. Olesya et al (2011), Michael and Aaron (2021) examined the value of the embedded option in the TIPS and showed the value of embedded options varies significantly over time correlating with periods of delationary expectations. Moreover, Yuriy & Jonanthan (2012) with the more recent market of options based on the CPI inflation used them to see how the PDF's respond to new announcements.

Additionally, the interest rates nearing zero and the high inflation putted pressure on the financial market. Yields of fixed-income bonds decreased, driving up bond prices as investors seek relatively better returns. However, this quest for higher yields can lead to asset bubble risks, with investors shifting towards riskier assets and potentially inflating prices unsustainably. The pressure on banks profit margins due to lower interest rates reduces their net interest margin and subsequently their banking product.

The FED also implemented the reintroduction of foreign exchange swap lines with other central banks. Foreign exchange swap lines are agreements between central banks to exchange currencies and provide liquidity during times of crisis. When the COVID-19 pandemic led to a widespread shortage of US dollars in international markets, the FED reinstated these agreements to ensure the supply of dollars and alleviate pressure in the global financial markets.

On other hand, on 12 March 2020, the ECB expanded the availability of large-scale liquidity and purchased large volumes of debt. By making this, they are ensuring the credit institutions have conditions to minimize the impacts of the pandemic and are ensuring credit reaches households and finance companies. The QE program implemented by the ECB encompassed various measures to stimulate the economy and maintain financial stability. Through asset purchases, the ECB injected liquidity into the markets, lowering borrowing costs and encouraging credit flow. By adopting negative interest rates and expanding the monetary base, the program aimed to make commercial banks pay to deposit money at the central bank. This incentivized banks to lend money instead of holding reserves, stimulating credit.

Due to inflation reaching historic levels, they have adopted a series of measures, which stand out the increase in interest rates. Interest rates have been successively increased and with that there is the probability of witnessing a crisis that could be one of the biggest of all time. With rising inflation, and rising interest rates, people who have credit will pay more, and since they won't be able to handle it, the real estate market and the stock market may suffer a crash never seen before.

The ECB also launched the PEPP, a large-scale asset purchase program aimed at combating deflationary risks and maintaining favorable financing conditions across the Eurozone. It acquires a variety of assets, including government bonds, corporate bonds, and asset-backed securities, as part of the program. Additionally, they introduced the TLTRO to ensure Eurozone banks had access to long-term funding under favorable terms, encouraging them to lend to businesses and households. The ECB provides loans to banks at low, and in some cases, even negative interest rates, depending on the fulfillment of certain conditions.

2.6 Relation between options and Risk-neutral densities

RND are distributions of asset prices at a future time, estimated today, considering a risk-neutral world. Considering a continuous time, option prices can be represented as:

$$C(X) = e^{-r(T-t)} \int_{x}^{+\infty} f(S_T)(S_T - X) dS_T \tag{19}$$

$$P(X) = e^{-r(T-t)} \int_{-\infty}^{x} f(S_T)(X - S_T) dS_T \tag{20}$$

Where f(S_T) represents the RND.

Breeden & Litzenberger (1978) were the first authors to identify a connection between option prices and risk-neutral densities. Their discovery opened the door to the possibility of calculating probabilities of different price movements of the underlying assets based on the prices of options available in the market, that is, estimating the probabilities associated with each scenario of asset prices at the options expiration date.

The authors employed the theoretical concept developed by Arrow & Debreu (1954) to create a type of asset that would pay a monetary amount only if the future price of the underlying asset reached a specific value K, otherwise, the payment would be zero. They demonstrated that this contingent asset structure could be replicated through a market strategy known as the Long Butterfly Effect. This implies that, by means of a specific set of options trades, it was feasible to establish a position that mimicked the payments of this contingent asset. The long butterfly effect operates as follows: involves three different strike prices and is composed of a combination of either call options or put options in equal quantities.

In this strategy, you would buy one call option with a lower strike price (below the current asset price), K_1, buy one call option with a higher strike price (above the current asset price), K_3, and sell two call options with a middle strike price (near the current asset price), K_2. The strategy achieves maximum profit when the underlying asset's price is at the middle strike price K_2 at the expiration date. In this scenario, the two sold options expire worthless, while the bought options hold the highest value. If the asset's price is above or below K_2 at the expiration date, the profit decreases.

Jens et al (2022) developed a method to use inflation options data to infer market probabilities for extreme inflation's events considering the options data does not reflect the real Arrow-Debreu probabilities.

