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MASTER'S FINAL WORK
DISSERTATION

GLOBAL CRISES AND MARKET TURMOIL:
EXPLORING FINANCIAL CONTAGION

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GLOSSARY

- ADF** Augmented Dickey-Fuller. 1, 6, 18, 19
- AIC** Akaike Information Criterion. 1, 27
- AR** Autoregressive. 1, 5, 28
- ARCH** Autoregressive Conditional Heteroskedasticity. 1, 31
- BEKK** Baba-Engle-Kraft-Kroner. 1, 30–32
- GARCH** General Autoregressive Conditional Heteroskedasticity. 1–3, 10, 30, 32
- HQ** Hannan-Quinn Information Criterion. 1, 27
- MGARCH** Multivariate General Autoregressive Conditional Heteroskedasticity. 1, 2, 5, 6, 8–13, 15, 17, 18, 26, 30–34, 36, 37, 42, 44
- MOEX** Moscow Exchange. 1, 2, 5, 6, 8, 9, 12, 14–17, 20–23, 26, 30, 32–35, 37, 38, 40, 42–45
- NIFTY50** National Stock Exchange Fifty. 1, 2, 5, 6, 8, 12, 15–17, 20–23, 26, 30, 32–34, 38
- QQ** Quantile-Quantile. 1, 5, 23–25
- SC** Schwarz Information Criterion. 1, 27
- SPX** S&P 500. 1, 2, 5, 6, 8, 12, 15–17, 20–23, 26, 30, 32–34, 38, 40, 43
- SSE** Shanghai Stock Exchange. 1, 2, 5, 6, 8, 12, 15–17, 20–23, 26, 30, 32–35, 37, 38, 40, 43
- SX5E** Euro Stoxx 50. 1, 2, 5, 6, 8, 12, 15–17, 20–23, 26, 30, 32–35, 38, 40, 43
- TARCH** Threshold Autoregressive Conditional Heteroskedasticity. 1, 30–32
- VAR** Vector Autoregressive. 1, 2, 5, 6, 8–13, 15, 17, 18, 26–30, 32–34, 36, 37, 42, 44
- VECH** Vectorization. 1, 30, 31

ABSTRACT

This study investigates the financial contagion effects of the COVID-19 pandemic and Russia's invasion of Ukraine on global stock markets. The research focuses on five key stock market indices: SSE (Shanghai Stock Exchange), MOEX (Moscow Exchange), SX5E (Euro Stoxx 50), SPX (S&P 500), and NIFTY50 (National Stock Exchange Fifty). Utilizing the Vector Autoregressive Multivariate General Autoregressive Conditional Heteroskedasticity (VAR-MGARCH) modelling approach, the study analyzes the conditional standard deviations and correlations between these indices to provide insights into their volatility and interdependencies.

The findings reveal significant increases in volatility across all indices during the initial outbreak and global spread of the COVID-19 virus and the subsequent Russian invasion of Ukraine. The study also observes distinct patterns in the correlations among the indices, shedding light on their interdependencies and the potential for financial spillover effects. The results underscore the interconnectedness of global financial markets and the potential for localized economic events to have far-reaching impacts, providing crucial insights for investors, policymakers, and financial regulators.

The study concludes by highlighting the importance of international cooperation in managing financial contagion and the need for effective risk management strategies in the face of global crises.

KEYWORDS: Financial Contagion; VAR Models; Multivariate GARCH Models; Market Shocks; Time Series Analysis.

JEL CODES: C32; C58; G01; G15; F30; F51.

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1 INTRODUCTION

The global financial landscape is characterized by a complex web of interconnections, with events in one market having the potential to influence others. This phenomenon, known as financial contagion, has been the subject of extensive research, particularly in the wake of major global events such as the COVID-19 pandemic and geopolitical conflicts. This study aims to contribute to this body of knowledge by examining the financial contagion effects of these events on selected global stock market indices.

The research questions guiding this study are:

- What is the impact of the COVID-19 pandemic and Russia's invasion of Ukraine on the volatility of global stock market indices?
- How did these events influence the correlations between these indices?
- What evidence of financial contagion can be observed from these impacts?

The study adds to the existing body of research on financial contagion, focusing on global events like the COVID-19 pandemic and the Russia-Ukraine conflict. Its findings may offer a in-depth understanding that could be useful for future research and preliminary policy discussions.

This study aligns largely with existing literature on financial contagion, particularly concerning the COVID-19 pandemic and the Russian invasion of Ukraine. Similar to previous works like those of Ghorbel and Jeribi (2021) and Chevallier (2020), this study finds significant spikes in volatility and correlation across key global stock indices during these crisis events. However, the scope and methodologies differ; while some studies incorporate additional variables such as commodity prices and exchange rates, or focus on different sets of indices and time periods, the core finding of increased volatility and financial interdependency remains consistent. These variations in methodology and focus underline the complexity of financial contagion and emphasize the necessity for ongoing research to provide a nuanced understanding.

The novelty of this study lies in its focus on recent global events and their impacts on a specific set of stock market indices. By employing a Vector Autoregressive (VAR) Multivariate General Autoregressive Conditional Heteroskedasticity (MGARCH) modelling approach, this study provides a detailed analysis of the volatility and correlation dynamics of these indices, offering new insights into the phenomenon of financial contagion.

We employ a VAR-MGARCH model to examine the dynamic interactions and volatility among selected global stock market indices: SSE (China), MOEX (Russia), SX5E (Europe), SPX (U.S.), and NIFTY50 (India). This model is able to capture time-varying

volatility and correlations among multiple time series, making it ideal for studying financial contagion in the context of the COVID-19 pandemic and the Russia-Ukraine conflict.

The indices were strategically chosen to cover a broad range of economies and to focus on the key players relevant to the global events under study. The VAR-MGARCH model's strength in handling multiple time series effectively, yet its limitation with too many variables, guided our selection. This ensures a comprehensive, yet manageable, analysis that avoids model overfitting.

Our findings revealed significant increases in market volatility during both global crises, indicating that shocks in one market often spilled over to others. The pandemic led to a heightened correlation among global markets, evidencing a synchronized reaction to the crisis. In contrast, the invasion resulted in a marked decrease in the correlation between the Russian MOEX index and other global indices, highlighting how geopolitical events can isolate specific markets.

Our analysis offers important insights for policymakers, financial regulators, and investors. It underlines the intricate interconnectedness of global financial markets, illustrating how localized economic or geopolitical events can have far-reaching impacts. The study recommends the adoption of robust risk management strategies and calls for international cooperation in stabilizing financial markets. This research lays the groundwork for future studies aiming to explore financial contagion through various other significant global events and variables.

The study is structured as follows: Section 2 provides a literature review, defining financial contagion and situating this research in the context of existing scholarly work. In Section 3 we first discuss why certain financial indices were selected for analysis. This is followed by an overview of the events that serve as market shocks for this study. We then describe the data sources and conclude with details on how the time series data were transformed for analysis. In Section 4 we present an initial descriptive analysis of the time series data, proceed to the estimation and diagnostic checking of the VAR model, and similarly estimate and apply diagnostic checking tools to the MGARCH model. This section also includes the computation and interpretation of conditional correlations and volatilities. Section 5 offers a comprehensive look at what these empirical results mean and how they relate to policy implications. Finally, Section 6 encapsulates the key findings of the study.

2 LITERATURE REVIEW

2.1 *Financial Contagion: Definition and Previous Studies*

Financial contagion refers to the phenomenon where financial shocks spread across markets and countries. This transmission of shocks can occur through various channels, such as trade links, financial connections, and investor behavior, among others.

Forbes and Rigobon (2002) define financial contagion as a notable escalation in the linkages between different markets in the aftermath of a shock in one or several countries. This definition emphasizes the change in the correlation structure of financial markets during periods of turbulence, distinguishing between interdependence and contagion.

The measurement of financial contagion has been a subject of extensive research. The most common methodologies include the correlation coefficients approach, the extreme value theory, the coexceedances approach, and the use of multivariate GARCH models. Each of these methodologies has its strengths and limitations, and the choice of method often depends on the specific research question and data availability.

The correlation coefficients approach, as discussed by Forbes and Rigobon (2002), is straightforward and easy to implement but may fail to capture nonlinear relationships and tail dependencies. Extreme value theory, as applied by Wu, Zhang, and Zhao (2012), focuses on extreme events and tail dependencies but may overlook subtler forms of contagion. The coexceedances approach, utilized by Vo (2014), considers the simultaneous occurrence of extreme returns in multiple markets. Lastly, multivariate GARCH models, such as the VAR-MGARCH model, have been extensively discussed by Andersen et al. (2009). These models allow for the examination of how shocks to one market can affect the volatility and correlation structure of other markets, providing a dynamic and comprehensive view of financial contagion.

In conclusion, understanding financial contagion is crucial for policymakers and investors alike. The choice of the VAR-MGARCH model in this study reflects its robustness and flexibility in capturing the complex dynamics of financial contagion, providing valuable insights into the interconnectedness of global financial markets in times of crisis.

2.2 *Comparison with Previous Studies*

The results of this study, which utilized a VAR-MGARCH model to analyze the conditional standard deviations and correlations of five stock market indices, can be compared to previous studies that have employed similar methodologies and/or examined similar events. This comparison allows for a comprehensive understanding of the findings within the broader context of financial contagion research.

Several studies have analyzed the financial contagion effects of the COVID-19 pandemic using VAR-MGARCH models. For instance, Ghorbel and Jeribi (2021) and Chevalier (2020) found significant increases in volatility and correlation across global financial markets during the initial outbreak and spread of the virus. However, these studies focused on a broader set of indices and included additional variables such as commodity prices and exchange rates.

In terms of the Russian invasion of Ukraine, fewer studies have examined its financial contagion effects due to the recency of the event. However, preliminary analyses such as Izzeldin et al. (2023) have also found significant increases in volatility, showing the impact of the invasion was more rapid while Covid's was enduring.

In terms of the use of VAR-MGARCH models to study financial contagion effects, Pan et al. (2019) and Saleem (2008) have used this class of models to analyze the volatility and correlation of various financial indices and found similar patterns of increased volatility and correlation during periods of financial crisis.

However, it is important to note that these studies have often used different specifications of the VAR-MGARCH model and different measures of volatility and correlation, which may lead to differences in the results. Furthermore, these studies have typically focused on different markets and time periods, which can influence the dynamics of volatility and correlation.

In conclusion, the findings of this study are largely consistent with previous research on financial contagion effects, particularly in relation to the COVID-19 pandemic and the Russian invasion of Ukraine. However, differences in the methodologies used and the contexts studied highlight the complexity of financial contagion and the need for further research in this area.

3 DATA DESCRIPTION AND EXPLORATORY ANALYSIS

3.1 *Preliminary: Selection of Financial indices*

The selection of financial indices for this analysis is a critical step, as it sets the foundation for the entire study. The indices chosen for this research are the SSE, MOEX, SX5E, SPX, and the NIFTY50. Each of these indices highlights a distinct and critical economic zone, playing an important role in the analysis of financial contagion from the COVID-19 pandemic and Russia's invasion of Ukraine.

Shanghai Stock Exchange (SSE) serves as a measure of China's economy and is critical for understanding the initial impact of the COVID-19 pandemic, which originated in China. Moscow Exchange (MOEX), Russia's primary financial market indicator, is indispensable for grasping the economic implications of Russia's invasion of Ukraine and its spillover effects on other markets. Euro Stoxx 50 (SX5E) index represents Europe's economic vitality and is particularly relevant given the economic ties between European countries and China, Russia and Ukraine. S&P 500 (SPX) measures the performance of major U.S. companies and is included to capture the substantial influence of U.S. markets on global financial systems. Lastly, National Stock Exchange Fifty (NIFTY50) offers a snapshot of India's market performance, providing insights into how an emerging economy responds to global crises. Each index was selected for its significance to a distinct economic region, thus allowing for a comprehensive study of financial contagion caused by the COVID-19 pandemic and Russia's invasion of Ukraine.

In summary, the chosen indices represent a mix of developed and emerging economies, providing a comprehensive view of the global financial landscape. They allow us to examine the financial contagion and spillover effects of the COVID-19 pandemic and Russia's invasion of Ukraine across different economic contexts. This selection of indices is expected to provide a robust analysis of the global impact of these significant events.

The choice of five stock market indices for the VAR-MGARCH model is strategic and methodologically sound for several reasons. Firstly, the VAR-MGARCH model can analyze multiple time series concurrently. However, an increase in variables augments model complexity, leading to computational challenges and interpretational difficulties, so a balance between diverse index inclusion and model manageability is vital. A selection of five indices offers a practical compromise, ensuring comprehensive yet manageable analysis. Secondly, the selection of five indices allows for a comprehensive examination of the global financial contagion effects. It includes representation from major economies across different regions and stages of development. Lastly, the use of five indices also helps to mitigate the risk of overfitting. By limiting the number of indices, we reduce the number of parameters that need to be estimated, thereby reducing the risk of overfitting.

In conclusion, the choice of five stock market indices for the VAR-MGARCH model in this analysis is a methodologically sound decision that balances the need for diversity and comprehensiveness with practical considerations of model complexity and overfitting.

3.2 Event Studies and Market Shocks

This section presents a detailed review of two significant global events that occurred in the recent years: the COVID-19 pandemic and the Russian invasion of Ukraine. The dates selected for these events were not arbitrary; they were chosen to encapsulate the global impact of these events, marked by their ripple effects on the world economy, international relations, and human welfare. Each event is laid out in the form of a timeline, helping to visualize the sequence of the events and their magnitude, as well as providing a context for the subsequent analysis of market shocks and reactions. Sources for the events are documented in the references section.

These timelines are instrumental in framing our empirical analysis. They provide the contextual breakpoints that allow us to segregate our data into distinct periods. These will be used to segment the data in the VAR-MGARCH model, enabling the study to analyze conditional standard deviations and correlations for each defined period. This temporal segmentation will offer a more detailed view on how market volatility and interdependencies evolved during these globally impactful events, facilitating a comprehensive investigation into the occurrence of financial contagion.

Table XVII outlines the timeline of the COVID-19 pandemic with 76 distinct dates. Each date corresponds to a significant event or milestone during the course of the pandemic.

Based on these dates, the following periods were defined to facilitate the further analysis of conditional standard deviations and correlations in Section 5:

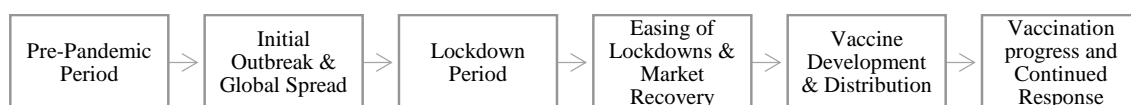


FIGURE 1: Timeline of COVID-19 Pandemic

Source: Own elaboration

1. **Pre-Pandemic Period (Yearly periods for each year from 2013 to 2019):** This period serves as a baseline to compare with the subsequent periods.
2. **Initial Outbreak and Global Spread (January 1, 2020 - March 10, 2020):** This period marks the initial global spread of COVID-19.
3. **Lockdown Period (March 11, 2020 - June 30, 2020):** Many countries initiated lockdowns during this period, which significantly impacted global markets.

