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# Setting up a framework for a notional financial instrument for impact investment in fisheries

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# Abstract

Social and environmental awareness has in recent years become an increasingly weighted factor in many investors' funding allocation decisions due to concerns that their investment could contribute to industries practicing human rights abuses, have deleterious impacts on nature and ecosystems or just be misaligned with their personal or political beliefs. This concept involves a close analysis of practices and commitments of the companies at stake in order to inform investment decisions. Impact investment is a form of ethical investment that takes action for improvements while generating financial returns. Fisheries are prime candidates for impact investment initiatives given their potential as ecologically sustainable sources of protein but also their widespread mismanagement, but they have not yet been widely targeted by impact investment. Overfished fisheries could provide both greater supply and economic returns but the respite needed by the fishery in order to recover constitutes a hardship to fishers that are dependent on annual incomes. Investors could bridge that gap, covering any initial losses while the fishery recovers and later benefiting from the gains when the fishery reaches its sustained maximum returns.

The aim of the current project is to set up a framework that allows for the definition of a market instrument aimed at correcting existing imbalances in the economic model for fisheries management. To this end, optimal bioeconomic utilization is explored, and a system proposed to internalize rights and responsibilities to the fishery. Within this framework, an instrument is outlined providing the required incentives for fisheries to finance themselves to ensure sustainability. The project includes considerations on the design of this instrument as well as some considerations on its pricing.

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# Contents

# Introduction

Social and environmental awareness has in recent years become an increasingly weighted factor in many investors' funding allocation decisions. Concerns that their investment could contribute to industries practicing human rights abuses, have deleterious impacts on nature and ecosystems or just be misaligned with their personal or political beliefs, along with associated reputation risks, has precipitated the use of the concept of ethical investment. This concept involves a close analysis of practices and commitments of the companies at stake in order to inform investment decisions.

Impact investment is a form of ethical investment that takes action for improvements while generating financial returns. Fisheries are prime candidates for impact investment initiatives given their potential as ecologically sustainable sources of protein but also their widespread mismanagement. They have not yet been widely targeted by impact investment, but the estimated 34.2% of fisheries that are overfished (FAO, 2020) are underserving populations. Conversely, the 6.2% of fisheries that are underfished could also contribute more to sources of food and to economies. Overfished fisheries could provide both greater supply and economic returns but the respite needed by the fishery in order to recover constitutes a hardship to fishers that are dependent on annual incomes. Investors could bridge that gap, covering any initial losses while the fishery recovers and later benefiting from the gains when the fishery reaches its sustained maximum returns.

Environmental resources like forests, fisheries or other ecosystems which provide ecosystem services – uses and benefits available to society from natural resources – are self-renewing with the potential to provide a steady stream of resources, and thus of revenue, if well managed.

The aim of the current project is to set up a framework that allows for the definition of a market instrument aimed at correcting existing imbalances in the economic model for fisheries management. To this end, optimal bioeconomic utilization is explored, and a system proposed to internalize rights and responsibilities to the fishery. Within this framework, an instrument is created providing the required incentives for fisheries to finance themselves to ensure sustainability. The project includes considerations on the design of this instrument as well as some considerations on its pricing.

## 1.1 Ethical investment

The concept of ethical investment involves basing investment decisions on an analysis of practices and commitments, generally via claims made by the companies or by ethical investment funds or auditors. Ethical investment may take the form of socially responsible investment or, alternatively, sustainable and responsible investment (both abbreviated as SRI) or environmental, social and governance investment (ESG) which generally involve selecting among publicly traded assets. These can include divestment as a strategy.

Impact investment goes further as it seeks financial returns but also measurable positive social or environmental impacts (UNDP, 2016) by investing in projects actively designed to generate positive impacts. The global impact investment market was estimated to be worth in the range of US$636 billion to $2.3 trillion in 2020 (Volk, 2021), but the wide range of estimates underscores that classification of companies and assets as to their objectives and strategies is still nascent.

Impact investment can take several forms, including to date fixed income, venture capital, private equity, social and development impact bonds (UNDP, 2016) and conditional debt relief/conversion (SSCOE, 2018). Private debt is the largest in value terms (UNDP, 2016). Development finance is considered in most definitions of impact investment, as social and economic development is generally a goal, and these publicly-managed investments still exceed privately-managed investments (Volk, 2021).

### 1.1.1 Impact instruments

Impact investment can take a wide variety of forms, but among structured debt instruments, green bonds are the largest class of impact bonds. They were first issued by the European Investment Bank (in 2007) and the World Bank (in 2008) (EIB, 2021; World Bank, 2019). The initial issuance aimed at funding projects with environmental benefits, mainly on climate change mitigation and adaptation or sustainable land or water use.

The green bond issuance in 2020 was US $267 billion representing over 2.5-fold the 2015 issuance, with further growth expected (Jones, 2021). According to the European Commission, the issuance in the EU has also grown quickly in recent years, with a five-fold increase over the last 5 years. In 2020, the EU was a global leader in this market, with 51% of global issuance in 2020 from EU companies and EU public bodies (European Commission, 2021).

Renewable energy leads the green bond pack at a global level. Along with energy efficiency initiatives it makes up over 50% of green bond investment. Sustainable land use and forestry makes up just 3%, at US $7.8 billion (Almeida, 2020). Similar instruments for investment in ocean-related environmental projects or sustainable ocean uses, blue bonds, also known as water bonds, lag behind. The Seychelles government issued the first blue bond in 2018 to fund the development of an integrated management plan for the Mahé Plateau, a small scale artisanal fishery (FAO, 2017; World Bank, 2017). The plan is expected to result in improved management, leading to a biologically and economically sustainable fishery, with associated social benefits to the community (IUCN, 2016). A second blue bond was issued in 2019 by the Nordic Investment Bank, covering a number of projects around the Baltic Sea targeting wastewater treatment, water pollution reduction and adaptation to climate change impacts (Nordic Investment Bank, 2019).