Alternatively and simpler, they showed that the RND can be obtained by taking the second derivative of the option price with respect to the option strike, considering the market is complete and with the absence of arbitrage opportunities, requiring a continuous European options with the same maturity and different strikes prices. For a set of options on the same underlying asset, assuming the maturity is the same but the strike prices are different, it is possible to obtain the probabilities associated with each of these prices on the expiration date.

Considering the value of the call option can be calculated by discounting the expected payoff of the option at its exercise date, the expected payoff is the positive difference between the asset's future value (S_T) and the exercise price. This reflects the potential gain for the option holder when the asset's price exceeds the exercise price.

Assuming the premium is paid before the exercise of the option, the premium of a call option with expiration $\tau = T - t$ corresponds to:

$$C(X,\tau) = e_{-r_t,\tau}\tau \int_0^{\infty} max[S_T - X, 0] q_t(S_T) dS_T \tag{21}$$

Where, r represents the relevant interest rate at time t (for the period τ), and $q_t(S_T)$ is the RND of the underlying asset price S_T, conditioned on the current asset price S_t.

Taking the derivative of the above expression (21) with respect to the exercise price it becomes:

$$\frac{dC(X,\tau)}{dX} = -e^{-r_t,\tau}\tau \int_x^{\infty} q_t(S_T) dS_T = -e^{-r_t,\tau}\tau(1 - \int_{-\infty}^{x} q_t(S_T) dS_T) \tag{22}$$

In other words,

$$1 + \frac{dC(X,\tau)}{dX} e^{-r_t,\tau}\tau = P_q[S_T \leq X] \tag{23}$$

Where P is the probability measure. Differentiating the last expression with respect to the exercise price emerges the expression below.

$$\frac{d^2C}{dX^2} = e^{-rt} g(S_T) \tag{24}$$

According to Breeden & Litzenberger (1978), the RND must be non-negative and integrate to 1, and the price of the call has to be equal to the expected future returns, considering no arbitrage in the markets.

However, when we are implementing their work, we must take in count that, when exists small number of options contracts, with the same maturity but with different strikes prices, the RND can be difficult to draw. To solve this problem, several works appeared with two main solutions, the first was to assume a lognormal mixture and the second was to make assumptions about the underlying diffusion process. Now the most used technique is the log-normal mixture and it will be explained detailed further ahead.

Moreover, according to Bahra (1997), it is only possible to use this result (24) if the function C is monotone decreasing and convex. If these assumptions are not met, it means there will be arbitrage opportunities and the risk-neutral probabilities assume negative values.

3 Data

The data used to estimate the probability density functions were obtained via Bloomberg. This data refers to the prices of 3-month Euribor futures contracts for a given day as well as the prices of options on these contracts with maturity date on December 2023. The criteria used for the options were determined based on research and were done to carry out a thorough and a current analysis of investor expectations, in the chosen period.

Another factor that was considered was the liquidity of the options. Therefore, it's chosen to analyze options that are traded frequently, as it gives more reliable information, than the ones with low volume. Additionally, it is discarded some options with strike prices that were very far from the current price of the underlying asset because these options were not traded frequently enough, which could affect the accuracy of the analysis.

The data range presented in this analysis goes from March 1, 2020, to March 1, 2023 (3 years of daily observations), including the period of high interest rates adopted by central banks, high inflation, and the COVID-19 pandemic. In these periods, markets experienced periods of stress with government bonds seeing a huge demand, pushing prices up and driving yields down, while corporate bonds experienced high volatility.

The Euribor, or the Euro Interbank Offered Rate, is an important rate which banks use for lending to each other. When the Euro was adopted as the currency in various European countries, the Euribor was created. This reference rate is an average of the interest rates that banks apply when they are lending money to each other, particularly for potential loans that don't have specific guarantees, like mortgages. On the other hand, the Eurodollar futures contract reflects interest rate expectations on dollar deposits outside the US.

Additionally, the Euribor futures and options contracts, are special financial agreements traded on the NYSE Liffe exchange, whose value depends on the Euribor. The expected gains or losses in these agreements are linked to the behavior of Euribor. The 3-month Euribor futures contracts have specific dates for conclusion. These dates are in March, June, September, and December. The last opportunity to trade them is two business days before the third Wednesday of each of these months. The agreed-upon delivery occurs on the first business day after this last trading day.