4. **Easing of Lockdowns and Market Recovery (July 1, 2020 - December 31, 2020):**
As countries eased lockdowns, many markets experienced a gradual recovery.
5. **Vaccine Development and Initial Distribution (January 1, 2021 - June 30, 2021):**
The development and distribution of COVID-19 vaccines marked a significant turning point in the pandemic.
6. **Vaccination Progress and Continued Pandemic Response (July 1, 2021 - end of data set):** This period involves ongoing vaccination efforts and the world's continued response to the pandemic.

Table XVIII provides a detailed timeline of the Russian invasion of Ukraine with 27 distinct dates. Each date signifies a critical moment or shift during the invasion.

The following periods were derived from these dates, for further analysis of conditional standard deviations and correlations in Section 5:

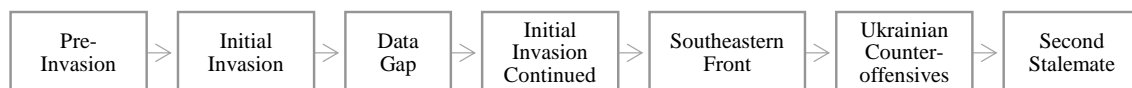


FIGURE 2: Timeline of Russia's Invasion of Ukraine

Source: Own elaboration

1. **Pre-Invasion (10 November 2021 - 23 February 2022):** Escalation of tensions between Russia and Ukraine, characterized by unusual Russian troop movements near Ukraine's border and warnings from Western nations about a potential invasion.
2. **Initial Invasion (24 February 2022 - 25 February 2022)** Russian President Vladimir Putin announced a "special military operation" in Ukraine on February 24, 2022, marking the start of the invasion.
3. **Data Gap (28 February 2022 - 23 March 2023)** No data available for MOEX during this period.
4. **Initial Invasion Continued (24 March - 7 April 2022):** Noteworthy events in this period include the stand-off at Snake Island and the battle at Antonov Airport.
5. **Southeastern Front (8 April - 26 August 2022):** This period was defined by a shift in the conflict to the southeastern region of Ukraine.
6. **Ukrainian Counteroffensives (29 August - 11 November 2022):** Ukrainian forces launched counteroffensives against the Russian invasion.
7. **Second Stalemate (12 November 2022 - end of dataset):** This period represents a phase of the conflict where neither side made significant territorial gains, and diplomatic efforts to resolve the conflict intensified.

In Section 3.3 we will look into the sources from which the data was obtained and present an initial analysis of the respective plots for each index, offering preliminary insights into the trends and patterns that will be further explored in our VAR-MGARCH model analysis.

3.3 Data Source and Description

Building on the selection of financial indices discussed in the previous section, we sourced our data from Yahoo Finance. This dataset comprises daily values for the SSE, MOEX, SX5E, SPX, and NIFTY50, spanning a decade from the start of 2013 through to the end of 2022.

The dataset begins in 2013 as this period provides a substantial window into the behavior of the financial markets during a relatively stable phase, prior to the significant events under research - the COVID-19 pandemic and Russia's ongoing invasion of Ukraine.

The dataset concludes in December 2022, a decision made to encapsulate the immediate and continuing effects of the aforementioned events. This end point facilitates a comprehensive analysis of financial contagion and spillover effects during and in the aftermath of these substantial geopolitical and global health crises.

The start dates for the earliest index values differ slightly among the five indices due to variations in market operations, national holidays, and data availability. SSE data begins on the 4th of January 2013, MOEX data is available from the 6th of March 2013, SX5E data starts on the 3rd of January 2013 and both NIFTY50 and NIFTY50 start on the 2nd of January 2013. All indices conclude with values on 30th of December 2022.

This dataset consists of a total of 12,321 observations, providing a robust base for the VAR-MGARCH model and the subsequent analysis of standard deviation and conditional correlation. Let's take a closer look at the evolution of each index.

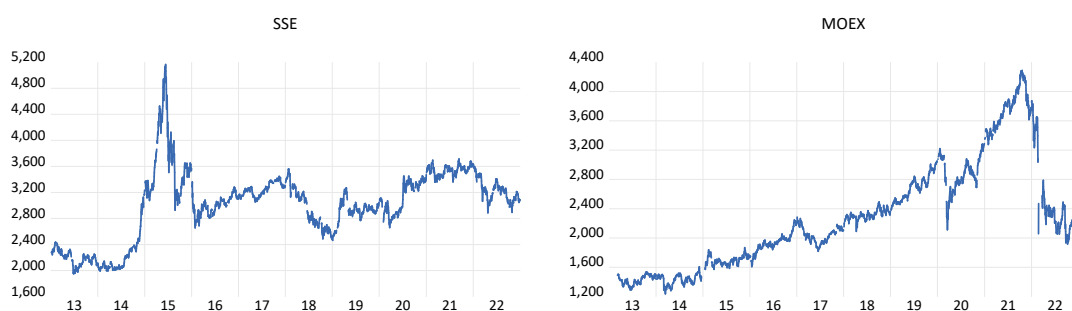


FIGURE 3: Time Series Plot of Daily Index Values for SSE, MOEX, SX5E, SPX and NIFTY50

Source: Data retrieved from Yahoo Finance; figure generated using EViews 13



FIGURE 3: (Continued) Time Series Plot of Daily Index Values for SSE, MOEX, SX5E, SPX and NIFTY50

Source: Data retrieved from Yahoo Finance; figure generated using EViews 13

Delving into the evolution of each index:

1. Shanghai Stock Exchange (SSE): The SSE index was relatively flat from 2013 to 2014, followed by a dramatic surge and crash in 2015. The index was relatively stable from 2016 to 2019, followed by a drop in early 2020 due to the COVID-19 pandemic. The index rebounded quickly and showed an upward trend in 2021. The index faced increased volatility in 2022 due to geopolitical tensions and domestic economic factors.
2. Moscow Exchange (MOEX): The MOEX index experienced a significant drop in 2014 due to the Ukraine crisis and international sanctions against Russia. It recovered and grew steadily from 2015 to 2020. The index was relatively resilient to the COVID-19 pandemic, but it faced a severe drop in 2022 due to Russia's invasion of Ukraine and subsequent international sanctions.
3. Euro Stoxx 50 (SX5E): The index showed a steady upward trend from 2013 to 2015, followed by a period of volatility due to the Greek debt crisis. It recovered and continued to rise until 2020, when it fell sharply due to the COVID-19 pandemic. The index rebounded in late 2020 and early 2021 but faced volatility again in 2022 due to Russia's invasion of Ukraine.

4. S&P 500 (SPX): The SPX index showed a strong and steady upward trend from 2013 to 2020, reflecting the robust growth of the US economy. The index experienced a sharp drop in early 2020 due to the COVID-19 pandemic but rebounded quickly and reached new highs in 2021. The index faced increased volatility in 2022 due to inflation concerns and geopolitical tensions.
5. National Stock Exchange Fifty (NIFTY50): The NIFTY50 index showed a strong upward trend from 2013 to 2020, with some volatility due to domestic economic factors and global events. The index dropped sharply in early 2020 due to the COVID-19 pandemic but rebounded quickly and continued its upward trend.

In conclusion, the selection of the five stock market indices - SSE, MOEX, SX5E, SPX, and NIFTY50 - provides a comprehensive and diverse representation of the global financial landscape. The data spans from 2013 to 2022, providing a substantial dataset for our VAR-MGARCH model. The preliminary analysis of the evolution of these indices offers valuable insights into their behavior and responses to significant global events. As we move forward to Section 3.4, we will delve into the necessary adjustments made to this time series data to facilitate a more detailed and quantitative study of its characteristics.

3.4 Time Series Transformation

Before proceeding, two significant challenges had to be addressed for the empirical analysis of the data: the presence of gaps, and non-stationarity.

The existence of gaps in the time series data can be attributed to a variety of factors, including non-trading days due to weekends and holidays, missing data due to technical glitches or errors in data collection, and periods of market closure due to extraordinary events. In the context of this study, the MOEX index, for instance, has a period between February 25th and March 25th, 2022, with no prices, likely due to the geopolitical events surrounding Russia's invasion of Ukraine.

The selected technique to solve this issue was the Cubic Spline interpolation, which is particularly apt for this analysis due to its capacity to encapsulate the inherent volatility dynamics in financial time series data (de Boor, 2001). Consequently, multiplicative interpolation was enabled to account for the proportional nature of these changes.

However, the application of Cubic Spline interpolation to the MOEX index resulted in an unexpected spike, with a significant deviation from the values recorded on February 25th and March 25th. This anomaly can be attributed to the characteristics of the Cubic Spline method, which, while attempting to fit a smooth curve through the data points, can overshoot actual values when confronted with large data gaps (de Boor, 2001).

Given this issue, an additional approach was considered for this period: Linear Interpolation. Although this method resolved the problem of the spike, it distorted the volatility dynamics for the period in question. It is important to underscore that any method employed to handle missing data invariably involves some degree of assumption and uncertainty so, the choice of interpolation method must balance the need for data continuity with the preservation of the inherent characteristics of the financial time series.

The second challenge we faced was the non-stationarity often present in financial time series data. To address this issue and to align with common empirical practices that use log-returns instead of prices, two steps were taken: differencing and logging.

Once the transformations were applied, the next step was to verify the stationarity of the series. This was accomplished using the Augmented Dickey-Fuller (ADF) test, a statistical test widely used to test the null hypothesis that a unit root is present in the data. The results of the ADF tests for each series are presented in table I.

The results provide strong statistical evidence that all the series are stationary. The confirmation of stationarity validates the suitability of the dataset to implement the VAR-MGARCH methodology, which requires the series to be stationary.

TABLE I: Augmented Dickey-Fuller (ADF) Test Results for Unit Root in Daily Log Returns of Selected Market indices

Null Hypothesis: DSX5E has a unit root		
Exogenous: Constant		
Lag Length: 0 (Automatic - based on SIC, maxlag=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-51.00233	0.0001
Test critical values: 1% level	-3.432667	
5% level	-2.862450	
10% level	-2.567299	

Source: Own elaboration with output from EViews

TABLE I: (Continued) Augmented Dickey-Fuller (ADF) Test Results for Unit Root in Daily Log Returns of Selected Market indices

Null Hypothesis: DSSE has a unit root		
Exogenous: Constant		
Lag Length: 0 (Automatic - based on SIC, maxlag=27)		
	t-Statistic	Prob.
Augmented Dickey-Fuller test statistic	-46.97656	0.0001
Test critical values:	1% level	-3.432668
	5% level	-2.862450
	10% level	-2.567299
Null Hypothesis: DMOEX has a unit root		
Exogenous: Constant		
Lag Length: 2 (Automatic - based on SIC, maxlag=26)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-27.75017	0.0000
Test critical values:	1% level	-3.432712
	5% level	-2.862469
	10% level	-2.567310
Null Hypothesis: DSPX has a unit root		
Exogenous: Constant		
Lag Length: 8 (Automatic - based on SIC, maxlag=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-15.92696	0.0000
Test critical values:	1% level	-3.432674
	5% level	-2.862453
	10% level	-2.567301
Null Hypothesis: DNIFTY has a unit root		
Exogenous: Constant		
Lag Length: 6 (Automatic - based on SIC, maxlag=27)		
	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-19.01541	0.0000
Test critical values:	1% level	-3.432672
	5% level	-2.862452
	10% level	-2.567300

Source: Own elaboration with output from EViews

4 EMPIRICAL ANALYSIS

4.1 Descriptive Analysis of the Time Series

The visual representation of the log returns for SSE, MOEX, SX5E, SPX, and NIFTY50 are depicted in Figure 4. A striking resemblance can be observed across these figures, which may indicate an inherent interconnection among these markets. Each series, in general, exhibits the characteristic volatility clustering effect, where periods of high volatility tend to cluster together, often followed by periods of tranquility, and vice versa.

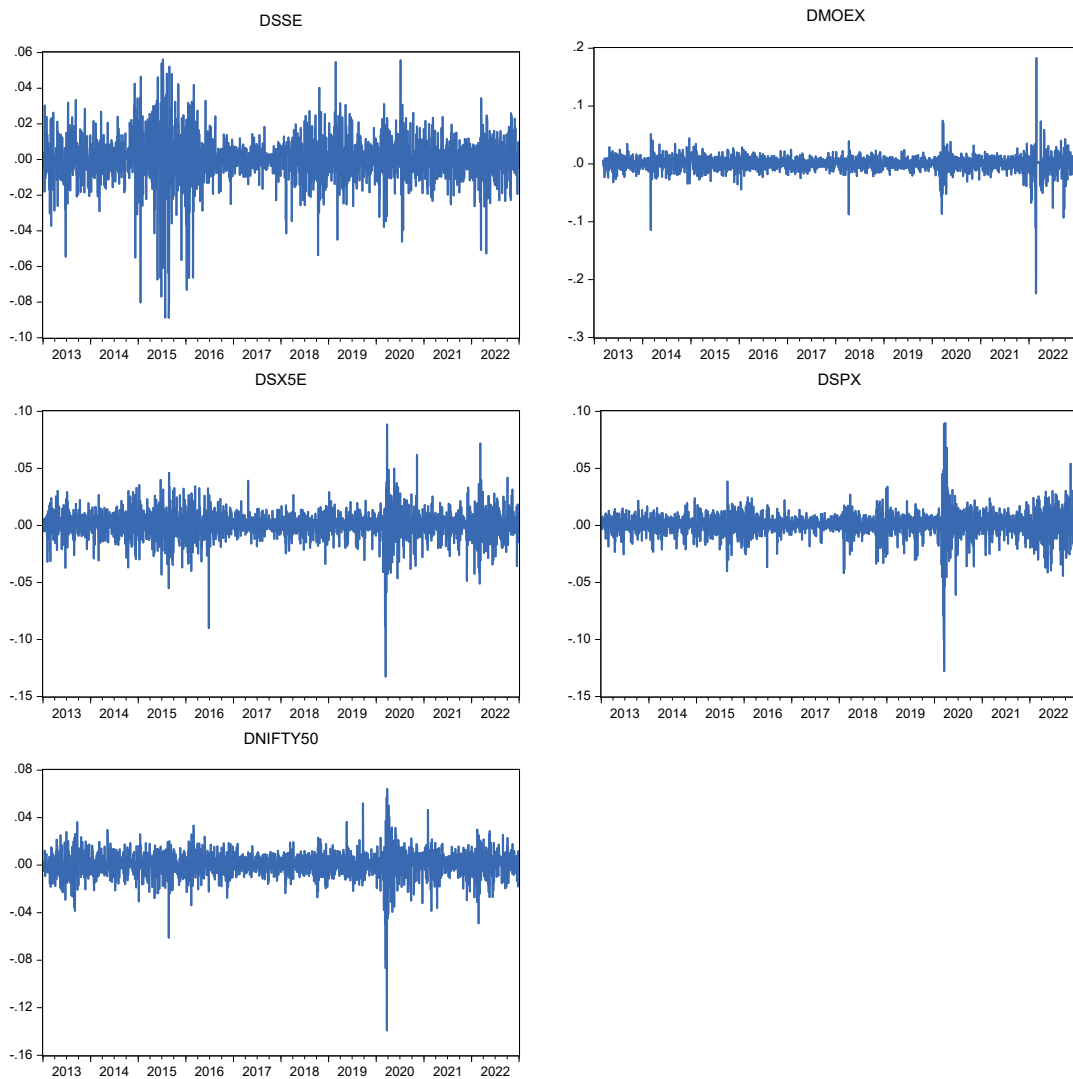


FIGURE 4: Time Series Plot of Daily Log Returns for SSE, MOEX, SX5E, SPX and NIFTY50

Source: Own elaboration with output from EViews

The MOEX, SX5E, SPX and NIFTY50 returns display a similar pattern with a noticeable increase in volatility during certain periods, while the SSE returns demonstrate pronounced volatility, with a significant number of observations diverging from average.