Social and development impact bonds, which include Environmental Impact Bonds (EIBs), differ from the established green bonds, with the latter being more traditionally structured bonds, paying coupons and principal (UNDP, n.d.). EIBs may diverge from this structure, not paying coupons if the returns are expected to be longer-term or the projects require a longer implementation time, and with rules for repayment of the principal dependent on the level of success of the project ("pay-for-success") (Quantifed Ventures, 2019; Nicola, 2013). There are indications that investors in environmental projects may not be willing to take on all the financial risk (European Commission, 2016; Mudaliar et al., 2018), but recent data suggests that returns from ESG investments may be higher than from traditional funds (Bioy and Boyadzhiev, 2020).

These instruments are modelled on Social Impact Bonds, and are characterized by an objective of funding improved management or restoration of a system or resource, which is expected to result in the creation of value, through reduced costs or increased income. The risk is taken on by the private investors and no public expenditure occurs until the scheme has been proven to be successful. In that case, investors receive returns based on measures of success or savings. The financing provides the initial investment in those changes, which a government may be reluctant to invest in if there is a degree of risk, such as in testing new strategies, and the users/industry may be too fragmented, or current income too low, to fund.

Forest resilience bonds are one of the best detailed EIBs in the pipeline. They work by applying funds raised into preventive forest management, which has been shown to reduce fire risks and represent significant savings to resource managers including water management, which is significantly affected by loss of soil and increased sedimentation due to fires. Savings to public services from reduced incidences of fires and thus reduction of costs of fire extinctions, and to water and hydroelectric providers from improved water quality and increased water quantity, are the source of payments to investors, dependent on the outcomes (Blue Forest Conservation, 2017; US EPA, 2021). Coupons can be paid in cash or in carbon credits from a programme that benefits from the reduction in carbon emissions from avoided fires (IFC, 2016) .

## 1.2 Natural resource property systems

In the past, fisheries were generally of undefined ownership and operated under an open access regime, as was the case with other natural resources. This leads to a situation where each individual's incentive is to fish as much as possible, before others do the same, and is economically inefficient as a system (Scott Gordon, 1954). It is also ecologically inefficient and open-access fisheries tend to decline in abundance (Seijo et al., 1998). A number of stock depletions and collapses have been documented through time starting from the 1st century AD (Pitcher and Lam, 2015).

In the pre-industrialization phase, the repercussions of this system were restricted, with the depletions described as localized and occasional. However, with the industrialization of fishing, including steam-powered vessels and then diesel engines, mechanization of gear, ice production/refrigeration, fish-locating technology and then a growing market for fish and fish products, came a series of fishery collapses or depletion events in many oceans (Pitcher and Lam, 2015). As a result, there was a change of paradigm reflecting the need to impose restrictions on access. Governments were forced to intervene to regulate access to ensure the long-term sustainable use of resources, ensuring their optimal use by society while assuring their continued supply in the future.

Currently, most fisheries operate under a common property regime, where some form of access rights and responsibilities are in place. Access is generally restricted by governments or agencies with delegated responsibilities, although increasingly communities are being involved in the process (Metzner, 2005), and benefits accrue to resource users (Bromley et al., 1992).

Maintaining the long-term health of the targeted fish population is the main objective of fisheries management both for ecological and economic considerations, as sustainable fishing ensures its own long-term viability as a source of income and employment. Conflicts between these two driving factors do arise. Fisheries suffer fluctuations typical of any natural system that is subject to a multiplicity of drivers, processes and interactions but economic interests favour stability. Managers have to properly balance these potentially divergent forces when deciding upon measures to implement. Other ecological considerations also factor into management, including direct and indirect impacts of fishing on the physical and other biological components of the ecosystem and the capture of other species along with the target species. Associated with the economic considerations are social factors, such as livelihood security and fairness and stability of income, particularly relevant when discussing subsistence or small-scale fishing where fishing is the only or main source of income of families or communities. Industrial fishing operations are not free from social impacts either, as fishing is among the most dangerous professions, and incidences of human rights abuse including slavery are being brought to light (Tickler et al., 2018).

### 1.2.1 Rights-based management

Management systems that employ catch shares characterized by setting a fishery-wide catch limit which is partitioned into limits allocated to individuals or communities, avoid a counterproductive increase in effort in the rush to fish (Scott Gordon, 1954). These rights-based management systems have proven effective at preventing fishery collapses but also at improving economic returns (Costello, Gaines, et al., 2008), and their use is increasing.

Guaranteeing fishers a stake in the fishery outcome increases their sense of ownership. In fact, rights-based management systems have demonstrated improved compliance with catch limits (Melnychuk et al., 2012; Grimm et al., 2012), suggesting a reduced need for enforcement. This is an important feature, as monitoring, control and surveillance (MCS) systems represent the largest portion of management costs (Mangin et al., 2018). For these reasons, a catch shares management system is expected to be the most economically efficient in terms of management costs (Mangin et al., 2018). Rights-based management has also been found to contribute to successful community co-management, which involves the fishers in the management process. That is itself associated with improved social and economic outcomes (Gutiérrez et al., 2011).

First sale value (prior to the escalation as it passes through the multiple components of seafood supply chains) of wild capture seafood is assessed at US$151 billion globally (FAO, 2020). In general fisheries are not being exploited to their maximum efficiency. It has been estimated that fisheries could yield an estimated additional US$53 billion in global profits annually if all fisheries transitioned to rights-based management (Costello, Ovando, et al., 2016). This is expected to take effect via three routes: reduction of costs, increase in prices and increased catches in the long-term.