The futures contract based on interest rates have a deposit as their underlying asset and their value varies according to changes in the interest rate. If interest rates rise, the buyer will pay to the seller, at maturity, the difference between the contracted interest rate and the actual interest rate multiplied by the notional amount. Otherwise, when interest rates are lower than those stipulated, the seller will have to compensate the buyer at maturity.

To determine, the profit or loss (price of liquidation) it can be used the following equation

$$PL = 100 - ORE \tag{25}$$

Where the last term is the Offered Rate of Euribor by EBF. It is important to mention that the International Money Market establishes that futures and options related to interest rates, such as 3-month Euribor futures contracts and Eurodollar futures contracts, are quoted in terms of a base of 100.

4 Methodology

In this chapter, it will be explained in detail the methodology used, more specifically, the method of mixture of two log-normals, for estimating RND's, as well as its advantages and disadvantages and how to overcome the possible disadvantages. This method consists of specifying a functional form for the RND and estimating its parameters through an optimization problem. The optimization problem aims to find the model parameters that yield the best possible fits to the observed data, by minimizing the discrepancy between theoretical prices and actual option prices.

Given the distribution used, it is assumed from now on that

$$q_t(S_T) = \sum_{i=1}^{k} \theta_i L(\alpha_i, \beta_i; S_T) \tag{26}$$

$$\sum_{i=1}^{k} \theta_i = 1, \theta_i > 0 \tag{27}$$

where $L(\alpha_i, \beta_i, S_i)$ is the i-th lognormal distribution with parameters α_i, β_i.

$$\alpha_i = \ln(S_t) + \left(u_i - \frac{1}{2}\sigma_i^2\right)\tau \tag{28}$$

$$\beta_i = \sigma_i \sqrt{\tau} \tag{29}$$

Thus, to estimate the parameters we want to obtain, we minimize the distance between the option premiums observed in the market and the prices obtained through the RND, through an optimization problem.

$$\begin{aligned} &= \min_{\alpha_1,\alpha_2,\beta_1,\beta_2,\theta} \sum_{i=1}^{M} [C(X_i,\tau) - C_i^0]^2 + \sum_{i=1}^{N} [P(X_i,\tau) - P_i^0]^2 \\ &\quad + [\theta e^{\alpha_1+(1/2)\beta_1^2} + (1-\theta) e^{\alpha_2+(1/2)\beta_2^2} - e^{rt} S]^2 \end{aligned} \tag{30}$$

such that $\beta_1, \beta_2 > 0$ and $0 \leq \theta \leq 1$, and

$$\begin{aligned} C(X_i,\tau), P(X_i,\tau) &= \text{Theoretical Option prices} \\ C_i^0, P_i^0 &= \text{Observed Option prices} \\ M &= \text{Number of call options in the data set} \\ N &= \text{Number of put options in the data set} \end{aligned}$$

$$\begin{aligned} C(X_i,\tau) &= \hat{C}(K_i) \\ &= e^{-rt} \int_{X_i}^{\infty} [\theta L(\alpha_1,\beta_1;S_T) + (1-\theta) L(\alpha_2,\beta_2;S_T)](S_T - X_i) dS_T \end{aligned} \tag{31}$$

$$P(X_i,\tau) = \hat{P}(K_i)$$
$$= e^{-rt}\int_0^{X_i}[\theta L(\alpha_1,\beta_1;S_T) + (1-\theta)L(\alpha_2,\beta_2;S_T)](X_i - S_T)dS_T \tag{32}$$

Where the first term of (30) is the sum of the squares of the deviations between the observed premiums and the estimated premiums of the call options, the second term is equivalent to the first one, but for put options and the third refers to the square of the difference between the mean estimated and future value.

According to Bahra (1997), options contracts are usually traded for only a small range of exercise prices being the parameters to be estimated limited. For that reason, it was decided to use the mixture of two log normals with five parameters to estimate, providing an attractive level of flexibility at the same time. Moreover, since many studies and real world data observations shows that underlying assets of options exhibit negative skewness, since the market tends to price options in a way that suggests a higher perceived risk, the log normal distribution does not capture this characteristic. In this context, it seems reasonable to assume that the RND can be modeled by a mixture of lognormals.

European options on futures and spot contracts with identical strike price and time to maturity are considered theoretical equal. Consequently, the probability distribution is also identical at maturity, resulting in equivalent parameters estimated.