In addition to observing daily fluctuations, analyzing the annualized returns is crucial for understanding the long-term performance trends of these indices. Annualized returns, calculated from the daily log returns, provide insight into the compounded effect of the markets' movements over each calendar year. These values are presented in Table II and they allow for a comparison against the visual volatility depicted in Figure 4.

TABLE II: Annualized Returns Estimates for SSE, MOEX, SX5E, SPX and NIFTY50

Year	Annualized Returns				
	SSE	MOEX	SX5E	SPX	NIFTY50
2013	-0.06939	0.01152	0.14195	0.25593	0.05041
2014	0.50648	-0.09552	0.00587	0.10977	0.30157
2015	0.09075	0.27380	0.03578	-0.00702	-0.03924
2016	-0.11908	0.26938	0.01849	0.09192	0.02908
2017	0.06350	-0.05343	0.06281	0.18770	0.27653
2018	-0.23800	0.11842	-0.14360	-0.06029	0.03041
2019	0.21371	0.27475	0.24398	0.27756	0.11584
2020	0.13304	0.08303	-0.04320	0.15592	0.14294
2021	0.04631	0.15034	0.20112	0.25855	0.23198
2022	-0.14696	-0.42698	-0.11841	-0.18905	0.04193

Source: Own elaboration

The annualized returns offer a comprehensive view of the indices' extended performance, revealing their responses to economic and geopolitical disturbances. For instance, the SSE index's notable upswing in 2014 can be partially attributed to the market bubble during that period, reflecting investor optimism and market overvaluation that later corrected in 2015.

In Europe, the SX5E index's performance reflects the region's sensitivity to political and economic crises. The volatility during the Greek debt crisis and subsequent events such as Brexit showcases the sensitivity of European markets to regional instability.

Alternatively, the SPX and NIFTY50 indices show resilience, indicating a shield against such crises, possibly due to diversified economic structures and responsive monetary policies.

The mixed responses across the indices highlight the complexity of global financial ecosystems. These insights validate the necessity for risk assessment strategies that can withstand the complex nature of market dynamics.

The next step is to examine the empirical distribution of the returns.

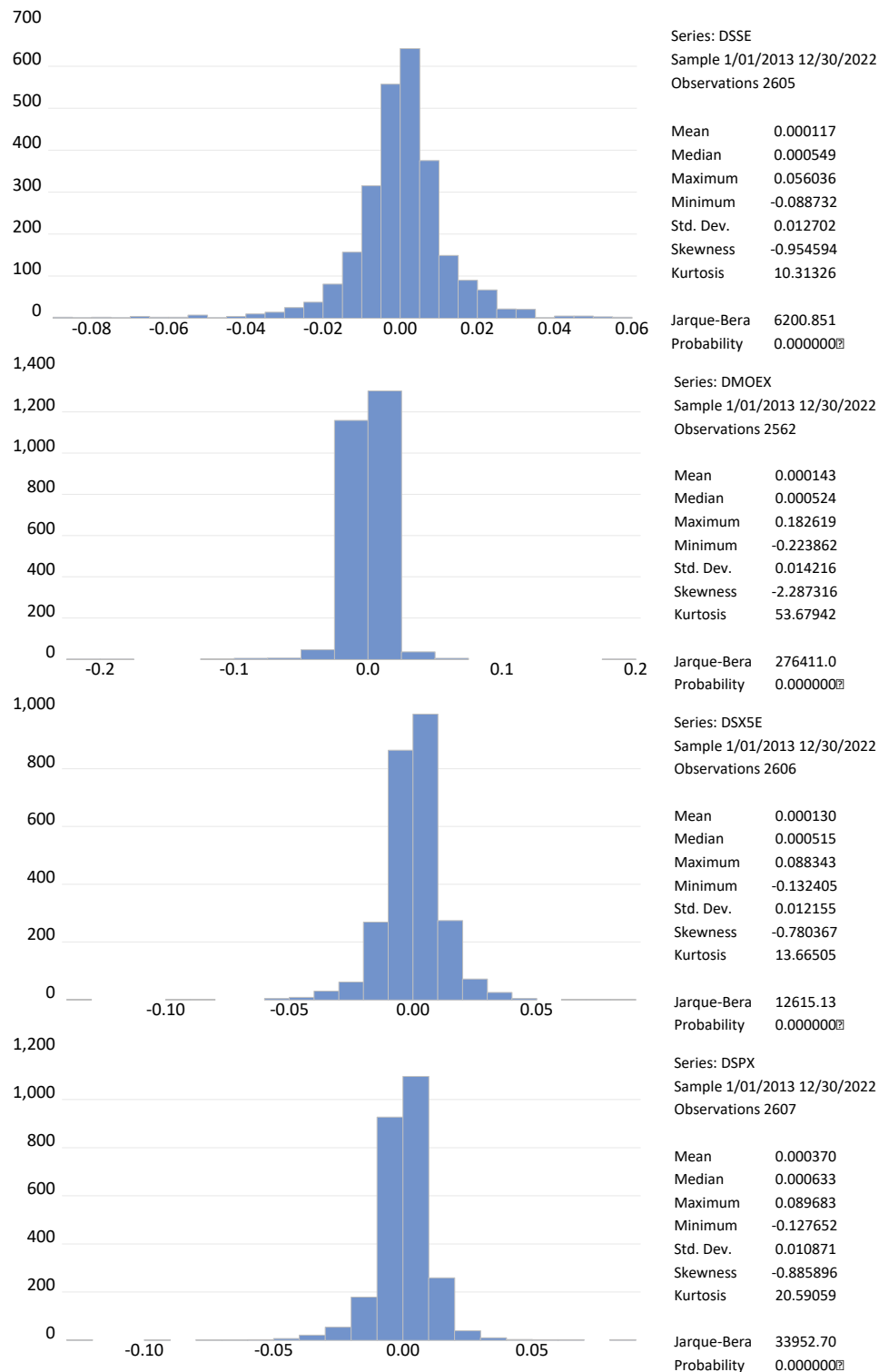


FIGURE 5: Histograms and Descriptive Statistics of Daily Log Returns for SSE, MOEX, SX5E, SPX and NIFTY50

(Continued on the following page)

Source: Own elaboration with output from EViews

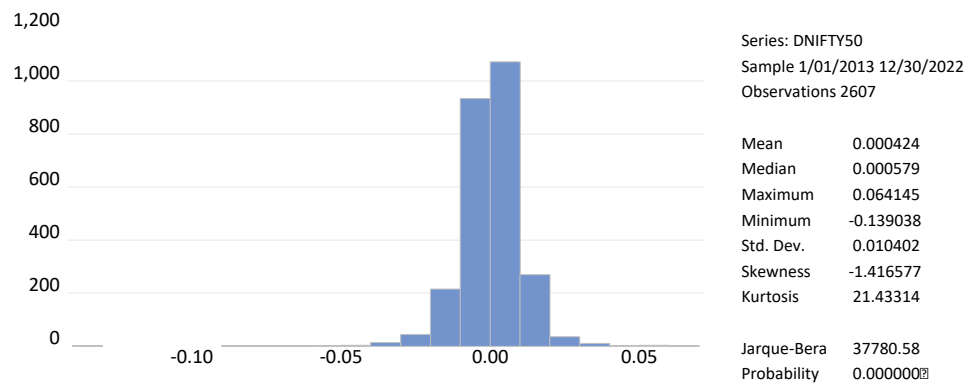


FIGURE 5: (Continued) Histograms and Descriptive Statistics of Daily Log Returns for SSE, MOEX, SX5E, SPX and NIFTY50
 Source: Own elaboration with output from EViews

The histograms and summary statistics of each market index are depicted in Figure 5 and provide valuable insights into their respective distributions.

As expected, these statistics indicate the presence of stylized facts of financial time series, such as skewness and asymmetry, high kurtosis (heavy tails), and volatility clustering. The heavy tails observed in the distribution of returns for all five indices suggest the presence of extreme events or outliers, which are often associated with financial crises or major market events. This evidence of fat tails suggests that a Student’s t-distribution may provide a better fit for these return series than a normal distribution.

The continuation of the analysis involves the application of Quantile-Quantile (QQ) plots, which are a powerful tool for visually inspecting the similarity of the distribution of a dataset to a theoretical distribution. In this case, the QQ plots are used to compare the distribution of the log returns of each market index to both a normal distribution and a Student’s t-distribution.

Figure 6 displays the QQ plots for the SSE, MOEX, SX5E, SPX, and NIFTY50 indices, respectively. Each line contains two plots: one comparing the distribution of log returns to a normal distribution, and the other to a Student’s t-distribution.

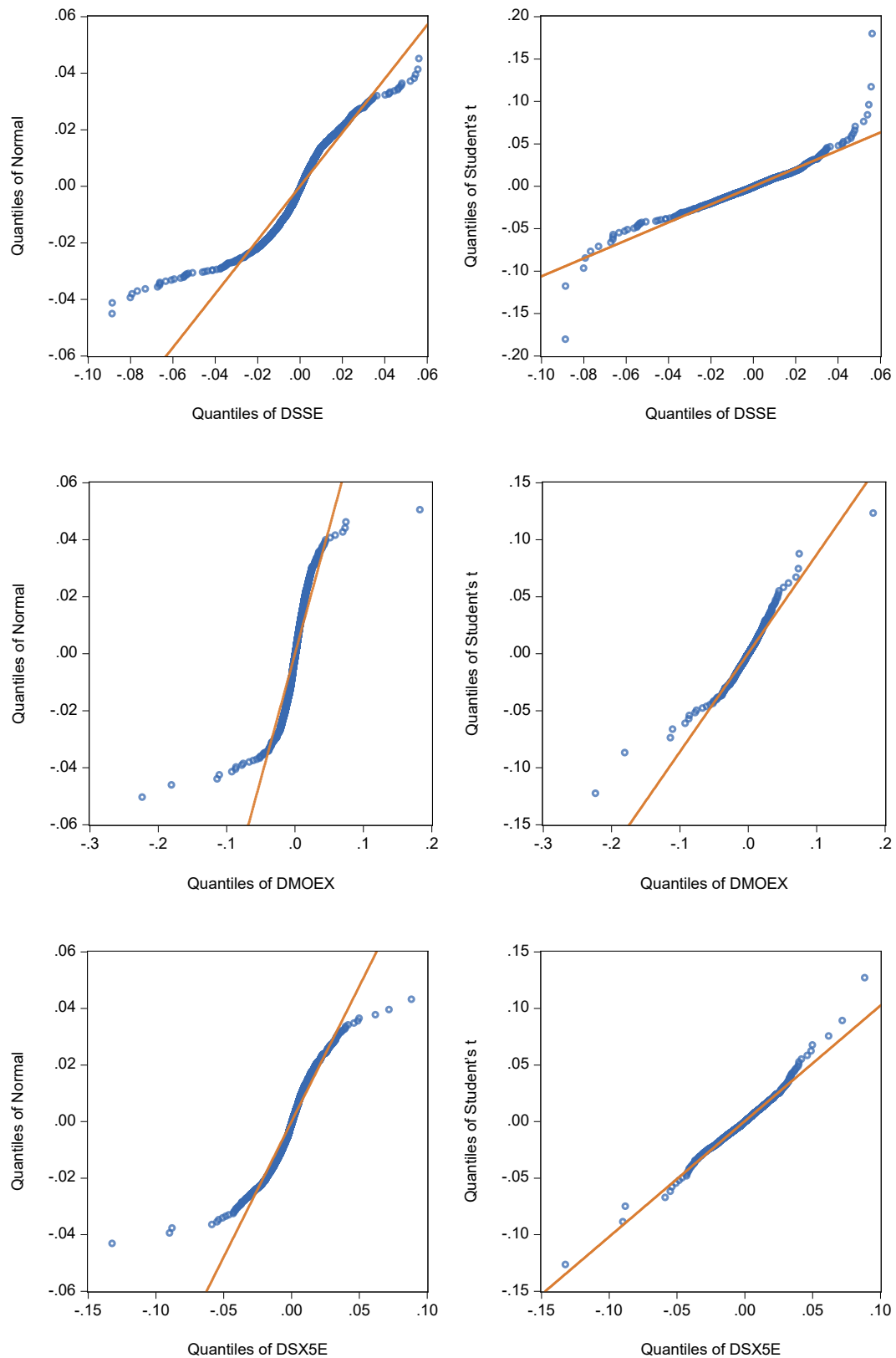


FIGURE 6: Quantile-Quantile Plots of Daily Log Returns for Selected Market indices, Comparing Against Normal Distribution (Left) and t-Distribution (Right)