# Methods

The concept of sustainable fishing is predicated on the knowledge that fish populations can be seen as renewable resources in an economic sense. As beings coexisting in unstable ecosystems along with a multitude of other naturally survival-oriented life-forms, fish are adapted to cope with a level of mortality, into which mortality due to fishing can be incorporated, if managed properly on an appropriate time scale (Mangel, 1985).

Determining how much fish can sustainably be removed is the basis of fisheries science, a field that models the dynamics of hard to sample populations. This has led to well-established concepts showing that a relatively stable rate of removals of individuals from a population through fishing is generally possible and sustainable in the long-term. A complex balancing act weighs the benefit of catching a fish now against leaving it in the sea to grow and reproduce and catch it or its descendants at a later point. Among the sustainable targets is the maximum economic yield (MEY), that integrates biological models with economic considerations in order to define the long-term model that maximizes sustainable returns and, potentially, the optimum path for reaching that state (Anderson and Seijo, 2011). The intent in effect is to maximize the sum of the present value of returns over time (Seijo et al., 1998).

If a fishery is not being fished to maximize economic returns due to overfishing, then an investment opportunity is present, as a short-term reduction in returns will be replaced by a long-term increase that more than covers the initial loss.

A fishery thus has the ability to generate a steady income stream, but it needs to be sufficient to also cover expenses related to ensuring sustainability. The financial instrument envisioned will provide an initial cash flow to set up or correct the system, but this system will then provide savings in the long term, namely through:

1. Stability of income. Lack of effective management leads to depletion events, which cost fishers their income, and may cost governments in subsidies for lost earnings.
2. Improved science. Investment in population surveys and other data collection and modelling of the population is an investment whose pay-offs increase year to year as model result estimates improve with each year of additional information.
3. Improved compliance. Illegal, unreported and unregulated (IUU) fishing hampers the science which in turns limits effective management. It also means bypassing of taxation and food safety regulations.
4. Adequate fishing capacity. Excess capacity in the fishery can lead to a 'race to fish' and to pressure on managers to not reduce catch limits to appropriate levels.

## 2.1 Bioeconomical model

A logistic growth model such as Pearl-Verhulst (2.1) is a simple and intuitive model to describe the growth of a population of organisms including fish, modulated by linear density-dependent effects (Haddon, 2001). The population is described by its biomass $B(t)$, intrinsic growth rate $r$ and carrying capacity $K$. The biomass is the total weight of fish present at any point in time.

The advantage over a count of fish is that larger fish contribute more to this index. The constant intrinsic growth rate considers fecundity, individual growth and mortality, independent of the size of the stock. The carrying capacity represents the maximum biomass the population can reach given the constraints of the ecosystem conditions, also assumed constant. The overall effect is that biomass growth slows at it nears $K$.

$$\frac{dB(t)}{dt} = rB(t)\left[1 - \frac{B(t)}{K}\right] \tag{2.1}$$

The catch rate, or yield, is given by

$$y(t) = qf(t)^{\alpha}B(t)^{\beta}. \tag{2.2}$$

Fishing effort at any point in time, $f(t)$, is the main variable under external control. It quantifies the fishing gear use per unit of time e.g., hours trawled per day or number of hooks set per day (FAO, 1997). The catchability coefficient, $q$, scales the fishing effort applied to the population size and the resulting catch, and can be considered to be a measure of the efficiency of the fishing gear (Hilborn and Walters, 2001). $\alpha$ is a constant $\in ]0,1]$, introducing diminishing marginal productivity to effort increases, and $\beta$ an integer either 0 or 1. If zero, catch is not directly a function of population size, which may be the case for fish with schooling behaviour (Anderson and Seijo, 2011). If 1, yield is a linear function of population size. For the purposes of this analysis, $\beta$ will be assumed to be =1 and will be omitted from the following steps.

Combining these two equations into eq. 2.3 allows for the introduction of initial bioeconomic considerations (Schaefer, 1954).

$$\frac{dB(t)}{dt} = rB(t)\left[1 - \frac{B(t)}{K}\right] - qf(t)^{\alpha}B(t)^{\beta} \tag{2.3}$$

The rate of biomass, $B(t)$, change over time of a population under exploitation is determined by the two terms on the right-hand side of eq. 2.3: the first represents the natural changes due to reproduction, individual growth and natural mortality, labelled the growth or production function, and the second the decrease due to removals by the fishery, representing the catch rate.

Scott Gordon (1954) brought a formal economic approach to fisheries, introducing a simplified cost function, linearly proportional to effort. Here an alternative is proposed, where costs include fixed costs $a$, such as the purchase of a fishing boat and gear, and a variable cost component that is linearly proportional to effort by a proportionality constant $v$, which includes fuel, staff and maintenance costs.

$$a + vf(t) \tag{2.4}$$

The model just introduced, attributed to Gordon-Schaefer, is considered the basic bioeconomic model. It is useful to perform an analysis of the states of static equilibrium, where the rate of population growth is equal to the catch rate. A state of equilibrium will occur at many distinct population sizes, and corresponding effort levels. It is possible to determine, from among all possible equilibrium states, that level where the removals are optimized to maximize the economic returns, termed the static maximum economic yield (MEY). This can be seen as the target state for the fishery.

An equilibrium analysis does not consider though the time taken to reach equilibrium while the system is responding to changes in fishing effort or other changes. In not considering a time dimension, it also cannot be used to optimize the economic returns over time. A dynamic model which does consider a time dimension, can be used to make a decision at each point in time as to how much to fish today and how much to leave unfished to grow, to potentially fish more in the future. That trajectory of optimal fishing maximizes economic returns not just in a static sense but at each point in time. Under an attempt to improve the status and management of a fishery, this sets up a framework where this optimal trajectory towards the dynamic MEY can be pursued.