In terms of advantages, Adão et al (1997), refers this method allows for more flexible density functions, allowing the simultaneous use of call and put option premiums. Also one of the advantages is to assume a priori distribution for the RND and not for the stochastic process that determines the underlying asset. Additionally the mixture of log normals is a way to incorporate different market scenarios into the pricing of options. The result of this option prices can be interpreted as a mixture of Black-Scholes prices. On the other hand, this method is less fast than the others and whose results are sensitive to the starting values. To overcome this problem, one can use a less extensive function and/or remove the factor from the approximation to the mean. (Adão et al, 1997)

In this methodology, one of the goals was to automate the process, as evidenced in the annexes. This approach significantly simplifies the process and delivers instantaneous results, enhancing efficiency and effectiveness in the financial analysis.

The code implementation was divided into two main parts. The initial part involves downloading information from Bloomberg as it is critical to provides the foundation for calculations and analysis. The second part of the code focuses on computing the risk-neutral density function. Due to the process having 13 VBA macros, it's decided to put a 5-part examples in this document (Annex A, Annex B, Annex C, Annex, D and Annex E).

5 Results

The results of the study on investor expectations implicit in option prices are presented through graphs of the risk neutral density functions.

As we can see, over time, (Figure 1) there is a slight shift of the curves to the left, reflecting a decrease in the expected value in the short term. Additionally, it is important to consider that market context plays a crucial role in interpreting skewness and other metrics associated with curves. During the period analyzed, the market demonstrated a clear perception that interest rates were on an upward trajectory, signaling a more restrictive monetary policy stance.

Market participants were alert to the possibility of an impending economic crisis. This understanding was evidenced when we analyzed the curves of the risk neutral functions on 09/01/2020 and 09/01/2021. The asymmetry of these curves has increased suggesting that the market was sensing extreme movements in interest rates.

However, a notable observation was the decrease in standard deviation between the dates, indicating a reduction in market uncertainty. This reduction in data dispersion reflects the consolidation of expectations and the growing confidence that interest rates would be rising. At the same time, the rise in the average curve indicates that the market was reviewing its expectations, anticipating a scenario of higher interest rates, and adjusting its investment strategies to align with this upward outlook.

In addition, from 09/01/2021 to 09/01/2022 the curve moved significantly to the left, reflecting the same thought above.

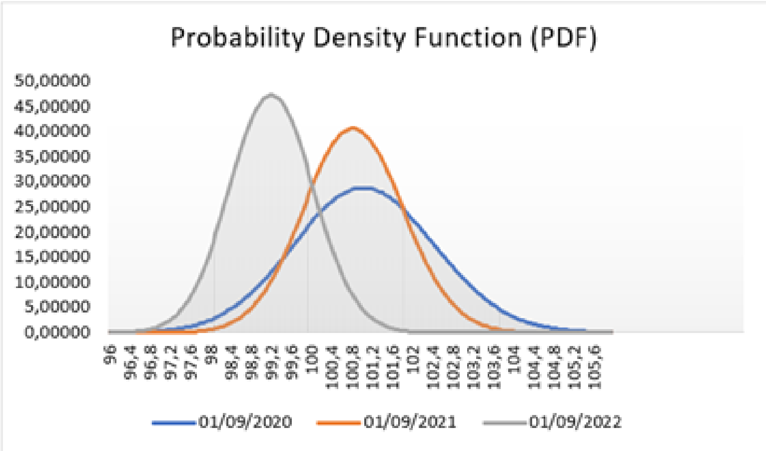


Figure 1: Probability Density Function for Euribor

Analyzing the evolution of the Eurodollar futures curves (Figure 2) over the span of one year, specifically observed at two different moments, 01/09/2021 and 01/09/2022, certain noteworthy trends emerge. Comparing the curves from these two key dates, a discernible shift to the left in the data distribution becomes apparent with a notable focus on lower strike prices compared to the previous period. Essentially, market participants seem to be aligning their expectations and operations towards a more conservative approach, anticipating lower future prices. Additionally, a decrease in the flatness of the function is observed. This reduction signifies a stabilization in market expectations regarding future prices. It implies that the market's perceptions are becoming more uniform and aligned, indicating a convergence of expectations among economic agents. The decrease in skewness, which indicates a more symmetrical distribution of data, is a significant observation. It suggests that extreme movements in the market are becoming less pronounced or less frequent, contributing to a more balanced and predictable outlook. Moreover, the increase in the average as the market revises its expectations upwards indicates a general positive trend. Market participants are progressively revising their outlook to anticipate higher prices, reflecting a growing confidence in the economic landscape.