Source: Own elaboration with output from EViews

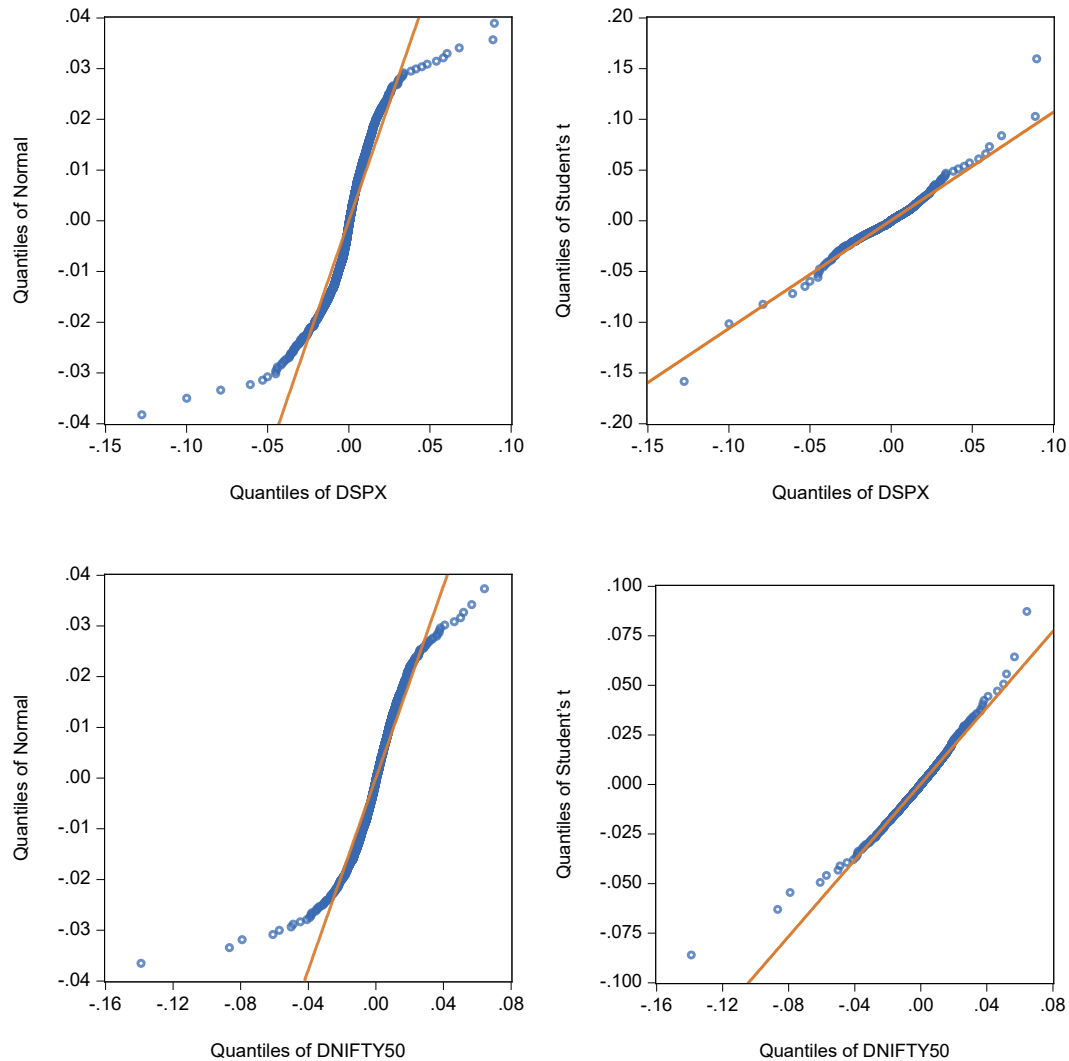


FIGURE 6: (Continued) Quantile-Quantile Plots of Daily Log Returns for Selected Market indices, Comparing Against Normal Distribution (Left) and t-Distribution (Right)

Source: Own elaboration with output from EViews

Upon inspection, it is evident that the Student's t-distribution provides a better fit for the data across all indices. This is consistent with our earlier observations of heavy tails in the distribution of returns, a characteristic that is better captured by the t-distribution than the normal distribution.

This finding underscores the importance of selecting the appropriate distribution when modeling financial time series data. The normal distribution, while mathematically convenient, often fails to capture the extreme events that are a common feature of financial markets. The Student's t-distribution, with its ability to model heavier tails, provides a more accurate representation of these events, leading to more robust risk management strategies.

Having established the descriptive characteristics of these series, the subsequent sections will focus on the application of the VAR model to these indices. The residuals from the VAR model will then be used to estimate a MGARCH model, allowing us to obtain the conditional standard deviations and correlations. This approach will provide a comprehensive understanding of the dynamics and interdependencies of these markets.

4.2 Estimation and Diagnostic Checking of VAR Model

In the preceding sections, we analyzed the log-returns of the five selected stock market indices - SSE, MOEX, SX5E, SPX, and NIFTY50. These indices exhibited a common pattern, indicative of the interconnected nature of these financial markets. To account for this interdependence, we employed the VAR methodology. The vector of time series y_t in this study is composed of the log-returns of these five stock market indices.

The VAR model for y_t can be succinctly described by the following equation:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + \epsilon_t, \quad \epsilon_t \stackrel{\text{iid}}{\sim} (0, \Sigma) \quad (1)$$

For a more comprehensive understanding of the VAR methodology, readers are referred to Tsay's *Analysis of Financial Time Series* (2010) and Lütkepohl's *New Introduction to Multiple Time Series Analysis* (2005).

The use of a VAR model on these five return series required two significant choices: the selection of the order p , indicative of the number of lags, and the validation of the VAR model's stability condition.

The selection of the lag order was guided by the minimization of multiple information criteria. Table III displays the values of different information criteria corresponding to the VAR lag order.

VAR Lag Order Selection Criteria

Endogenous variables: DMOEX DNIFTY50 DSPX DSSE DSX5E

Exogenous variables: C

Sample: 1/01/2013 12/30/2022

Included observations: 2554

Lag	AIC	SC	HQ
0	-30.91794	-30.90650	-30.91379
1	-31.12036	-31.05170*	-31.09546
2	-31.14992	-31.02404	-31.10427*
3	-31.15850	-30.97541	-31.09210
4	-31.16552	-30.92520	-31.07837
5	-31.17026	-30.87272	-31.06235
6	-31.19398	-30.83922	-31.06532
7	-31.19528	-30.78331	-31.04588
8	-31.20013*	-30.73094	-31.02998

TABLE III: VAR Lag Order Selection Criteria

Source: Own elaboration with output from EViews

* indicates lag order selected by the criterion

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

Upon examination of the information criteria, we opted for a VAR model with two lags. This decision was informed by the Hannan-Quinn Information Criterion (HQ), which recommended 2 lags. The choice of two lags also provides a middle ground, striking a balance between the Akaike Information Criterion (AIC) that suggested a higher number of lags (8 lags) and the Schwarz Information Criterion (SC) that recommended a lower number (1 lag). This approach ensures a compromise between model complexity, as indicated by AIC, and parsimony, as suggested by SC.

Next, we verified the stability condition of the VAR model. This process involved the verification of whether the roots of the VAR(2) characteristic polynomial equation were located within the unit circle. Our computations confirmed that no root lies outside the unit circle, thus satisfying the stability condition of the VAR model. The results of this test are presented in the table IV and figure 7.

Roots of Characteristic Polynomial

Endogenous variables: DMOEX DNIFTY50 DSPX DSSE DSX5E

Exogenous variables: C

Lag specification: 1 2

Root	Modulus
-0.163423	0.366089
-0.163423	0.366089
0.036145	0.303140
0.036145	0.303140
-0.264947	0.264946
0.207389	0.238425
0.207389	0.238425
0.010612	0.230481
0.010612	0.230481
-0.035595	0.035594

Inverse Roots of AR Characteristic Polynomial

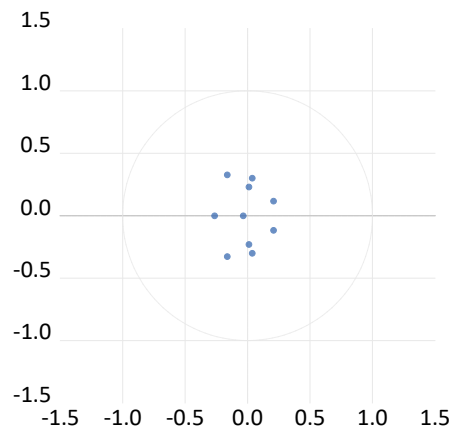


FIGURE 7: Inverse Roots of the AR Characteristic Polynomial

TABLE IV: Roots of the Characteristic Polynomial of the VAR Model

Source: Own elaboration with output from EViews

As for the detection of autocorrelation in the residuals, we employed the Portmanteau (Q) test, which is a common choice in the VAR analysis. The Portmanteau test is a general test for the null hypothesis of no autocorrelation up to a certain number of lags. Given that our VAR model is of order 2, we conducted the Portmanteau test up to 4 lags, which is twice the order of the VAR model. This is a standard practice in time series analysis to ensure that the autocorrelation at the VAR order and at possible seasonal lags is adequately captured.

The results of the Portmanteau test, presented in Table V, indicate the presence of autocorrelation in the residuals of the VAR model.

VAR Residual Portmanteau Tests for Autocorrelations
 Null Hypothesis: No residual autocorrelations up to lag h
 Sample: 1/01/2013 12/30/2022
 Included observations: 2560

TABLE V: VAR Residual Portmanteau Q-Statistics for Testing Autocorrelations

Lags	Q-Stat	Prob.*	Adj Q-Stat	Prob.*	df
1	1.120052		1.120490	—	
2	4.893815		4.897203	—	
3	72.34928	0.0000	72.43181	0.0000	25
4	130.0150	0.0000	130.1878	0.0000	50

Source: Own elaboration with output from EViews

*Test is valid only for lags larger than the VAR lag order.
 df is degrees of freedom for (approximate) chi-square distribution

Various strategies were employed to tackle the persistent issue of autocorrelation in the VAR model. While multiple methods were attempted, the problem remained, suggesting that it might be a structural feature of the data rather than a result of model specification.

Model validation involved several robustness checks to ensure the adequacy of the chosen VAR model, crucial for the subsequent GARCH model estimation. Initially, we varied the lag structures based on AIC, SC, and HQ criteria, testing lags of 1, 2, and 8. Despite these adjustments, autocorrelation issues persisted. This is aligned with observations from Bauwens et al. (1997), confirming that changing the lag structure alone isn't sufficient. We also introduced dummy variables for significant events like the COVID-19 pandemic and Russia's invasion, but they did not prove statistically significant. Incorporating structural breaks, a method endorsed by Yue Liu et al. (2020), was also ineffective. A version of the VAR model that excluded SSE data until the pandemic's onset eliminated autocorrelation, but notably, autocorrelation remained in any model that included data after 2020, regardless of the inclusion of SSE. This suggests that the influence of significant events such as the pandemic and invasion also contribute to the problem.

While the persistent autocorrelation issue in the VAR model is still not fully resolved, it doesn't undermine its usefulness for the subsequent GARCH model estimation. This unresolved issue highlights the complexity of financial market modeling but does not compromise the insights gained from our analysis. Importantly, the persistence of autocorrelation after 2020 indicates that market events have a lasting impact on the data, adding another layer of complexity.

In conclusion, despite the presence of autocorrelation in the residuals, the VAR(2) model was found to be stable and suitable for further analysis. The complete output of the VAR(2) estimation is available in the Annexes section.

Having established the VAR model, we will now use its residuals to estimate a GARCH model in Section 4.3. This will allow us to examine the conditional volatility and correlation of the selected stock market indices, thereby providing insights into the occurrence of financial spillover.

4.3 Estimation and Diagnostic Checking of GARCH Model

To assess the financial contagion effects of the COVID-19 pandemic and Russia's invasion of Ukraine on global stock markets, this study employs a VAR-MGARCH model. The VAR-MGARCH model is advantageous in that it captures both the interdependencies between multiple time series and their conditional volatilities.

In the VAR-MGARCH, y_t represents the vector of the five stock market indices of interest: SSE, MOEX, SX5E, SPX, and NIFTY50. For the MGARCH component, the conditional covariance matrix of y_t is denoted as Σ_t , and can be represented as:

$$\Sigma_t = f(\epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-q}; \theta) \quad (2)$$

For a deeper insight into the MGARCH model framework utilized in the current study, it is recommended to consult foundational texts in the field of time series analysis, specifically Tsay's *Analysis of Financial Time Series* (2010) and Lütkepohl's *New Introduction to Multiple Time Series Analysis* (2005).

In the process of model selection, we considered four different MGARCH models. The choice of the best-fitting model was based on information criteria coefficients, summarized in Table VI. The four models are as follows:

- MGARCH(1,1) with Diagonal VECH
- MGARCH(1,1) with Diagonal VECH and TARCH(1)
- MGARCH(1,1) with Diagonal BEKK
- MGARCH(1,1) with Diagonal BEKK and TARCH(1)

TABLE VI: Information Criteria for Selecting the Optimal MGARCH Model

System: MGARCH(1,1) with Diagonal VECH			
Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)			
Covariance specification: Diagonal VECH			
Log likelihood	42651.48	Schwarz criterion	-33.1804
Avg. log likelihood	3.33214	Hannan-Quinn criter.	-33.2474
Akaike info criterion	-33.2855		
System: MGARCH(1,1) with Diagonal BEKK			
Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)			
Covariance specification: Diagonal BEKK			
Log likelihood	42499.78	Schwarz criterion	-33.1232
Avg. log likelihood	3.32029	Hannan-Quinn criter.	-33.1611
Akaike info criterion	-33.1826		
System: MGARCH(1,1) with Diagonal VECH and TARCH(1)			
Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)			
Covariance specification: Diagonal VECH			
Log likelihood	42258.75	Schwarz criterion	-32.8276
Avg. log likelihood	3.30146	Hannan-Quinn criter.	-32.9164
Akaike info criterion	-32.9669		
System: MGARCH(1,1) with Diagonal BEKK and TARCH(1)			
Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)			
Covariance specification: Diagonal BEKK			
Log likelihood	42666.17	Schwarz criterion	-33.2379
Avg. log likelihood	3.33329	Hannan-Quinn criter.	-33.2830
Akaike info criterion	-33.3087		

Source: Own elaboration with output from EViews

Among the four models, the MGARCH(1,1) TARCH(1) Diagonal BEKK model emerged as the most suitable, as it minimized most of the information criteria. The detailed specification of this model can be found in the Annexes section.

Following the model estimation, a diagnostic check was performed on the standardized residuals of the model to ensure its adequacy for further econometric analysis. The diagnostic check involved employing the Portmanteau test, coupled with Cholesky orthogonalization, to ascertain the existence or non-existence of residual ARCH effects.

The results of the Portmanteau test are presented in Table VII.

System Residual Portmanteau Tests for Autocorrelations
 Null Hypothesis: no residual autocorrelations up to lag h
 Orthogonalization: Cholesky (Lutkepohl)
 Sample: 3/11/2013 12/30/2022
 Included observations: 2560

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	71.71460	0.0000	71.74262	0.0000	25
2	122.21296	0.0000	122.28046	0.0000	50
3	151.24979	0.0000	151.35136	0.0000	75
4	186.62647	0.0000	186.78340	0.0000	100
5	204.19231	0.0000	204.38363	0.0000	125
6	225.81412	0.0001	226.05623	0.0001	150
7	255.46088	0.0001	255.78428	0.0001	175
8	284.51956	0.0001	284.93405	0.0001	200
9	306.71984	0.0002	307.21265	0.0002	225
10	328.99507	0.0006	329.57524	0.0005	250
11	363.38306	0.0003	364.11162	0.0003	275
12	384.14222	0.0007	384.96855	0.0007	300

TABLE VII: MGARCH
 System Residual
 Portmanteau Tests for
 Autocorrelations

Source: Own elaboration with
 output from EViews

*The test is valid only for lags larger than the System lag order.
 df is degrees of freedom for (approximate) chi-square distribution

As expected, the Portmanteau test indicated the presence of autocorrelation in the residuals. This was anticipated given the autocorrelation observed in the VAR(2) model that was used to estimate the GARCH model. The presence of autocorrelation in the residuals implies that there is some predictable pattern in the volatility that is not captured by the model. However, this does not necessarily invalidate the model, especially since we are primarily interested in the conditional standard deviation and correlation, which are based on historical data.

Despite the presence of autocorrelation, we decided to proceed with the MGARCH(1,1) TARCH(1) Diagonal BEKK model as this model minimized most of the information criteria, suggesting a good fit to the data, and because the model's assumptions align well with the characteristics of our data.