### 2.1.1 Costs

The costs involved with fishing include not just the cost of vessels, fishing gear, fuel and maintenance, salaries, depreciation, "opportunity cost of labour and capital [...] and a margin for risks being faced" (OECD, 2003), but there are also costs associated with "fisheries services" undertaken by the public sector that benefit the whole fishery: research including surveys, other data collection, stock assessments; management and administrative services; and enforcement, monitoring, surveillance and control activities (OECD, 2003; Mangin et al., 2018). These social costs are in most OECD countries covered by general taxation revenues. Arguments for this arrangement focus on the entities responsible for providing the services being generally government ministries or agencies, and the need for independence of these operations from influences from the fishing industry. This structuring may undercut economic (Wallis and Flaaten, 2001), financial (Garcia and Boncoeur, 2005) and social efficiency however (OECD, 2003). Should fishers bear these costs, the same authors say that those fishers will push for more efficient and cost-effective services. Several countries are recouping a proportion of the costs from the fishing industry through cost recovery programmes, and others do it indirectly through user fees or in-kind contribution of fishing effort to research surveys (Mangin et al., 2018; OECD, 2003). Whereas in developed countries many of the necessary services will be provided even if indirectly funded by taxation, this may not be true in the less commercially important fisheries. Similarly in developing countries, lack of the necessary funding for assessment, management and enforcement means many fishery are conducted without adequate services, leading to poor outcomes: potentially stock collapse but as a rule, economically inefficient fishing.

Should the costs of the fisheries services be redirected to the fishing sector, and accompanied by greater input of fishers/stakeholders via co-management, this could "lead to higher compliance rates, more effective management outcomes, longer lasting returns on management inputs and, potentially, lower overall costs of management" (OECD, 2003).

Internalizing these costs to the fishery revenues ensures that assessment and management costs can be covered indefinitely. It also removes the artificial inflation of profits created by not directly taxing these services. Charging these taxes creates an immediate income stream for the government agencies to assess, manage and monitor compliance, which are necessary steps for long-term sustainability.

Taxes can be charged on the inputs to the fishery such as fishing effort, or alternatively on outputs such as a levy charged per catch weight. Taxation of inputs rather than outputs lowers incentives for underreporting of harvests and does not discourage increases in fishing efficiency. Inputs are generally easier to monitor and control than outputs.

An input tax $z$ proportional to effort is thus proposed and added to the cost function, with a new parameter $b$ defined as the sum of the variable cost and taxation components

$$C(f) = a + vf(t) + zf(t) = a + bf(t). \tag{2.5}$$

### 2.1.2 Objectives for the proposed instrument

1. The long-term objective is to reach a state of dynamic optimal utilization, in terms of inputs (fishing effort, stock biomass) and maximizing outputs (catch levels and profits)

2. The short-term objective is to find a path to that state which maximizes net present value of harvest

3. It is also an objective to guarantee a measure of economic stability to fishers should harvest need to be reduced in the short-term in order to maximize revenue, and particularly in small-scale or subsistence fisheries.

### 2.1.3 Optimal bioeconomic utilization

#### Optimal static utilization

A static analysis can be used to determine the theoretical steady-state (stable equilibrium) solution, where $B$ reaches its static maximum economic yield point. This state of equilibrium lacks realism, as parameters change over time and thus a dynamic system is more accurate. However, this equilibrium analysis can be used to determine the state around which the system should be maintained via careful management, if one does not consider a time dimension, and thus no discounting.

A condition for sustainable equilibrium is that the natural growth rate of the biomass is equal to the catch rate so eq. 2.3 with $\beta = 1$ becomes

$$\frac{dB}{dt} = rB\left[1 - \frac{B}{K}\right] - qf^{\alpha}B = 0. \tag{2.6}$$

As noted, this sustainable equilibrium occurs for a whole range of different biomass and effort combinations.The term $f^{\alpha}$ is substituted by $u$ so that the equilibrium condition becomes an affine equation. That population equilibrium curve (of $B$ and $f$) can then be expressed as

$$rB\left[1 - \frac{B}{K}\right] = qf^{\alpha}B = quB = 0. \tag{2.7}$$

This yields the equilibrium biomass, $B_{eq}$, as a function of the fishing effort,

$$B_{eq} = K\left(1 - \frac{q}{r}u\right). \tag{2.8}$$

The profit, $\Pi$, resulting from a fishery can be determined by subtracting costs from revenue

$$\Pi = Py(f, B) - C(f) \tag{2.9}$$

where $P$ is price per unit catch. $P$ is known not to be constant but is treated as an exogenous variable in this analysis. Global price elasticity of demand is generally negative (Delgado et al., 2003) but changes in the harvest within a single population as treated here are unlikely to have an effect on the market price given the considerable fungibility and scale of the effect. On the other hand, rights-based management has been shown to increase price, but we consider that will be a longer-term and larger-scale effect.

Replacing equation 2.2 into the equation for profit 2.9 using equation 2.5 for expressing costs, and replacing $f^\alpha$ with $u$, we have profit in terms of biomass and $u$

$$\Pi = PqBu - a - bu^{\frac{1}{\alpha}}. \tag{2.10}$$

Replacing in equilibrium biomass from eq. 2.8,

$$\Pi_{eq} = PqKu - \frac{Pq^2K}{r}u^2 - a - bu^{\frac{1}{\alpha}}. \tag{2.11}$$

Determining its first and second order derivatives

$$\frac{\partial \Pi_{eq}}{\partial u} = PqK\left(1 - \frac{2q}{r}u\right) - \frac{b}{\alpha}u^{\frac{1-\alpha}{\alpha}}, \tag{2.12}$$

$$\frac{\partial^2 \Pi_{eq}}{\partial u^2} = -\frac{2Pq^2K}{r} - \frac{b\,(1-\alpha)}{\alpha^2}u^{\frac{1-2\alpha}{\alpha}}. \tag{2.13}$$