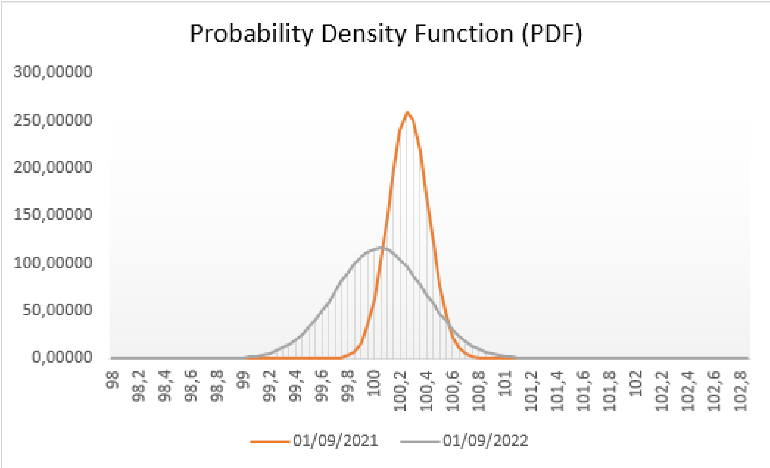


Figure 2: Probability Density Function for Euro-dollar futures

6 Conclusion

Financial derivatives like futures and options serve as vital tools for investors to navigate risk and speculate on price movements. Futures prices offer insights into the predicted future value of an asset, whereas option premiums offer a more comprehensive outlook by incorporating multiple factors that influence option pricing.

The reduction in interest rates can be seen as a direct response to the economic crisis triggered by the pandemic. Central banks worldwide took measures to lower interest rates, aiming to invigorate the economy and facilitate access to credit. This reduction had side effects like the escalation of inflation. In the US, in the end of 2020 inflation was 1,23% and in the final of 2021 skyrocket to 4,70%. Now in 2022 is 8%.

During the pandemic's early stages, two significant themes emerged: concerns over liquidity and escalated risk associated with loans and investments. The most affected loans were in the European region, given that in the American region, the most of the issued credits are at fixed rates. With the abrupt increase in rates, these credits were not significantly impacted. The same cannot be said for Europe, where the most issued credits are at variable rates.

In light of these concerns, the inflation options for example saw heightened demand. Many studies emerged where they computed the probability density functions using this type of derivative to see if the market was waiting for extreme movements in inflation or not.

Looking to the graphs (Figure 1 and 2) we can see that in both of them the probability density function moves to the left with investors anticipating a trajectory of higher interest rates. In both cases the standard deviation reduced, meaning the consolidation of expectations and a growing confidence of rising interest rates in the future.

With this stated, it is expected that the central banks will raise the short-term interest rates again at least one more time, affecting in large scale the people who have loans at variable rates. Taking into account the stress effects these macro-economic trends have on personal savings and finances, another interest rate hike, even by a small percentage, continues to raise the probability of people being forced to deliver their houses to the banks. On mass, as it may be the case in our day and age, it's possible to see a full blown crisis emerging as a consequence, one we might have never seen before.

7 References

Arrow, K. J., & Debreu, G. (1954). Existence of an equilibrium for a competitive economy. Econometrica: Journal of the Econometric Society, pp. 251-274.

Adão, B., Cassola, N., & Luís, B. (1997). Extracção de Informação a Partir dos Prémios das Opções: O Caso da Reentrada da Lira Italiana no Mtc do Sme; Boletim. Económico/Banco de Portugal.

Allan M. Malz (2014). A Simple and Reliable Way to Compute Option-Based Risk-Neutral Distribution (No. 677). Staff Report, Federal Reserve Bank of New York.

Antoni Vaello-SebastiÃ & M.Magdalena Vich-Llompart (2017). Can We Really Discard Forecasting Ability of Option Implied Risk-Neutral Distributions?; Social Science Research Network, 6-17.

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of state-contingent claims implicit in option prices. Journal of business, pp. 600-640.

Bahra, B. (1997). Implied risk-neutral probability density functions from option prices: theory and application. UK: Bank of England.

Cox, J. C., & Ross, S. A. (1976). The valuation of options for alternative stochastic processes. Journal of financial economics, 3(1), pp. 130-160.

Cornuejols & Tütüncü (2006). Optimization Methods in Finance. Carnegie Mellon University.

Correia, I., Farhi, E., Nicolini, J. P., & Teles, P. (2011). Unconventional fiscal policy at the zero bound. National Bureau of Economic Research.

Jens Hilscher, Alon Raviv & Ricardo Reis (2022). How Likely is an Inflation Disaster?. Social Science Research Network, 2-19.