In the next section, we will compute the standard deviations and correlations based on this MGARCH model and present them in graphical form. These graphs will provide a visual representation of the volatility dynamics and correlation structure in the data, which are key aspects of financial contagion.

4.4 Computation and Interpretation of Conditional Correlations and Volatilities

The introductory analysis of conditional correlations and volatilities of the five market indices: SSE, MOEX, SX5E, SPX, and NIFTY50, is the focus of this section. These metrics, derived from the VAR-MGARCH model, offer insights into the financial spillover effects of the COVID-19 pandemic and Russia's invasion of Ukraine on these markets.

The conditional standard deviations, indicative of market risk, reveal considerable fluctuations throughout the period under study. Figure 8, a graphical representation of these standard deviations, provides a more detailed view of these fluctuations.

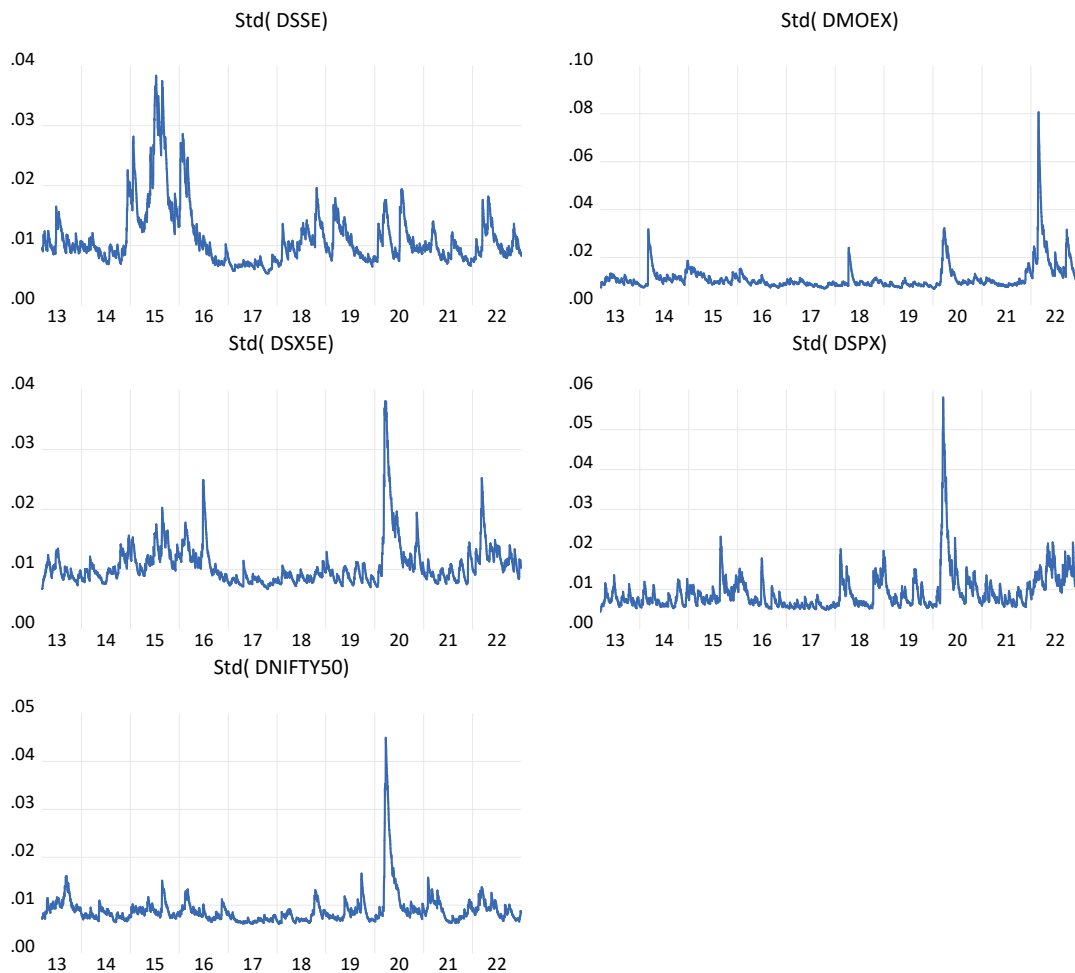


FIGURE 8: Conditional Standard Deviation Estimates from the VAR-MGARCH Model for the Daily Log Returns of SSE, MOEX, SX5E, SPX, and NIFTY50

Source: Own elaboration with output from EViews

The conditional correlations observed among the five stock market indices unveil distinct patterns that shed light on their interdependencies. These correlations, which serve as indicators of the relationship and simultaneous movements between the indices, demonstrate notable variances throughout the analyzed period. To provide a comprehensive perspective on these fluctuations, Figure 9 graphically presents the conditional correlations, enabling a detailed examination of the observed patterns.

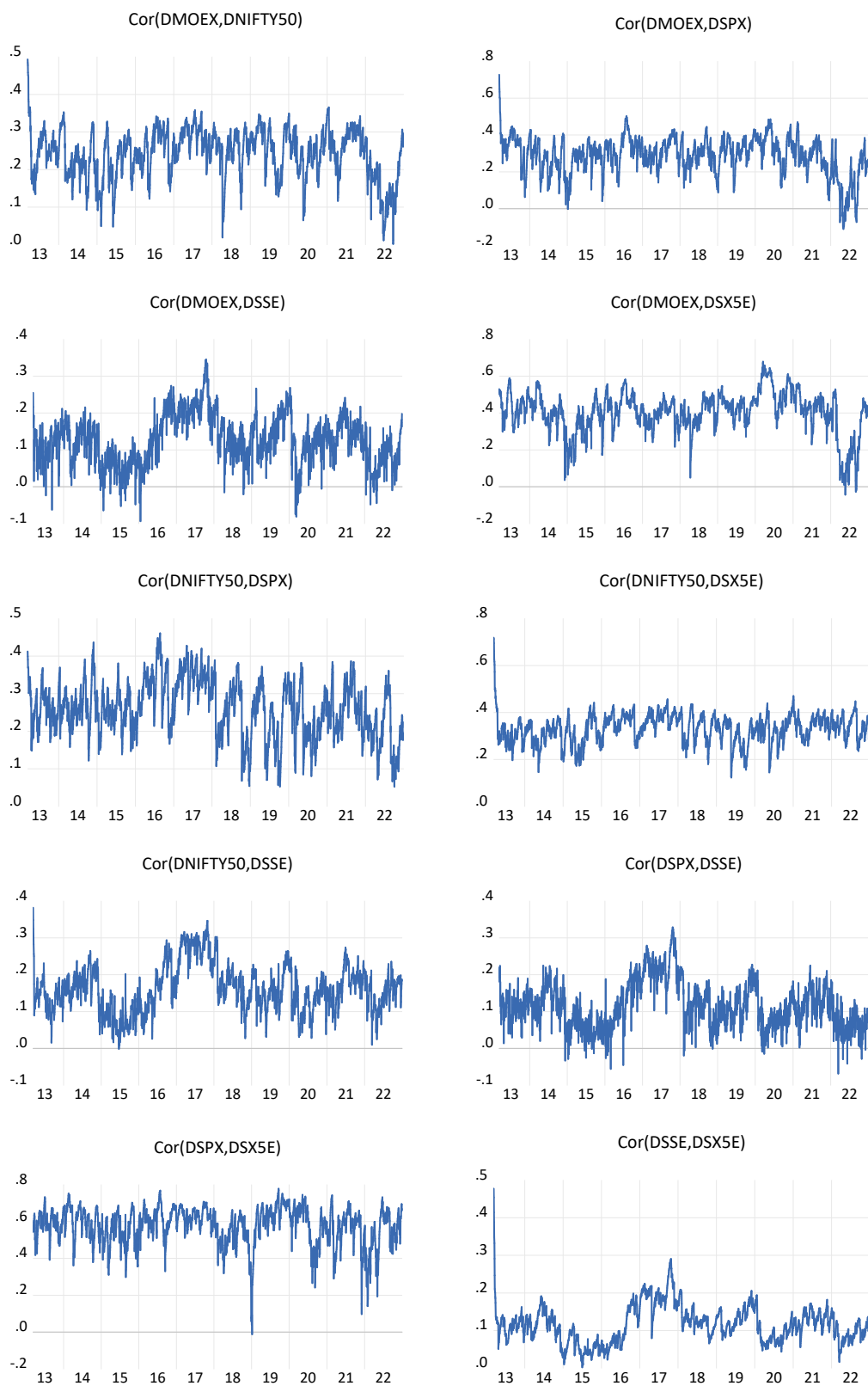


FIGURE 9: Conditional Correlation Estimates from the VAR-MGARCH Model for the Daily Log Returns of SSE, MOEX, SX5E, SPX and NIFTY50

Source: Own elaboration with output from EViews

In 2020 a significant surge in the standard deviations for all the indices is evident. This surge is a reflection of the escalated market uncertainty and risk during this period. In February 2022, the commencement of Russia's invasion of Ukraine is also marked by a significant increase in volatility, with MOEX, the Russian index, experiencing a particularly large spike. These increases in standard deviations underscore the heightened market risk during these periods of geopolitical and health crises.

In addition to these events, the graph reveals other periods of increased volatility. Notably, the SX5E index shows a period of heightened volatility around 2015-2016. Similarly, the SSE index also experiences a significant increase in volatility during the same period. These spikes, although not directly related to the events under study, highlight the sensitivity of markets to global events and their potential for financial spillover effects.

The subsequent analysis focuses on the examination of conditional correlation, aiming to identify periods characterized by noticeable changes in correlation. Around 2016-2017, a correlation between the SSE index and other indices becomes apparent. This correlation may be attributed to the stabilization of the Chinese economy following the burst of the Chinese stock market bubble. As the Chinese economy regained stability, it potentially influenced the movements of the SSE index, leading to a higher correlation with other indices during this period.

Interestingly, in 2020, despite a significant surge in standard deviations across all indices, there are no notable spikes in correlations graphically. This observation suggests that despite the widespread impact of the global crisis during that year, the correlation patterns may have remained relatively similar to the pre-crisis period. In fact, there seems to be a decline in correlations in 2020, indicating a potential decrease in the synchronicity of movements between the stock market indices during this turbulent period.

In 2022, there is a discernible decrease in the correlation of MOEX, the Russian index, with other markets, indicating reduced correlation between MOEX and the other indices during this period. This decline in correlation can be attributed to the commencement of Russia's invasion of Ukraine, which likely created distinct circumstances impacting the Russian market and causing it to deviate from general market trends.

The correlation patterns and dynamics provide initial evidence of potential financial spillover effects during periods of crisis. These insights underscore the importance of considering not only the individual movements of stock market indices but also their interdependencies and correlations when assessing market dynamics and risk exposure.

These observations, while preliminary, provide a foundation for the more in-depth analysis to be conducted in Section 5.

5 DISCUSSION AND INTERPRETATION

5.1 Interpretation of Results

This section employs the same VAR-MGARCH model as before to analyze the selected stock indices' volatility and interdependencies. It starts by evaluating the conditional standard deviations for each index. Table VIII and Figure 10 offer a detailed look at the average standard deviations during significant global events to provide more comprehensive insights into market behaviors.

TABLE VIII: Conditional Standard Deviation Estimates from the VAR-MGARCH Model for the Daily Log Returns of Selected Market indices

Period	Average Conditional Standard Deviation				
	SSE	MOEX	SX5E	SPX	NIFTY50
11/03/2013 - 31/12/2013	0.01068	0.01015	0.00939	0.00749	0.01039
01/01/2014 - 31/12/2014	0.00861	0.01248	0.00982	0.00757	0.01006
01/01/2015 - 31/12/2015	0.02071	0.01139	0.01262	0.00954	0.00967
01/01/2016 - 30/12/2016	0.01254	0.00966	0.01285	0.00828	0.00894
02/01/2017 - 29/12/2017	0.00686	0.00909	0.00751	0.00569	0.00702
01/01/2018 - 31/12/2018	0.01101	0.01014	0.00914	0.00990	0.00815
01/01/2019 - 31/12/2019	0.01029	0.00850	0.00885	0.00829	0.00829
01/01/2020 - 10/03/2020	0.01054	0.00967	0.01078	0.01107	0.00876
11/03/2020 - 30/06/2020	0.01150	0.01897	0.02688	0.02588	0.02421
01/07/2020 - 31/12/2020	0.01152	0.00994	0.01176	0.00986	0.00908
01/01/2021 - 30/06/2021	0.00963	0.00931	0.00865	0.00817	0.00999
01/07/2021 - 09/11/2021	0.00925	0.00885	0.00956	0.00727	0.00699
10/11/2021 - 23/02/2022	0.00811	0.01695	0.01134	0.01047	0.00991
24/02/2022 - 25/02/2022	0.00884	0.07365	0.01348	0.01557	0.01208
28/02/2022 - 23/03/2022	0.01128		0.02034	0.01456	0.01289
24/03/2022 - 07/04/2022	0.01324	0.03686	0.01671	0.01110	0.01002
08/04/2022 - 26/08/2022	0.01198	0.01927	0.01305	0.01511	0.00903
29/08/2022 - 11/11/2022	0.00956	0.02004	0.01130	0.01620	0.00849
12/11/2022 - 30/12/2022	0.01001	0.01166	0.00888	0.01270	0.00758
Period Average	0.01132	0.01165	0.01087	0.00944	0.00916
2013 - 2019	0.01172	0.01020	0.01006	0.00812	0.00854
01/01/2020 - 09/11/2021	0.01047	0.01102	0.01290	0.01168	0.01138
10/11/2021 - 31/12/2022	0.01030	0.02132	0.01238	0.01366	0.00927

Source: Own elaboration with output from EViews

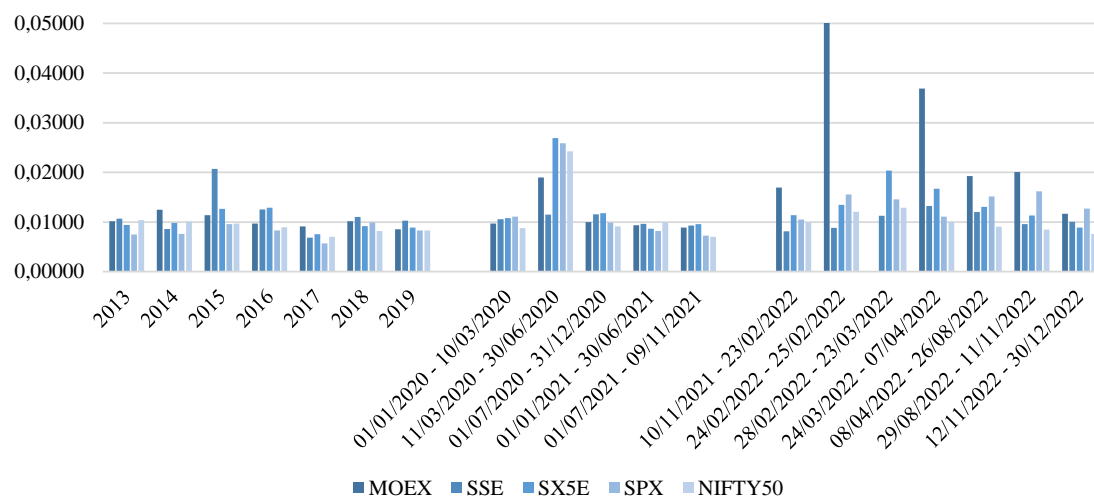


FIGURE 10: Comparative Analysis of Periodic Volatility for Selected Stock Indices Using Conditional Standard Deviation Estimates from the VAR-MGARCH Model
 Source: Own elaboration

During the pre-pandemic period, the standard deviations of all indices were relatively stable, with the exception of the SSE index. The SSE index experienced a higher level of volatility due to the stock market bubble in China during this period. This is reflected in the corresponding rows of Table VIII and respective bars of the graph depicted in Figure 10.