As all parameters are non-negative and $\alpha \in ]0,1]$, both terms on the right-hand side of eq. 2.13 are negative, and thus the second derivative is always negative. From the first order equation (2.12), the sufficient condition for optimality is

$$PqK\left(1 - \frac{2q}{r}u\right) - \frac{b}{\alpha}u^{\frac{1-\alpha}{\alpha}} = 0. \tag{2.14}$$

In the case $\alpha = 1$ the optimal effort, $\hat{u}$, is

$$\hat{u} = \begin{cases} \dfrac{(PqK - b)\, r}{2Pq^2K}, & \text{if } b < PqK \quad (2.15a) \\ 0, & \text{if } b \geq PqK. \quad (2.15b) \end{cases}$$

In the case $\alpha \in ]0, 1[$, the left-hand side of eq. 2.14 is positive when $u = 0$ and tends to $-\infty$ as $u \to +\infty$. Therefore eq. 2.14 has one unique solution, which in general cannot be explicitly expressed. One exception is the case where $\alpha = \frac{1}{2}$, where the solution is given by

$$\hat{u} = \frac{PqK}{2\left(\frac{Pq^2K}{r} + b\right)}. \tag{2.16}$$

Equation 2.14 also gives a useful bound for the optimal effort. Indeed eq. 2.14 can be rewritten as

$$\hat{u} = \frac{r}{2q}\left(1 - \frac{b}{\alpha PqK}\hat{u}^{\frac{1-\alpha}{\alpha}}\right). \tag{2.17}$$

Since the subtracted term is positive, we can state that $0 < \hat{u} < \frac{r}{2q}$ if $\alpha \in ]0, 1[$, and $0 \leq \hat{u} < \frac{r}{2q}$ if $\alpha = 1$. Another form of eq. 2.14 is

$$\hat{u}^{\frac{1-\alpha}{\alpha}} = \frac{\alpha PqK}{br}\left(r - 2q\hat{u}\right), \tag{2.18}$$

which we will use below.

For generic $\alpha \in ]0, 1[$, eq. 2.14 cannot be explicitly solved. However, the sensitivity of the optimal effort with respect to the model parameters can be studied via its elastcicity, by applying the implicit function theorem, given eq. 2.17 is continuously differentiable. In particular,

$$\frac{\partial \hat{u}}{\partial P} = -\frac{\frac{\partial^2 \pi_{eq}}{\partial P \partial u}}{\frac{\partial^2 \pi_{eq}}{\partial u^2}}. \tag{2.19}$$

Using equality 2.18, this reduces to

$$\frac{\partial \hat{u}}{\partial P} = \frac{\hat{u}}{P} \frac{\alpha \left(r - 2q\hat{u}\right)}{\left(1-\alpha\right) r - \left(1-2\alpha\right) 2q\hat{u}}. \tag{2.20}$$

The elasticity of $\hat{u}$ with respect to $P$, $E_P^{\hat{u}}$, is

$$E_P^{\hat{u}} = \frac{\partial \hat{u}/\hat{u}}{\partial P/P} = \frac{\alpha \left(r - 2q\hat{u}\right)}{\left(1-\alpha\right) r - \left(1-2\alpha\right) 2q\hat{u}} = \frac{\alpha}{1-\alpha} \frac{r - 2q\hat{u}}{r - 2q\hat{u} + \frac{\alpha}{1-\alpha} 2q\hat{u}}. \tag{2.21}$$

As the second fraction on the right-hand side is $\in ]0,1[$ , $0 < E_P^{\hat{u}} < \frac{\alpha}{1-\alpha}$.

Similarly for the catchability coefficient $q$:

$$E_q^{\hat{u}} = \frac{\alpha}{1-\alpha} \frac{r - 4q\hat{u}}{r - 2q\hat{u} + \frac{\alpha}{1-\alpha} 2q\hat{u}}. \tag{2.22}$$

Thus the elasticity with respect to the catchability coefficient has the same upper bound ($E_q^{\hat{u}} < \frac{\alpha}{1-\alpha}$) as the elasticity with respect to price. However, $E_q^{\hat{u}}$ can be negative, as for $\alpha = \frac{1}{2}$

$$\hat{u} = \frac{1}{1 + \frac{b}{Pq^2K} r} \frac{r}{2q} \tag{2.23}$$

thus $r - 4q\hat{u} < 0$ if and only if $br < Pq^2K$.

For the intrinsic growth rate $r$,

$$E_r^{\hat{u}} = \frac{\alpha}{1-\alpha} \frac{2q\hat{u}}{r - 2q\hat{u} + \frac{\alpha}{1-\alpha} 2q\hat{u}}. \tag{2.24}$$

Thus $0 < E_r^{\hat{u}} < 1$ .

For carrying capacity $K$,

$$E_K^{\hat{u}} = \frac{\alpha}{1-\alpha} \frac{r - 2q\hat{u}}{r - 2q\hat{u} + \frac{\alpha}{1-\alpha} 2q\hat{u}}, \tag{2.25}$$

that is, $E_K^{\hat{u}} = E_P^{\hat{u}}$ .

The elasticity with respect to $\alpha$ is

$$E_\alpha^{\hat{u}} = \frac{br}{PqK} \frac{\hat{u}^{\frac{1-\alpha}{\alpha}} \left(1 + \frac{1}{\alpha}\ln\hat{u}\right)}{(1-\alpha) r - (1-2\alpha) 2q\hat{u}}. \tag{2.26}$$

The denominator is positive given eq. 2.17, and the numerator is negative if $u < e^{-\alpha} < 1$, in which case elasticity is negative, with optimal effort declining with increasing $\alpha$. If $u > e^{-\alpha}$ then elasticity is positive with optimal effort increasing with $\alpha$.