Michael Hanke, Maria Kosolapova & Alex Weissensteiner (2020). COVID-19 and Market Expectations: Evidence from Option-Implied Densities. Economics Letters.

Michael McCracken & Aaron Amburgey (2021). Market-Based Measures of inflation Risks. St. Louis Fed.

Nicolau, J. (2011). Econometria Financeira. ISEG/UTL e CEMAPRE.

Olesya V. Grishchenko, Joel M. Vanden, & Jianing Zhang (2011). The Information Content of the Embedded Deflation Option in TIPS. Finance and Economics Discussion Series Divisions of Research and Statistics and Monetary Affairs Federal Reserve Board, Washington, D.C, 7-39.

Pascal Albert, Michael Herold & Matthias Muck (2022). Estimation of Rare Disaster Concerns from Option Prices – An Arbitrage-free RND-based Smile Construction Approach; The Journal of Computational Finance.

Rupert de Vincent-Humphreys & Joseph Noss (2012). Estimating Probability Distributions Of Future Asset Prices: Empirical Transformations From Option-Implied Risk-Neutral to Real-World Density Functions. Bank of England (No 455), pp. 19-28.

Yuriy Kitsul & Jonanthan H. Wright (2012). The Economics of Options-Implied Inflation Probability Density Function; Journal of Financial Eco-

nomics.

8 Annexes

```
Sub Calls_Put_Bid()

Sheets("Automatização").Activate

    Strike_Call = Cells(4, 9)
    Strike_Put = Cells(4, 14)


    Sheets("Call_Bid").Activate
    Cells(1, 2) = Strike_Call
    For i = 3 To 33
        Cells(1, i) = Cells(1, i - 1) + 0.125
    Next i

    Sheets("Put_Bid").Activate
    Cells(1, 2) = Strike_Put
    For i = 3 To 33
        Cells(1, i) = Cells(1, i - 1) + 0.125
    Next i

    Sheets("Call_Bid").Activate
    Range("B2").Select
    ActiveCell.FormulaR1C1 = _
        "=+CONCAT(Automatização!R8C9,""C "", R[-1]C, "" Comdty"")"
    Range("B2").Copy
    Range("C2:AG2").PasteSpecial


    Sheets("Put_Bid").Activate
    Range("B2").Select
    ActiveCell.FormulaR1C1 = _
        "=+CONCAT(Automatização!R8C14,""P "", R[-1]C, "" Comdty"")"
    Range("B2").Copy
    Range("C2:AG2").PasteSpecial
    Application.CutCopyMode = False

    Sheets("Call_Bid").Select
    Range("A1").Select
    ActiveCell.FormulaR1C1 = "Px_Bid"

    Sheets("Put_Bid").Select
    Range("A1").Select
    ActiveCell.FormulaR1C1 = "Px_Bid"

    Sheets("Call_Bid").Select
    Range("A3").Select
    ActiveCell.FormulaR1C1 = "=@BDH(R2C2,R1C1,Automatização!R6C9,Automatização!R10C9,""Dir=V"",""CDR=5D"",""Days=A"",""Dts=H"",""cols=1;rows=44"")"

    Sheets("Put_Bid").Select
    Range("A3").Select
    ActiveCell.FormulaR1C1 = "=@BDH(R2C2,R1C1,Automatização!R6C9,Automatização!R10C9,""Dir=V"",""CDR=5D"",""Days=A"",""Dts=H"",""cols=1;rows=44"")"
```

Annex A - Parts of the automatization of the process

```
Application.DisplayAlerts = False

    Dim Ws As Worksheet
    Dim lastColumn As Long
    Dim col As Long

    Dim ws1 As Worksheet
    Dim lastColumn1 As Long
    Dim col1 As Long

    Set Ws = ThisWorkbook.Sheets("Call_Bid")
    Set ws1 = ThisWorkbook.Sheets("Put_Bid")


    lastColumn = Ws.Cells(1, Ws.Columns.Count).End(xlToLeft).Column
    lastColumn1 = ws1.Cells(1, ws1.Columns.Count).End(xlToLeft).Column


    For col = 3 To lastColumn
        If Not IsEmpty(Ws.Cells(1, col)) Then
            |
            Ws.Cells(3, col).Formula = "=@BDH(R2C,R1C1,Automatização!R6C9,Automatização!R10C9,""Dir=V"",""CDR=5D"",""Days=A"",""Dts=H"",""cols=1;rows=44"")"
        End If
    Next col

    For col1 = 3 To lastColumn1
        If Not IsEmpty(ws1.Cells(1, col1)) Then
            ws1.Cells(3, col1).Formula = "=@BDH(R2C,R1C1,Automatização!R6C9,Automatização!R10C9,""Dir=V"",""CDR=5D"",""Days=A"",""Dts=H"",""cols=1;rows=44"")"
        End If
    Next col1

Application.DisplayAlerts = True

Sheets("Automatização").Activate
```