With the onset of the COVID-19 pandemic in early 2020, there was a significant increase in volatility across all indices. This increase was particularly pronounced during the initial outbreak and global spread of the virus (January 1, 2020 - March 10, 2020) and the subsequent lockdown period (March 11, 2020 - June 30, 2020). These periods correspond to a time of heightened uncertainty and disruption in global markets, as countries around the world grappled with the unprecedented challenges posed by the pandemic.

The volatility gradually decreased as countries began to ease lockdown restrictions and markets started to recover. However, the volatility remained high compared to the pre-pandemic period, reflecting the ongoing impact of the pandemic on global markets.

The period preceding the Russian invasion of Ukraine, which marked the escalation of tensions between Russia and Ukraine, reignited an increase in volatility after months of recovery. This is particularly evident in the MOEX index, which experienced a significant increase in volatility during this period.

The Russian invasion of Ukraine in February 2022 triggered an exceptional spike in market volatility, especially observed in the MOEX index with standard deviations escalating significantly during 24-25 February 2022. This spike reflects the immediate turmoil in the Russian market, with the high volatility levels persisting, indicating sustained market unease. For visual clarity in Figure 10, the y-axis scale was limited to 0.05 despite

the MOEX peaking at 0.07365, ensuring the bar graph remains interpretable. A data gap for the MOEX from 28 February to 23 March 2022 means no standard deviation data is present for this interval.

Towards the end of 2022, most indices seem to be returning to normal, with an overall decrease in volatility. This suggests the markets may be beginning to stabilize after the significant disruptions caused by the COVID pandemic and Russia's invasion of Ukraine.

To complement the conditional standard deviations discussed, the study also calculates annualized volatility to contextualize the volatility over a consistent time period. Annualized volatility is derived from daily standard deviations, scaled to an annual measure, providing a normalized view of volatility that facilitates year-over-year comparisons. This calculation is particularly useful for assessing the long-term volatility trends of each index, beyond the immediate fluctuations caused by specific events.

TABLE IX: Annualized Volatility Estimates for SSE, MOEX, SX5E, SPX and NIFTY50

Year	Annualized Volatility				
	SSE	MOEX	SX5E	SPX	NIFTY50
2013	0.15552	0.14776	0.13668	0.10899	0.15133
2014	0.15689	0.20155	0.16042	0.12118	0.12352
2015	0.33461	0.18403	0.20393	0.15415	0.15619
2016	0.20262	0.15599	0.20760	0.13382	0.14450
2017	0.11068	0.14664	0.12116	0.09168	0.11319
2018	0.17781	0.16382	0.14763	0.15993	0.13166
2019	0.16623	0.13736	0.14292	0.13394	0.13396
2020	0.18335	0.20474	0.26201	0.24257	0.22076
2021	0.14839	0.15970	0.15050	0.12762	0.14441
2022	0.17259	0.36015	0.20293	0.23240	0.14733

Source: Own elaboration

The annualized volatilities highlight the surge in market uncertainty during the early stages of the COVID-19 pandemic and the following Russian-Ukrainian conflict. Notably, MOEX index's annualized volatility reveals the impact of the conflict on the Russian markets, consistent with the spikes observed in the conditional standard deviations.

Furthering our exploration of market dynamics, the study assesses the correlation between squared volatilities and the correlation between squared returns. This analysis is key to understanding the extent to which volatility within individual markets is echoed across others and how this oscillation contributes to the collective movement during periods of financial turbulence. In this analysis, larger market fluctuations are accentuated over smaller ones, since squaring inherently magnifies larger values to a greater extent.

Covariance Analysis: Ordinary
 Sample: 3/11/2013 12/31/2019
 Included observations: 1777

Correlation Probability	SD2 DSSE	SD2 DMOEX	SD2 DSX5E	SD2 DSPX	SD2 DNIFTY50
SD2 DSSE	1.00000	—			
SD2 DMOEX	0.08307	1.00000			
	0.0005	—			
SD2 DSX5E	0.53562	0.12945	1.00000		
	0.0000	0.0000	—		
SD2 DSPX	0.37224	0.07338	0.49465	1.00000	
	0.0000	0.0020	0.0000	—	
SD2 DNIFTY50	0.37894	-0.00638	0.36998	0.41087	1.00000
	0.0000	0.7882	0.0000	0.0000	—

TABLE X: Correlation Estimates for the Squared Volatilities of Selected Market Indices for the Pre-Pandemic Period

Source: Own elaboration with output from EViews

Covariance Analysis: Ordinary
 Sample: 1/01/2020 11/09/2021
 Included observations: 485

Correlation Probability	SD2 DSSE	SD2 DMOEX	SD2 DSX5E	SD2 DSPX	SD2 DNIFTY50
SD2 DSSE	1.00000	—			
SD2 DMOEX	0.50754	1.00000			
	0.0000	—			
SD2 DSX5E	0.51532	0.98257	1.00000		
	0.0000	0.0000	—		
SD2 DSPX	0.51005	0.89839	0.94324	1.00000	
	0.0000	0.0000	0.0000	—	
SD2 DNIFTY50	0.48978	0.91539	0.90435	0.83392	1.00000
	0.0000	0.0000	0.0000	0.0000	—

TABLE XI: Correlation Estimates for the Squared Volatilities of Selected Market Indices for COVID-19 Pandemic Period

Source: Own elaboration with output from EViews

Covariance Analysis: Ordinary
 Sample: 11/10/2021 12/30/2022
 Included observations: 298

Correlation Probability	SD2 DSSE	SD2 DMOEX	SD2 DSX5E	SD2 DSPX	SD2 DNIFTY50
SD2 DSSE	1.00000	—			
SD2 DMOEX	0.09307	1.00000			
	0.1088	—			
SD2 DSX5E	0.30416	0.64696	1.00000		
	0.0000	0.0000	—		
SD2 DSPX	0.45947	0.06011	0.29384	1.00000	
	0.0000	0.3010	0.0000	—	
SD2 DNIFTY50	0.09677	0.62574	0.75821	0.27067	1.00000
	0.0954	0.0000	0.0000	0.0000	—

TABLE XII: Correlation Estimates for the Squared Volatilities of Selected Market Indices for Russia's invasion of Ukraine Period

Source: Own elaboration with output from EViews

Tables X, XI, and XII present correlations between squared volatilities of market indices. The analysis of the correlation estimates for the squared volatilities provides insight into the degree of synchronicity in market behaviors during the Pre-Pandemic Period, the COVID-19 Pandemic, and Russia's invasion of Ukraine.

In the pre-crisis period, correlations among squared volatilities of the indices were moderate, indicating a baseline of market co-movements under normal economic conditions. Notably, the interlinkages between European and US markets (SX5E and SPX) were more pronounced, reflecting established economic connections. In contrast, Russian and Chinese markets (MOEX and SSE) exhibited lower synchronicity, potentially due to disparate market drivers and economic environments.

The COVID-19 pandemic saw a noticeable elevation in these correlations, signifying a unified market response to the global shock. The persistence of high correlation between European and US indices was evident, while the Chinese market demonstrated an increase in correlation with other indices, though to a lesser extent, which could be attributed to its different trajectory during the pandemic.

During the period of Russia's invasion of Ukraine, the correlation pattern diverged, particularly for the Russian market. The MOEX index showed a distinct reduction in correlation with other global indices, likely a reflection of the unique geopolitical risks and economic sanctions impacting Russia. Meanwhile, the remaining indices continued to exhibit strong interdependencies.

These correlation dynamics across different periods suggest varied manifestations of financial contagion and market interdependence. Having established the landscape of squared volatility correlations across different market indices, we now turn our focus to the correlation of squared returns. This metric will further highlight the interconnectedness of market behaviors, particularly how individual market movements echo collectively during times of financial stress.

Covariance Analysis: Ordinary
 Sample: 3/11/2013 12/31/2019
 Included observations: 1779
 Balanced sample (listwise missing value deletion)

Correlation Probability	DSSE2	DMOEX2	DSX5E2	DSPX2	DNIFTY502
DSSE2	1.0000 0.0000				
DMOEX2	0.0236 0.3189	1.0000 0.0000			
DSX5E2	0.2052 0.0000	0.1311 0.0000	1.0000 0.0000		
DSPX2	0.1611 0.0000	0.0575 0.0152	0.4231 0.0000	1.0000 0.0000	
DNIFTY502	0.2183 0.0000	0.0353 0.1368	0.2744 0.0000	0.2423 0.0000	1.0000 0.0000

TABLE XIII: Correlation Estimates for the Squared Returns of Selected Market Indices for Pre-Pandemic Period

Source: Own elaboration with output from EViews

Covariance Analysis: Ordinary
 Sample: 1/01/2020 11/09/2021
 Included observations: 485
 Balanced sample (listwise missing value deletion)

Correlation Probability	DSSE2	DMOEX2	DSX5E2	DSPX2	DNIFTY502
DSSE2	1.0000 0.0000				
DMOEX2	0.1379 0.0023	1.0000 0.0000			
DSX5E2	0.1477 0.0011	0.8180 0.0000	1.0000 0.0000		
DSPX2	0.2107 0.0000	0.5014 0.0000	0.6609 0.0000	1.0000 0.0000	
DNIFTY502	0.2437 0.0000	0.3737 0.0000	0.4008 0.0000	0.4282 0.0000	1.0000 0.0000

TABLE XIV: Correlation Estimates for the Squared Returns of Selected Market Indices for COVID-19 Pandemic Period

Source: Own elaboration with output from EViews

Covariance Analysis: Ordinary
 Sample: 11/10/2021 12/30/2022
 Included observations: 298
 Balanced sample (listwise missing value deletion)

Correlation Probability	DSSE2	DMOEX2	DSX5E2	DSPX2	DNIFTY502
DSSE2	1.0000 0.0000				
DMOEX2	-0.0021 0.9707	1.0000 0.0000			
DSX5E2	0.0373 0.5217	0.1276 0.0276	1.0000 0.0000		
DSPX2	0.0167 0.7741	0.0340 0.5593	0.2586 0.0000	1.0000 0.0000	
DNIFTY502	0.0801 0.1679	0.3485 0.0000	0.3790 0.0000	0.0731 0.2080	1.0000 0.0000

TABLE XV: Correlation Estimates for the Squared Returns of Selected Market Indices for Russia's invasion of Ukraine Period

Source: Own elaboration with output from EViews

Tables XIII, XIV, and XV present correlations between squared returns of market indices, reflecting the intensity of return fluctuations across the Pre-Pandemic Period, the COVID-19 Pandemic, and Russia's Invasion of Ukraine.

Pre-pandemic correlations suggest typical market interdependence. The COVID-19 pandemic shows a significant increase in correlations, indicating intensified co-movements during the global crisis, with statistically significant p-values underscoring this trend.

During Russia's invasion of Ukraine, the correlation pattern shifts, with MOEX Index showing varied correlations with other indices, indicating a differentiated impact of regional tensions on market behaviors.

The preceding analyses of annualized volatility and the correlations of squared volatilities and returns explain the complex nature of market volatility. The study now turns to Table XVI which summarizes the average conditional correlations per period.

TABLE XVI: Conditional Correlation Estimates from the VAR-MGARCH Model for the Daily Log Returns of Selected Market indices

Period	Average Conditional Correlation									
	MOEX NIFTY50	MOEX SPX	MOEX SSE	MOEX SX5E	NIFTY50 SPX	NIFTY50 SSE	NIFTY50 SX5E	SPX SSE	SPX SX5E	SSE SX5E
11/03/2013 - 31/12/2013	0.3420	0.3302	0.0825	0.4379	0.3145	0.1253	0.4070	0.1385	0.5769	0.1424
01/01/2014 - 31/12/2014	0.2148	0.2296	0.1848	0.2643	0.3255	0.2144	0.2348	0.1952	0.3439	0.3429
01/01/2015 - 31/12/2015	0.2759	0.2790	0.0518	0.3415	0.3117	0.0961	0.3495	0.1385	0.5930	0.1019
01/01/2016 - 30/12/2016	0.2744	0.3329	0.1311	0.4426	0.3402	0.1326	0.4009	0.1492	0.5934	0.1169
02/01/2017 - 29/12/2017	0.1956	0.2805	0.1936	0.3515	0.2031	0.2607	0.2381	0.1278	0.5572	0.1310
01/01/2018 - 31/12/2018	0.2223	0.2664	0.1003	0.3684	0.2384	0.1607	0.3199	0.1505	0.5475	0.1533
01/01/2019 - 31/12/2019	0.2264	0.2824	0.1264	0.3745	0.2500	0.1715	0.2611	0.1495	0.6008	0.1173
01/01/2020 - 10/03/2020	0.2818	0.3443	0.1388	0.4946	0.2509	0.1681	0.3336	0.1616	0.6439	0.1324
11/03/2020 - 30/06/2020	0.4221	0.3416	-0.0082	0.6334	0.4379	0.0346	0.5244	0.2450	0.7315	0.1529
01/07/2020 - 31/12/2020	0.2891	0.2472	0.1078	0.4676	0.2223	0.1214	0.4289	0.1716	0.5318	0.0981
01/01/2021 - 30/06/2021	0.3185	0.2637	0.1251	0.4277	0.2645	0.1580	0.3486	0.1374	0.5263	0.1025
01/07/2021 - 09/11/2021	0.2667	0.2997	0.1144	0.4052	0.2294	0.1955	0.3036	0.1086	0.5982	0.1092
10/11/2021 - 23/02/2022	0.4460	0.3141	0.1160	0.4574	0.3073	0.1931	0.4577	0.1460	0.4493	0.1007
24/02/2022 - 25/02/2022	0.2882	0.3904	0.1449	0.3891	0.2304	0.1175	0.5460	0.0920	0.3756	0.1141
28/02/2022 - 23/03/2022					0.2938	0.1221	0.5536	0.1428	0.4639	0.1060
24/03/2022 - 07/04/2022	0.2952	0.1690	0.0334	0.2157	0.2702	0.1058	0.4563	0.0875	0.4961	0.0761
08/04/2022 - 26/08/2022	0.1822	0.2306	0.0816	0.2643	0.3301	0.1506	0.4048	0.1023	0.5960	0.1251
29/08/2022 - 11/11/2022	0.1971	0.2329	0.0880	0.3112	0.2673	0.1756	0.3634	0.0690	0.5944	0.1222
12/11/2022 - 30/12/2022	0.2599	0.2395	0.0667	0.3745	0.2523	0.1351	0.2828	0.0684	0.5357	0.1051
Period Average	0.2777	0.2819	0.1044	0.3901	0.2810	0.1494	0.3797	0.1359	0.5450	0.1290

Source: Own elaboration with output from EViews

The period from 11/03/2013 to 31/12/2019 serves as our reference point for the analysis. During this time, the average correlation was approximately 0.26, reflecting a moderate level of interdependence between the indices. The correlations remained relatively consistent throughout this period, with the highest correlation observed between the SPX and SX5E indices in 2019 (0.60) and the lowest between the MOEX and SSE indices in 2015 (0.05). These values provide a snapshot of the market dynamics prior to the onset of the COVID-19 pandemic and the Russian invasion of Ukraine.