For the variable cost parameter $b$, again making use of eq. 2.18

$$E_b^{\hat{u}} = -\frac{r - 2q\hat{u}}{(1-\alpha)(r - 2q\hat{u}) + \alpha 2q\hat{u}}, \tag{2.27}$$

that is, $E_b^{\hat{u}} = -\frac{1}{\alpha} E_K^{\hat{u}} = -\frac{1}{\alpha} E_P^{\hat{u}}$

### Optimal dynamic utilization

By adding a time dimension, a dynamic analysis considers the optimal path to reaching long-term optimal usage. At each point in time, the fishing effort maximizing the total discounted profit is sought, considering the optimal amount to fish at each point, or to leave unfished to grow and fish more at a later point.

The dynamic optimization problem can be framed in a finite or infinite time horizon. If finite, a target biomass level at the end of the time interval can be set.

Taking a time-dependent fishing effort $u(t) = f^\alpha(t)$, equation 2.3 becomes a one-dimensional control system:

$$\frac{dB(t)}{dt} = rB(t)\left(1 - \frac{B(t)}{K}\right) - qB(t)u(t). \tag{2.28}$$

Initial biomass is considered to be known,

$$B(0) = B_0.$$

If the time horizon is finite and we wish to introduce a constraint on the final biomass, we can add a terminal boundary condition

$$B(T) \geq B_T \tag{2.29}$$

with

$$u(t) \in [0, u_{max}] \; \forall \, t \geq 0.$$

The intention is to adjust the state by varying $u(t)$ in order to maximize the total present value of the profit over time. The profit rate is given, as in eq. 2.10, by

$$\Pi(t) = Pqu(t)B(t) - a - bu(t)^{\frac{1}{\alpha}}. \tag{2.30}$$

Thus we have an optimal control problem with objective functional

$$\int_0^T e^{-\delta t}\left(Pqu(t)B(t) - a - bu(t)^{\frac{1}{\alpha}}\right) dt \rightarrow max \tag{2.31}$$

where $\delta$ is the social discount rate (assumed constant) and $T \in ]0, +\infty[$ is the time horizon of the problem. We limit $\alpha$ to $\in ]0, 1[$. Since $\int_0^T e^{-\delta t} a \; dt$ is a constant independent of $u$, $a$ can be omitted, and eq. 2.31 is equivalent to the minimization problem

$$\int_0^T e^{-\delta t}\left(bu(t)^{\frac{1}{\alpha}} - Pqu(t)B(t)\right) dt \rightarrow min. \tag{2.32}$$

This is a nonlinear optimal control problem in $u(t)$, where the $\alpha$-root of the fishing effort which maximizes the objective functional over time is $\hat{u}(t), t \geq 0$.

If $\hat{u}$ is an optimal fishing strategy then, at almost every $t \in [0, T]$, $\hat{u}$ must be equal to or less than the minimizer of the function $u \mapsto (bu^{\frac{1}{\alpha}} - PqBu)$, otherwise there would be a strategy achieving a larger total present value and leaving a larger population of fish, therefore,

$$0 \leq \hat{u}(t) \leq \left(\frac{\alpha q}{b} PB(t)\right)^{\frac{\alpha}{1-\alpha}} \tag{2.33}$$

for almost every $t$.

An optimal strategy can be proven to exist as per e.g. Cesari, 1983, Chap.9, via the Arzelà-Ascoli theorem and the Filippov measurable selection theorem, if the following three conditions are met:

i. For any $t \in [0, T]$ and $B \geq 0$, the set $Q(t, B) = \left\{ \left(l, rB\left(1 - \frac{B}{K}\right) - qBu\right) : u \geq 0,\ l \geq e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) \right\}$ is convex.

ii. For any $K < +\infty$, $\lim_{u \to +\infty} e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) = +\infty$ uniformly in relation to $t \in [0, T], B \in [0, K]$.

iii. A function $\phi : [0, +\infty[ \mapsto \mathbb{R}$ exists such that:

   a. $\phi$ is bounded from below,
   b. $\lim_{x \to +\infty} \frac{\phi(x)}{x} = +\infty$,
   c. $e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) \geq \phi\left(\left|rB\left(1 - \frac{B}{K}\right) - qBu\right|\right)$.

The set under (i) can be shown to be convex by demonstrating that any point on a line segment joining two elements of the set is also contained in the set. Taking two elements $(l_1, u_1)$, $u_1 \geq 0, l_1 \geq e^{-\delta t}\left(bu_1^{\frac{1}{\alpha}} - PqBu_1\right)$ and $(l_2, u_2)$ $u_2 \geq 0, l_2 \geq e^{-\delta t}\left(bu_2^{\frac{1}{\alpha}} - PqBu_2\right)$, an element $(l_3, u_3)$ defined as an affine combination $\lambda(l_1, u_1) + (1-\lambda)(l_2, u_2)$, with $\lambda \in ]0,1[$, is also an element of $Q(t, B)$.

Starting with $u$,

$$\lambda\left(rB\left(1-\frac{B}{K}\right) - qBu_1\right) + (1-\lambda)\left(rB\left(1-\frac{B}{K}\right) - qBu_2\right) =$$

$$\begin{aligned} &= rB\left(1-\frac{B}{K}\right) - qBu_3 \\ &\Leftrightarrow u_3 = \lambda u_1 + (1-\lambda)\, u_2 \end{aligned} \tag{2.34}$$

which shows inclusion. Taking now $l$,

$$\lambda l_1 + (1-\lambda)\, l_2 \geq e^{-\delta t}\left(bu_3^{\frac{1}{\alpha}} - PqBu_3\right).$$

Replacing in the result from eq. 2.34 and rearranging,

$$\begin{aligned} = \lambda e^{-\delta t}\left(bu_1^{\frac{1}{\alpha}} - PqBu_1\right) + (1-\lambda)\, e^{-\delta t}\left(bu_2^{\frac{1}{\alpha}} - PqBu_2\right) + \\ + e^{-\delta t} b\left[(\lambda u_1 + (1-\lambda)\, u_2)^{\frac{1}{\alpha}} - \lambda u_1^{\frac{1}{\alpha}} - (1-\lambda)\, u_2^{\frac{1}{\alpha}}\right]. \end{aligned} \tag{2.35}$$

The first term is by definition, $\leq \lambda l_1$, the second $\leq (1-\lambda)\, l_2$, and the third is $\leq 0$, which proves the inequality.