Annex B - Parts of the automatization of the process

```
Sub PDF()

Sheets("Automatização").Activate

    Strike_Call = Cells(12, 9)

Sheets("PDF").Activate
    Cells(1, 2) = Strike_Call
    For i = 3 To 201
        Cells(1, i) = Cells(1, i - 1) + 0.05
    Next i

Sheets("PDF").Activate

Range("A1").Select
    ActiveCell.FormulaR1C1 = "Strikes"

Range("A2").Select
    ActiveCell.FormulaR1C1 = "=+Call_Bid!RC"
Range("A2").Copy
Range("A3:A1000").PasteSpecial

Columns("A:A").Select
    Range("A15").Activate
    Selection.Copy
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
    Application.CutCopyMode = False

Columns("A:A").AutoFilter Field:=1, Criteria1:="00/01/1900": Columns("A:A").SpecialCells(xlCellTypeVisible).ClearContents
Range("A1").Select
    ActiveCell.FormulaR1C1 = "Strikes"

Range("B2").Select
    ActiveCell.FormulaR1C1 = "=Param!RC2*NORMDIST(LN(PDF!R1C),Param!RC7,Param!RC8,FALSE)+(1-Param!RC2)*NORMDIST(LN(PDF!R1C),Param!RC9,Param!RC10,FALSE)"
Selection.Copy
Range("C2:GS2").PasteSpecial
Range("B2:GS2").Select
Range("B3:GS1000").PasteSpecial

On Error Resume Next
Columns("A").SpecialCells(xlCellTypeBlanks).EntireRow.Delete

Sheets("Automatização").Activate

End Sub
```

Annex C - Parts of the automatization of the process

```
    Dim Ws As Worksheet
    Dim col As Long
    Dim nomes() As Variant
    Dim i As Long
    Set Ws = ThisWorkbook.Sheets("Param")
    nomes = Array("q", "miu_1", "sigma_1", "miu_2", "sigma_2", "alfa_1", "beta_1", "alfa_2", "beta_2", "Mean1", "Mean2", "Weighted Mean", "StDev1", "StDev2", "Weighted StDev", "SSR")
    Ws.Cells(1, 2).Value = "q"
    For i = 1 To UBound(nomes)
        Ws.Cells(1, i + 2).Value = nomes(i)
    Next i
Sheets("Param").Activate
Range("A1").Select
    ActiveCell.FormulaR1C1 = "Strikes"
Range("A2").Select
    ActiveCell.FormulaR1C1 = "=+Call_Bid!RC"
Range("A2").Copy
Range("A3:A1000").PasteSpecial
Columns("A:A").Select
    Range("A15").Activate
    Selection.Copy
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
    Application.CutCopyMode = False
Columns("A:A").AutoFilter Field:=1, Criteria1:="00/01/1900": Columns("A:A").SpecialCells(xlCellTypeVisible).ClearContents
Range("A1").Select
    ActiveCell.FormulaR1C1 = "Strikes"
Range("G2").Select
    ActiveCell.FormulaR1C1 = "=LN(Other!RC[-5])+(Param!RC[-4]-Param!RC[-3]^2/2)*Other!RC18/360"
Range("H2").Select
    ActiveCell.FormulaR1C1 = "=RC[-4]*SQRT(Other!RC18/360)"
Range("I2").Select
    ActiveCell.FormulaR1C1 = "=LN(Other!RC[-7])+(Param!RC[-4]-Param!RC[-3]^2/2)*Other!RC18/360"
Range("J2").Select
    ActiveCell.FormulaR1C1 = "=RC[-4]*SQRT(Other!RC18/360)"
Range("K2").Select
    ActiveCell.FormulaR1C1 = "=EXP(RC[-4]+0.5*RC[-3]^2)"
Range("L2").Select
    ActiveCell.FormulaR1C1 = "=EXP(RC[-3]+0.5*RC[-2]^2)"
Range("M2").Select
    ActiveCell.FormulaR1C1 = "=RC[-11]*RC[-2]+(1-RC[-11])*RC[-1]"
Range("N2").Select
    ActiveCell.FormulaR1C1 = "=SQRT(EXP(2*RC[-11]+2*RC[-10]^2)-EXP(2*RC[-11]+RC[-10]^2))"
Range("O2").Select
    ActiveCell.FormulaR1C1 = "=SQRT(EXP(2*RC[-10]+2*RC[-9]^2)-EXP(2*RC[-10]+RC[-9]^2))"
Range("P2").Select
    ActiveCell.FormulaR1C1 = "=RC[-14]*RC[-2]+(1-RC[-14])*RC[-1]"
Range("Q2").Select
    ActiveCell.FormulaR1C1 = "=SUM(SQR_calls!RC[-15]:RC[16])+SUM(SQR_puts!RC[-15]:RC[16])"

Range("B2:Q2").Copy
Range("B3:Q1000").PasteSpecial

On Error Resume Next
Columns("A").SpecialCells(xlCellTypeBlanks).EntireRow.Delete

Sheets("Automatização").Activate
```