The global economic impact of the COVID-19 pandemic, particularly during the period of 11/03/2020 - 30/06/2020, led to a significant increase in correlation across the indices, with the average correlation surging to approximately 0.35. This heightened correlation reflects the synchronized response of global markets to the unprecedented crisis, as countries around the world grappled with the economic fallout of the pandemic.

In the subsequent period, from 01/07/2020 to the pre-invasion period of late 2021, the correlation began to normalize, reflecting the gradual adaptation of global markets to the new economic realities imposed by the pandemic. This period saw a gradual return to pre-pandemic correlation levels, indicating the resilience of global markets. However, the correlation remained slightly high compared to the pre-pandemic period, suggesting that the effects of the pandemic continued to influence market interdependencies to some extent.

The geopolitical events that unfolded with the Russian invasion of Ukraine had a noticeable impact on the correlation of the MOEX index with other indices. The onset of the invasion and the ensuing sanctions and geopolitical tensions led to a significant decrease in the correlation between the MOEX and other indices. For instance, the correlation between MOEX and SX5E plummeted from 0.63 during the period of 11/03/2020 - 30/06/2020 to 0.22 in the period of 24/03/2022 - 07/04/2022. This decrease underscores the isolating effect of geopolitical conflicts on market interdependencies.

In the most recent period of 12/11/2022 - 30/12/2022, the average correlation returned to a more moderate level of approximately 0.23. This suggests a degree of stabilization in the markets, albeit at a level of correlation that remains influenced by ongoing global events and market conditions.

In conclusion, the comprehensive analysis covering conditional standard deviations and correlations, annualized volatilities and correlation of squared volatilities and returns, provides compelling evidence of financial contagion. The analysis revealed marked increases in volatility, signifying intensified market fluctuations that went beyond geographical borders. This was mirrored by a simultaneous elevation in the correlations across multiple markets, further supporting the notion of financial contagion. The periods of the COVID-19 pandemic and the Russian invasion of Ukraine were particularly telling, as

they provoked sudden and significant changes in these metrics. These shifts in correlations and volatilities during such critical periods underscore the deep interconnectedness of global financial markets and the capacity of localized economic events to spread wide-reaching impacts.

5.2 Policy Implications

This research offers some important considerations for policy development in the realms of financial regulation and risk management.

First, the data suggests that financial markets are interconnected, with particular attention to the COVID-19 pandemic and the Russian invasion of Ukraine. This interconnectedness could imply that policy decisions in one market might have ripple effects elsewhere. While the findings are not definitive, they invite policymakers to think globally when considering financial regulations.

Second, the observed volatility during crisis periods signals the relevance of risk management. Institutions and investors might consider having strategies for such times, like diversifying portfolios or employing financial derivatives as hedges. However, these are suggestions and should be adapted to specific conditions.

Third, geopolitical events, as demonstrated by the Russian invasion affecting the MOEX index, can have an important impact on financial markets. Policymakers might consider this when making geopolitical decisions, although it's important to note that this is one study among many that could inform such considerations.

In summary, this study offers some points for reflection for those involved in policy-making and financial regulation. It should not be seen as definitive but adds to the body of work aimed at understanding financial market dynamics.

6 CONCLUSION

This study aimed to investigate the financial contagion effects of the COVID-19 pandemic and Russia's invasion of Ukraine on five selected stock market indices. Employing a VAR-MGARCH model, the study analyzed not only the conditional standard deviations and correlations of these indices but also incorporated the assessment of annualized volatility and the correlation of squared returns and volatilities. This approach provided deeper insights into their volatility, interdependencies, and the intricate dynamics of market movements under the impact of these global events.

The findings of this study provide compelling evidence of financial contagion. The heightened levels of volatility that spilled over from one market to another, coupled with the simultaneous increase in correlations across multiple markets, are indicative of finan-

cial contagion. The abrupt and sensitive changes in correlations and volatilities observed during the periods of the COVID-19 pandemic and the Russian invasion of Ukraine further underscore this phenomenon.

Specifically, the study found significant increases in volatility across all indices during the initial outbreak and global spread of the COVID-19 virus, and the subsequent lockdown period. The volatility gradually decreased as countries began to ease lockdown restrictions and markets started to recover, but remained elevated compared to the pre-pandemic period.

The Russian invasion of Ukraine marked another period of increased volatility, particularly for the MOEX index. The standard deviations for this index increased significantly in the days following the start of the invasion, reflecting the immediate impact of the invasion on the Russian financial market.

In terms of correlations, the study found a significant increase across the indices during the COVID-19 pandemic, reflecting the synchronized response of global markets to the crisis. However, the Russian invasion of Ukraine led to a significant decrease in the correlation between the MOEX and other indices, underscoring the isolating effect of geopolitical conflicts on market interdependencies.

While this study provides valuable insights into the financial contagion effects of the COVID-19 pandemic and the Russian invasion of Ukraine, it is not without limitations. The study focused on a specific set of stock market indices, and the findings may not be generalizable to other markets or indices. Furthermore, the study relied on a VAR-MGARCH model, which, while robust, may not capture all the complexities of financial contagion.

Future research could extend this study in several ways. First, researchers could examine a broader set of indices or include additional variables such as commodity prices and exchange rates to provide a more comprehensive picture of financial contagion. Second, researchers could employ different econometric models or methodologies to analyze financial contagion, which could provide complementary insights. Finally, future research could focus on other significant global events and their impacts on financial markets, further enriching our understanding of financial contagion.

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A EVENTS TIMELINE

A.1 COVID-19 Pandemic

TABLE XVII: Timeline of COVID-19 Pandemic

Date	Description
December 12, 2019	A cluster of patients in the city of Wuhan, begin to experience symptoms of an atypical pneumonia-like illness that does not respond well to standard treatments.
January 1, 2020	The Huanan Seafood Wholesale Market in Wuhan is closed amid worries in China of a reprise of the 2002–2004 SARS-CoV-1 outbreak.
January 7, 2020	Public health officials in China identify a novel coronavirus as the causative agent of the outbreak.
January 11, 2020	Chinese media reported the first death from the novel coronavirus.
January 13, 2020	The Thailand Ministry of Public Health confirms the first case of the SARS-CoV-2 virus outside of China.
January 19, 2020	Worldwide, 282 laboratory-confirmed cases of the 2019 Novel Coronavirus have been reported in four countries.
January 20, 2020	CDC reports the first laboratory-confirmed case of the 2019 Novel Coronavirus in the U.S..
January 23, 2020	Wuhan, China— a city of 11 million people — is placed under lockdown due to the 2019 Novel Coronavirus outbreak.
January 30, 2020	The W.H.O. declared a global health emergency.
February 2, 2020	The first coronavirus death was reported outside China.
February 10, 2020	Worldwide deaths from the 2019 Novel Coronavirus reach 1,013. The SARS-CoV-2 virus has now killed more people than the severe acute respiratory syndrome (SARS-CoV-1) outbreak.
February 11, 2020	WHO announces the official name for the disease that is causing the 2019 Novel Coronavirus outbreak: “COVID-19.”

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Date	Description
February 14, 2020	France announced the first coronavirus death in Europe.
February 21, 2020	A significant increase of COVID-19 cases are registered in Northern Italy.
February 23, 2020	As Italy becomes a global COVID-19 hotspot, the Italian government issues Decree-Law No. 6, containing urgent measures to contain and manage the epidemiological emergency caused by COVID-19, effectively locking down the country.
February 25, 2020	CDC's Dr. Nancy Messonnier braces the nation to expect mitigation efforts to contain the SARS-CoV-2 virus in the U.S. that may include school closings, workplace shutdowns, and the canceling of large gatherings and public events.
February 29, 2020	The authorities announce a patient died in what was believed to be the first coronavirus death in the U.S. at the time.
March 7, 2020	To mark the number of confirmed COVID-19 cases surpassing 100 000 globally.
March 11, 2020	After more than 118,000 cases in 114 countries and 4,291 deaths, the WHO declares COVID-19 a pandemic.
March 13, 2020	Europe become the epicentre of the pandemic with more reported cases and deaths than the rest of the world combined, apart from the People's Republic of China. The Trump Administration declares a nationwide emergency and issues an additional travel ban.
March 15, 2020	States begin to implement shutdowns in order to prevent the spread of COVID-19.
March 18, 2020	EU member states join forces to keep priority traffic moving.
March 24, 2020	India announced a 21-day lockdown.

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TABLE XVII – continued from previous page

Date	Description
March 27, 2020	The Trump Administration signs the Coronavirus Aid, Relief, and Economic Security (CARES) Act into law. The act includes funding for \$1,200 per adult, expanded unemployment benefits, forgivable small business loans, loans to major industries and corporations, and expanded funding in response to the economic crisis caused by COVID-19.
March 28, 2020	To prevent the spread of COVID-19, the White House extends all social distancing measures.
March 30, 2020	The EU adopted legislation to quickly release funding from the EU budget to tackle the COVID-19 crisis.
April 4, 2020	WHO reported that over 1 million cases of COVID-19 had been confirmed worldwide.
April 9, 2020	Eurogroup puts forward €500 billion support package.
April 10, 2020	Cases surged in Russia. U.S. is the country with the most reported COVID-19 cases and deaths.
April 23, 2020	EU leaders to work on a recovery fund
May 21, 2020	AstraZeneca receives more than \$1 billion from the U.S. government in funding for the development of the COVID-19 vaccine, with the first doses due to arrive in September.
May 28, 2020	The recorded death toll from COVID-19 in the U.S. surpasses 100,000.
June 8, 2020	The World Bank states that the COVID-19 pandemic will plunge the global economy into the worst recession since World War II.
June 10, 2020	Confirmed COVID-19 cases in the U.S. surpass 2 million.
June 30, 2020	The E.U. said it would reopen borders. The European Union prepared to open to visitors from 15 countries on July 1. The United States begins to reopen its economy.

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TABLE XVII – continued from previous page

Date	Description
July 7, 2020	The number of confirmed COVID-19 cases in the U.S. surpasses 3 million.
July 13, 2020	The first COVID-19 vaccine candidate is approved for human trials.
July 17, 2020	India reached a million coronavirus cases, and lockdowns were reimposed.
July 21, 2020	European leaders agreed on a \$857 billion stimulus package.
August 17, 2020	COVID-19 becomes the 3rd leading cause of death in the U.S..
August 24, 2020	The first documented case of COVID-19 reinfection is confirmed.
September 6, 2020	India became the country with the second-highest number of cases with more than 4 million.
September 21, 2020	Johnson & Johnson begins phase 3 clinical trials of its COVID-19 vaccine.
September 28, 2020	The reported death toll from COVID-19 reaches more than 1 million worldwide.
October 7, 2020	New Zealand lifts restrictions and declares COVID-19 “beaten” after a cluster of 179 cases is fully contained.
November 16, 2020	Moderna’s COVID-19 vaccine is found to be 95.4% effective in its clinical trial.
November 18, 2020	Pfizer-BioNTech’s COVID-19 vaccine is found to be 95% effective in their trial.
November 19, 2020	EU leaders step up coordination on mutual recognition of tests and vaccines deployment
December 11, 2020	FDA issues an EUA for the Pfizer-BioNTech COVID-19 vaccine.
December 14, 2020	The recorded death toll from COVID-19 in the U.S surpasses 300,000.

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TABLE XVII – continued from previous page

Date	Description
December 18, 2020	FDA issues an EUA for the Moderna COVID-19 vaccine.
December 21, 2020	First COVID-19 vaccine authorised for use in the EU.
December 24, 2020	More than 1 million COVID-19 vaccine doses have been administered in the U.S. in just 10 days.
December 30, 2020	The AstraZeneca COVID-19 vaccine is authorized for emergency use in the U.K.
December 31, 2020	WHO issued its first emergency use validation for a COVID-19 vaccine and emphasized the need for equitable global access.
January 6, 2021	The EU granted a conditional marketing authorisation for the Moderna COVID-19 vaccine, the second COVID-19 vaccine authorised for use in EU countries.
January 18, 2021	The reported death toll from COVID-19 in the U.S. surpasses 400,000.
January 26, 2021	More than 23 million COVID-19 vaccine doses have been administered in the U.S.
January 29, 2021	EU authorises third COVID-19 vaccine for use in the EU
February 21, 2021	The recorded COVID-19 death toll in the U.S. surpasses 500,000.
March 8, 2021	CDC recommends that people who are fully vaccinated against COVID-19 can safely gather with other fully vaccinated people indoors without masks and without socially distancing.
March 11, 2021	Fourth COVID-19 vaccine authorised for use in the EU
March 13, 2021	More than 100 million COVID-19 vaccine doses have been administered in the U.S.
March 29, 2021	A CDC study finds that COVID-19 vaccines are highly effective at preventing infection with the SARS-CoV-2 virus in real-world conditions.

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TABLE XVII – continued from previous page

Date	Description
April 2, 2021	CDC recommends that people who are fully vaccinated against COVID-19 can safely travel at lower-risk to themselves.
April 21, 2021	More than 200 million COVID-19 vaccine doses have been administered in the U.S.
May 14, 2021	CDC finds that mRNA COVID-19 vaccines reduce the risk of infection with the SARS-CoV-2 virus by approximately 94%.
May 20, 2021	Agreement on EU digital COVID certificate
June 30, 2021	Over 1 billion doses of COVID-19 vaccine have been administered worldwide.
July 13, 2021	Green light for first EU recovery funds to reach 12 EU countries
July 28, 2021	Green light for EU recovery funds given to four more EU countries
January 3, 2022	The U.S. reports nearly 1 million new COVID-19 infections—the highest daily total of any country in the world.
January 31, 2022	FDA fully approves the Moderna COVID-19 vaccine for all people ages 18 years and older.
February 11, 2022	CDC releases data showing that COVID-19 vaccine boosters remain safe and were highly effective against severe disease during the Omicron and Delta variant surges.
March 14, 2022	Several regions in China face new lockdowns under the “COVID Zero” policy when cases of the Omicron variant are found. Tens of millions of people are required to stay inside their homes, key technology manufacturers like Foxconn and Unimicron close factories, and the production and distribution of goods is disrupted throughout the world.
May 31, 2022	Authorities in Shanghai announce that they are partially re-opening China’s largest city after two months of a COVID-19 lockdown.