Addressing next condition (ii), this requires that $\forall N \in ]0, +\infty[\ \exists\ V < +\infty :$ $\forall t \in [0, T],\ B \in [0, K],\ u > V \Rightarrow e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) > N.$

It can be shown that

$$e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) \geq e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqKu\right) = e^{-\delta t}\left(bu^{\frac{1-\alpha}{\alpha}} - PqK\right) u. \tag{2.36}$$

The expression on right-hand side of the equality is positive for a sufficiently large $u$, so we can define $u > \left(\frac{PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}$ or $u > 1$, as $u > \max\left\{1, \left(\frac{PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}\right\}$

such that

$$e^{-\delta t}\left(bu^{\frac{1-\alpha}{\alpha}} - PqK\right)u \geq e^{-\delta T}\left(bu^{\frac{1-\alpha}{\alpha}} - PqK\right) > N \tag{2.37}$$
$$\Leftrightarrow bu^{\frac{1-\alpha}{\alpha}} - PqK > e^{\delta T}N$$
$$\Leftrightarrow u > \left(\frac{e^{\delta T}N + PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}$$

for any $N$ and any $u$ such that

$$u > \max\left\{1, \left(\frac{PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}, \left(\frac{e^{\delta T}N + PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}\right\} =$$

$$= \max\left\{1, \left(\frac{e^{\delta T}N + PqK}{b}\right)^{\frac{\alpha}{1-\alpha}}\right\} = V.$$

For $u$ defined in this way in terms of constants, we have shown that

$$e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) > N \quad \forall t \in [0,T],\ B \in [0,K]. \tag{2.38}$$

In order to address condition (iii), a function of the form

$$\phi(t) = -A_1 + A_2 t^{\frac{1}{\alpha}} \tag{2.39}$$

is defined. It can be shown that $B \in [0,K]$. As above, for a sufficiently large $u$,

$$e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqBu\right) \geq e^{-\delta t}\left(bu^{\frac{1}{\alpha}} - PqKu\right) \geq e^{-\delta T}\left(bu^{\frac{1}{\alpha}} - PqKu\right) \tag{2.40}$$

$$\lim_{u\to+\infty} \frac{\left|rB\left(1-\frac{B}{K}\right) - qBu\right|^{\frac{1}{\alpha}}}{e^{-\delta T}\left(bu^{\frac{1}{\alpha}} - PqKu\right)} = \frac{(qB)^{\frac{1}{\alpha}}}{e^{-\delta T}b} \leq \frac{(qK)^{\frac{1}{\alpha}}}{e^{-\delta T}b} < \frac{2\,(qK)^{\frac{1}{\alpha}}}{e^{-\delta T}b}. \tag{2.41}$$

There is a $\tilde{u} < +\infty$, such that $u > \tilde{u}$ implies

$$\frac{e^{-\delta T} b}{2 (qK)^{\frac{1}{\alpha}}} \left| rB \left(1 - \frac{B}{K}\right) - qBu \right|^{\frac{1}{\alpha}} < e^{-\delta T} \left(bu^{\frac{1}{\alpha}} - PqKu\right). \tag{2.42}$$

There may exist a $u \in [0, \tilde{u}]$ such that

$$\frac{e^{-\delta T} b}{2 (qK)^{\frac{1}{\alpha}}} \left| rB \left(1 - \frac{B}{K}\right) - qBu \right|^{\frac{1}{\alpha}} - e^{-\delta T} \left(bu^{\frac{1}{\alpha}} - PqKu\right) \geq 0. \tag{2.43}$$

As such, we can define

$$A_1 = \max \left\{ \frac{e^{-\delta T} b}{2 (qK)^{\frac{1}{\alpha}}} \left| rB \left(1 - \frac{B}{K}\right) - qBu \right|^{\frac{1}{\alpha}} - e^{-\delta T} \left(bu^{\frac{1}{\alpha}} - PqKu\right), \right. \\ \left. u \in [0, \tilde{u}],\ B \in [0, K] \right\} \tag{2.44}$$

and

$$A_2 = \frac{e^{-\delta T} b}{2 (qK)^{\frac{1}{\alpha}}}. \tag{2.45}$$

The function $\phi(t)$ defined in this way can be shown to be $\leq e^{-\delta t} \left(bu^{\frac{1}{\alpha}} - PqBu\right)$. As $A_1$ and $A_2$ are constants it is bounded from below and condition (iii.b) is met.

These results guarantee the existence of a solution.

To arrive at an expression for the optimum dynamic utilization, we find solutions for the Pontryagin maximum principle. The Hamiltonian function is:

$$H(t, \lambda_0, \lambda_1, B, u) = \lambda_0 e^{-\delta t} \left( b u^{\frac{1}{\alpha}} - PqBu \right) + \lambda_1 \left[ rB \left( 1 - \frac{B}{K} \right) - qBu \right] \quad (2.46)$$

where $\lambda_0 \in \{0, -1\}$ and $\lambda_1$ is the costate variable. The costate variable is the shadow value of fish in the ocean - the discounted increase in economic value that results from leaving a unit of fish in the ocean to grow (Anderson and Seijo, 2011), "shadow" referring to it being a theoretical price that fish in the sea would sell for if it were instead in the market.

$$\lambda_1(T) = 0$$

to satisfy the terminal transversality condition. $T$ is chosen to be sufficiently large that the dynamic MEY biomass is approached within a chosen tolerance.