Annex D - Parts of the automatization of the process

```
Sub Solver()
    Dim m As Integer
    Dim setCell As String

    For m = 2 To 1000

        setCell = "$Q$" & m

        Application.SolverClear

        SolverAdd CellRef:="$B$" & m, Relation:=3, FormulaText:="1/1000"
        SolverAdd CellRef:="$B$" & m, Relation:=1, FormulaText:="999/1000"
        SolverAdd CellRef:="$C$" & m, Relation:=1, FormulaText:="1"
        SolverAdd CellRef:="$D$" & m, Relation:=3, FormulaText:="1/1000"
        SolverAdd CellRef:="$F$" & m, Relation:=3, FormulaText:="1/1000"

        SolverOk setCell:=setCell, MaxMinVal:=2, ValueOf:=0, ByChange:="$B$" & m & ":$F$" & m, _
            Engine:=1, EngineDesc:="GRG Nonlinear"

        SolverSolve
    Next m
End Sub
```

Annex E - Parts of the automatization of the process

D2 =@BDH($E$1;$B$1;Automatização!$I$6;Automatização!$I$10;"Dir=V";"CDR=5D";"Days=A";"Dts=H";"cols=1;rows=44")

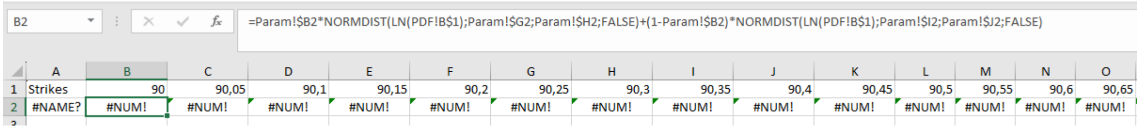
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	ERZ3 Comdty	Px last			EONIA Index			EUR001M Index	EUR002M Index	EUR003M Index	EUR004M Index	EUR005M Index	EUR006M Index	EUR007M Index	EUR008M Index	EUR009M Index	EUR012M Index
2	#NAME?			#NAME?			#NAME?		#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?
3																	

Annex F - Parts of Excel after applying the automatic procedure through VBA

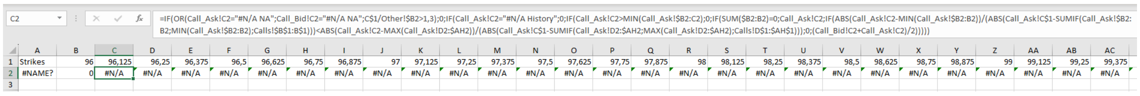
C3 =@BDH(C$2;$A$1;Automatização!$I$6;Automatização!$I$10;"Dir=V";"CDR=5D";"Days=A";"Dts=H";"cols=1;rows=44")

	A	B	C	D	E	F	G	H	I
1	Px_Bid	96	96,125	96,25	96,375	96,5	96,625	96,75	96,875
2		ERZ3C 96 Comdty	ERZ3C 96,125 Comdty	ERZ3C 96,25 Comdty	ERZ3C 96,375 Comdty	ERZ3C 96,5 Comdty	ERZ3C 96,625 Comdty	ERZ3C 96,75 Comdty	ERZ3C 96,875 Comdty
3	#NAME?		#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?	#NAME?

Annex G - Parts of Excel after applying the automatic procedure through VBA



Annex H - Parts of Excel after applying the automatic procedure through VBA



Annex I - Parts of Excel after applying the automatic procedure through VBA