TABLE XVII: Timeline of COVID 19 Pandemic (continued)

A.2 *Russia Invasion of Ukraine*

TABLE XVIII: Timeline of Russian Invasion of Ukraine

Date	Description
November 10, 2021	Washington reports unusual Russian troop movements near the Ukrainian border.
November 28, 2021	Ukraine says Russia is massing nearly 92,000 troops for an offensive at the end of January or early February.
December 7, 2021	US President Biden threatens Russia with "strong economic and other measures" if he invades Ukraine.
January 17, 2022	Russian troops begin arriving Belarus for military drills, which Moscow says are aimed at "thwarting external aggression".
January 24, 2022	NATO puts troops on standby and sends ships and fighter jets to bolster Europe's eastern defences.
January 25, 2022	Moscow begins military exercises involving some 6,000 troops and at least 60 fighter jets in southern Russia near Ukraine and in Moscow-annexed Crimea.
February 2, 2022	U.S. sends 3,000 troops to fortify NATO forces in eastern Europe.
February 17, 2022	Shellfire intensifies along the frontline of Russian-backed enclaves in eastern Ukraine.
February 19, 2022	Ukraine says two of its soldiers died in attacks on the frontline with Russian-backed separatists.
February 23, 2022	In a televised address on February 22, Putin recognises the independence of two separatist regions in eastern Ukraine. The EU vows sanctions. Putin orders Russian troops into separatist areas in eastern Ukraine on a "peacekeeping" mission.
February 24, 2022	Putin announces the launch a "special military operation" to Ukraine. In the early hours of February 24, Russian President Putin ordered his troops into Ukraine.

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TABLE XVIII – continued from previous page

Date	Description
February 2022	Western nations impose a widespread package of sanctions on Moscow, including stopping transactions with Russia's central bank, banning new investment, and freezing the assets of Russian political and business leaders.
March 2, 2022	The United Nations' refugee agency said at least 100,000 people had left their homes in the first 24 hours of the military assault with thousands of people waiting to cross into Poland.
April 7, 2022	The United States Congress began passing a bill that would make it easier to send weapons to Ukraine. The United Nations General Assembly expelled Russia from the UN Human Rights Council. Russian troops deployed to the northern front by the Russian Eastern Military District pulled back from the Kyiv offensive, apparently to resupply then redeploy to the Donbas region to reinforce the renewed invasion of southeastern Ukraine.
April 14, 2022	The flagship of Russia's Black Sea fleet sank. Ukraine said it hit the Moskva with anti-ship cruise missiles, sparking a fire that detonated stored ammunition.
April 19, 2022	The conflict escalates the global food crisis and Ukraine's government announces a ban on a wide range of agricultural exports. World food prices reach a record high. The Russian Federation and Ukraine, combined, supply around 30% of global wheat exports and around a fifth of the world's maize.
May 31, 2022	EU Bloc agrees to cut 90% of Russian oil imports by year-end
June 7, 2022	World Bank approves \$1.49 billion in funds for Ukraine
July 25, 2022	Russian energy giant Gazprom says it will halve gas supplies to Europe through the Nord Stream 1 pipeline. Prior to the war, Europe imported more than 40% of its gas from Russia.
August 29, 2022	Ukraine launches counter-offensive in south as Russia shells port city.

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TABLE XVIII – continued from previous page

Date	Description
September 1, 2022	Russian forces flee Kharkiv region. A Ukrainian counteroffensive in eastern Ukraine recaptured large swaths of territory and forced Russian troops to flee Kharkiv.
September 21, 2022	Russia's partial mobilization starts. Putin announced Russia's first mobilization since World War II. The partial mobilization produced fighters that were poorly equipped and largely untrained but it significantly increased Russia's troop numbers.
September 2022	European gas prices spike by as much as 30% after Russia says one of its main gas supply pipelines to Europe will remain closed indefinitely.
October 8, 2022	Crimea bridge attack. The only bridge connecting Russia with the Crimean Peninsula was severely damaged by an explosion.
October 10, 2022	Kyiv blackout. A new phase of the war began when Russia launched the first of several waves of missile strikes on Ukraine's critical energy infrastructure. Moscow began targeting Ukrainian power facilities, leaving large areas of the country without power and water.
November 11, 2022	Ukrainian troops entered the city of Kherson, while the front line reached the west bank of the Dnipro River.
November 12, 2022	After months of Russian occupation, Kherson was liberated. Russia's withdrawal from the west bank of the Dnipro River was another bleak moment for Moscow, as Kherson was the only Ukrainian regional capital that Russia had captured.
December 21, 2022	The Biden administration announced it was sending nearly \$2 billion in additional security assistance to Ukraine.
December 2022	The European Central Bank says it expects inflation to remain above its 2% target for the next three years. Several factors, including the war in Ukraine, caused inflation to spike at 10.6% in October across the 19 countries that use the euro.

TABLE XVIII: Timeline of Russian Invasion of Ukraine (continued)

B VAR-MGARCH MODEL SPECIFICATIONS

B.1 VAR Model

Vector Autoregression Estimates
 Sample (adjusted): 3/11/2013 12/30/2022
 Included observations: 2560 after adjustments
 Standard errors in () & t-statistics in []

	DMOEX	DNIFTY50	DSPX	DSSE	DSX5E
DMOEX(-1)	0.091389 (0.02171) [4.20803]	0.053543 (0.01551) [3.45188]	-0.01373 (0.01663) [-0.82515]	0.016479 (0.01940) [0.84915]	0.018696 (0.01856) [1.00707]
DMOEX(-2)	-0.07294 (0.02174) [-3.35395]	-0.01712 (0.01553) [-1.10246]	-0.00542 (0.01665) [-0.32522]	0.005776 (0.01943) [0.29725]	-0.00502 (0.01859) [-0.26998]
DNIFTY50(-1)	-0.10457 (0.03123) [-3.34824]	-0.06407 (0.02230) [-2.87214]	-0.11728 (0.02392) [-4.90202]	-0.07135 (0.02790) [-2.55669]	-0.13385 (0.02669) [-5.01343]
DNIFTY50(-2)	-0.01689 (0.03035) [-0.55641]	-0.04729 (0.02168) [-2.18109]	0.022009 (0.02325) [0.94644]	0.031667 (0.02712) [1.16744]	0.006655 (0.02594) [0.25648]
DSPX(-1)	0.24511 (0.03279) [7.47345]	0.290351 (0.02342) [12.3951]	-0.11336 (0.02512) [-4.51184]	0.145213 (0.02930) [4.95495]	0.25362 (0.02803) [9.04641]
DSPX(-2)	0.081881 (0.03361) [2.43585]	0.051049 (0.02400) [2.12631]	-0.0151 (0.02575) [-0.58646]	0.019013 (0.03003) [0.63298]	0.081744 (0.02873) [2.84486]
DSSE(-1)	-0.02521 (0.02269) [-1.11069]	-0.03575 (0.01620) [-2.20562]	0.00482 (0.01738) [0.27725]	0.060274 (0.02027) [2.97226]	-0.02378 (0.01939) [-1.22596]
DSSE(-2)	0.015641 (0.02254) [0.69368]	-0.01361 (0.01610) [-0.84541]	-0.03038 (0.01727) [-1.75861]	-0.02685 (0.02014) [-1.33286]	-0.03854 (0.01927) [-1.99956]
DSX5E(-1)	-0.12688 (0.03146) [-4.03294]	-0.03174 (0.0224) [-1.41263]	0.054522 (0.02410) [2.26221]	0.06562 (0.02811) [2.33417]	-0.09333 (0.02689) [-3.47045]
DSX5E(-2)	0.03395 (0.03125) [1.08637]	0.076383 (0.02232) [3.42218]	0.137327 (0.02394) [5.73628]	0.044875 (0.02792) [1.60699]	0.011515 (0.02671) [0.43107]
C	9.04E-05 (0.00027) [0.32619]	0.000357 (0.00019) [1.80252]	0.000421 (0.00021) [1.98155]	4.89E-05 (0.00024) [0.19745]	8.20E-05 (0.00023) [0.34617]
R-squared	0.036298	0.090479	0.044745	0.03495	0.03895
Adj. R-squared	0.032517	0.08691	0.040997	0.031164	0.03518
Sum sq. resids	0.498776	0.254429	0.292703	0.398249	0.364451
S.E. equation	0.013988	0.009991	0.010716	0.012499	0.011957
F-statistic	9.600737	25.35728	11.93976	9.231392	10.33075
Log likelihood	7303.019	8164.631	7985.258	7591.121	7704.639
Akaike AIC	-5.69689	-6.37002	-6.22989	-5.92197	-6.01066
Schwarz SC	-5.67176	-6.3449	-6.20476	-5.89684	-5.98553
Mean dependent	0.000141	0.000435	0.000354	0.000112	0.000129
S.D. dependent	0.014222	0.010455	0.010943	0.012699	0.012173
Determinant resid covariance (dof adj.)			1.98E-20		
Determinant resid covariance			1.94E-20		
Log likelihood			39936.63		
Akaike information criterion			-31.1575		
Schwarz criterion			-31.0319		
Number of coefficients			55		

TABLE XIX: VAR(2) Model

B.2 MGARCH Model

System: MGARCH111DBEKK
 Estimation Method: ARCH Maximum Likelihood (BFGS / Marquardt steps)
 Covariance specification: Diagonal BEKK
 Sample: 3/11/2013 12/30/2022
 Included observations: 2560
 Total system (balanced) observations 12800
 Disturbance assumption: Student's t distribution
 Presample covariance: backcast (parameter =0.7)
 Convergence achieved after 166 iterations
 Coefficient covariance computed using outer product of gradients

	Coefficient	Std. Error	z-Statistic	Prob.
Variance Equation Coefficients				
C(1)	3.20E-06	4.49E-07	7.129920	0.0000
C(2)	3.36E-07	1.62E-07	2.072185	0.0382
C(3)	9.83E-07	1.88E-07	5.231725	0.0000
C(4)	7.69E-07	1.72E-07	4.468411	0.0000
C(5)	9.56E-07	1.85E-07	5.166735	0.0000
C(6)	2.80E-06	4.68E-07	5.989880	0.0000
C(7)	5.19E-07	1.59E-07	3.266758	0.0011
C(8)	9.09E-07	1.73E-07	5.256625	0.0000
C(9)	4.10E-07	1.58E-07	2.595816	0.0094
C(10)	2.36E-06	2.84E-07	8.290501	0.0000
C(11)	2.99E-07	1.65E-07	1.812828	0.0699
C(12)	1.40E-06	1.97E-07	7.094735	0.0000
C(13)	1.56E-06	3.05E-07	5.126462	0.0000
C(14)	3.05E-07	1.50E-07	2.030475	0.0423
C(15)	2.19E-06	3.12E-07	7.006752	0.0000
C(16)	-0.125341	0.016342	-7.669787	0.0000
C(17)	-0.103410	0.018743	-5.517242	0.0000
C(18)	0.122088	0.020632	5.917395	0.0000
C(19)	0.216609	0.012950	16.726606	0.0000
C(20)	0.001447	0.016067	0.090066	0.9282
C(21)	0.232222	0.016800	13.822440	0.0000
C(22)	0.294899	0.017309	17.037734	0.0000
C(23)	0.403367	0.019685	20.490925	0.0000
C(24)	0.088511	0.023494	3.767348	0.0002
C(25)	0.291216	0.012907	22.562992	0.0000
C(26)	0.963770	0.003440	280.173253	0.0000
C(27)	0.955326	0.004807	198.747910	0.0000
C(28)	0.937575	0.004643	201.934903	0.0000
C(29)	0.968054	0.003304	292.984813	0.0000
C(30)	0.967900	0.002506	386.302788	0.0000
t-Distribution (Degree of Freedom)				
C(31)	7.615984	0.399976	19.041115	7.79E-81
Log likelihood	42666.179253	Schwarz criterion		-33.237921
Avg. log likelihood	3.333295	Hannan-Quinn criter.		-33.283056
Akaike info criterion	-33.305098	Schwarz criterion		-33.237921
S.E. of regression	0.072212	Sum squared resid		159.219867
Squared resid	0.002214	Durbin-Watson stat		1.986721

TABLE XX: MGARCH(1,1) TARCH(1) Diagonal BEKK Summary

Covariance specification: Diagonal BEKK
 $GARCH = M + A1 * RESID(-1) * RESID(-1)' * A1 + D1 * (RESID(-1) * (RESID(-1) < 0)) * (RESID(-1) * (RESID(-1) < 0))' * D1 + B1 * GARCH(-1) * B1$
M is an indefinite matrix
A1 is a diagonal matrix
D1 is a diagonal matrix
B1 is a diagonal matrix

Transformed Variance Coefficients				
	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	3.20E-06	4.49E-07	7.129920	0.0000
M(1,2)	3.36E-07	1.62E-07	2.072185	0.0382
M(1,3)	9.83E-07	1.88E-07	5.231725	0.0000
M(1,4)	7.69E-07	1.72E-07	4.468411	0.0000
M(1,5)	9.56E-07	1.85E-07	5.166735	0.0000
M(2,2)	2.80E-06	4.68E-07	5.989880	0.0000
M(2,3)	5.19E-07	1.59E-07	3.266758	0.0011
M(2,4)	9.09E-07	1.73E-07	5.256625	0.0000
M(2,5)	4.10E-07	1.58E-07	2.595816	0.0094
M(3,3)	2.36E-06	2.84E-07	8.290501	0.0000
M(3,4)	2.99E-07	1.65E-07	1.812828	0.0699
M(3,5)	1.40E-06	1.97E-07	7.094735	0.0000
M(4,4)	1.56E-06	3.05E-07	5.126462	0.0000
M(4,5)	3.05E-07	1.50E-07	2.030476	0.0423
M(5,5)	2.19E-06	3.12E-07	7.006752	0.0000
A1(1,1)	-0.125341	0.016342	-7.669787	0.0000
A1(2,2)	-0.103410	0.018743	-5.517242	0.0000
A1(3,3)	0.122088	0.020632	5.917395	0.0000
A1(4,4)	0.216609	0.012950	16.726606	0.0000
A1(5,5)	0.001447	0.016067	0.090066	0.9282
D1(1,1)	0.232222	0.016800	13.822440	0.0000
D1(2,2)	0.294899	0.017309	17.037734	0.0000
D1(3,3)	0.403367	0.019685	20.490925	0.0000
D1(4,4)	8.85E-02	2.35E-02	3.767348	0.0002
D1(4,4)	8.85E-02	2.35E-02	3.767348	0.0002
D1(5,5)	2.91E-01	1.29E-02	22.5630	0.0000
B1(1,1)	9.64E-01	3.44E-03	280.173	0.0000
B1(2,2)	9.55E-01	4.81E-03	198.748	0.0000
B1(3,3)	9.38E-01	4.64E-03	201.935	0.0000
B1(4,4)	9.68E-01	3.30E-03	292.985	0.0000
B1(5,5)	9.68E-01	2.51E-03	386.303	0.0000

TABLE XXI: Diagonal BEKK Covariance Specification and Transformed Variance Coefficients