To find the first-order condition for the Hamiltonian in relation to the control variable, $u(t)$,

$$\frac{\partial H}{\partial u} = \lambda_0 e^{-\delta t} \left( \frac{b}{\alpha} u^{\frac{1-\alpha}{\alpha}} - PqB \right) - \lambda_1 qB = 0. \quad (2.47)$$

The case $\lambda_0 = 0$ can be excluded since in this case

$$\frac{\partial H}{\partial u} = -\lambda_1 qB \neq 0. \quad (2.48)$$

A positive derivative would mean there is no optimal solution, and the stock should be fished instantaneously to extinction, which is not optimal since this would drive the functional 2.32 to $+\infty$. A negative derivative would mean $u = 0$ which cannot be optimal, unless this is the unique strategy satisfying the terminal condition 2.29.

Taking then the case $\lambda_0 = -1$,

$$H = -e^{-\delta t} b u^{\frac{1}{\alpha}} + \left(e^{-\delta t} P - \lambda_1\right) qBu + \lambda_1 rB \left(1 - \frac{B}{K}\right) \tag{2.49}$$

$$\frac{\partial H}{\partial u} = -\frac{e^{-\delta t} b}{\alpha} u^{\frac{1-\alpha}{\alpha}} + qB \left(e^{-\delta t} P - \lambda_1\right) \tag{2.50}$$

$$\frac{\partial^2 H}{\partial u^2} = -\frac{1-\alpha}{\alpha^2} e^{-\delta t} b u^{\frac{1-2\alpha}{\alpha}}. \tag{2.51}$$

Eq. 2.51 is strictly negative so eq. 2.49 is strictly concave. The maximum is given by

$$\frac{\partial H}{\partial u} = 0 \Leftrightarrow \frac{e^{-\delta t} b}{\alpha} u^{\frac{1-\alpha}{\alpha}} = qB \left(e^{-\delta t} P - \lambda_1\right) \tag{2.52}$$

and provides a unique solution to the system.

Eq. 2.52, in conjunction with eq. 2.33, leads to two possible cases,

$$\hat{u} = \begin{cases} \left[\frac{\alpha q}{b} e^{\delta t} B \left(e^{-\delta t} P - \lambda_1\right)\right]^{\frac{\alpha}{1-\alpha}}, & \text{if } 0 < e^{-\delta t} P - \lambda_1 \leq e^{-\delta t} P \quad (2.53a) \\ 0, & \text{if } e^{-\delta t} P - \lambda_1 \leq 0. \quad (2.53b) \end{cases}$$

According to the maximum principle,

$$\frac{dB}{dt} = \frac{\partial \hat{H}}{\partial \lambda_1} = rB \left(1 - \frac{B}{K}\right) - qB\hat{u} \tag{2.54}$$

and

$$\frac{d\lambda_1}{dt} = -\frac{\partial \hat{H}}{\partial B} = -r\lambda_1 \left(1 - \frac{2}{K} B\right) - q \left(Pe^{-\delta t} - \lambda_1\right) \hat{u}. \tag{2.55}$$

A change of variable is made, denoting $\gamma = Pe^{-\delta t} - \lambda_1$, with $\hat{u}$ becoming

$$\hat{u} = \begin{cases} \left(\frac{\alpha q}{b} e^{\delta t} \gamma B\right)^{\frac{\alpha}{1-\alpha}}, & \text{if } 0 < \gamma \leq e^{-\delta t} P \quad (2.56a) \\ 0, & \text{if } \gamma \leq 0 \quad (2.56b) \end{cases}$$

and the modified system is described by eq. 2.54 and

$$\frac{d\gamma}{dt} = r\gamma\left(\frac{2}{K}B - 1\right) + Pe^{-\delta t}\left[r\left(1 - \frac{2}{K}B\right) - \delta\right] + q\gamma\hat{u}. \tag{2.57}$$

For case 2.56b, $\gamma \leq 0$,

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) > 0 \tag{2.58}$$

and

$$\frac{d\gamma}{dt} = r\gamma\left(\frac{2}{K}B - 1\right) + Pe^{-\delta t}\left[r\left(1 - \frac{2}{K}B\right) - \delta\right]. \tag{2.59}$$

From eq. 2.59, it can be shown that

$$\begin{cases} \dfrac{d\gamma}{dt} > 0, & \text{if } B < \left(1 - \frac{\delta}{r}\frac{e^{-\delta t}P}{e^{-\delta t}P - \gamma}\right)\frac{K}{2} \quad (2.60a) \\ \dfrac{d\gamma}{dt} < 0, & \text{if } B > \left(1 - \frac{\delta}{r}\frac{e^{-\delta t}P}{e^{-\delta t}P - \gamma}\right)\frac{K}{2} \quad (2.60b) \end{cases}$$

For case 2.56a, $0 < \gamma \leq e^{-\delta t}P$

$$\frac{dB}{dt} = rB\left(1 - \frac{B}{K}\right) - qB\left(\frac{\alpha q}{b}e^{\delta t}\gamma B\right)^{\frac{\alpha}{1-\alpha}} \tag{2.61}$$

from which

$$\begin{cases} \dfrac{dB}{dt} > 0, & \text{if } \gamma < \frac{b}{\alpha q}\left(\frac{r}{q}\right)^{\frac{1-\alpha}{\alpha}} e^{-\delta t}\frac{\left(1 - \frac{B}{K}\right)^{\frac{1-\alpha}{\alpha}}}{B} \quad (2.62a) \\ \dfrac{dB}{dt} < 0, & \text{if } \gamma > \frac{b}{\alpha q}\left(\frac{r}{q}\right)^{\frac{1-\alpha}{\alpha}} e^{-\delta t}\frac{\left(1 - \frac{B}{K}\right)^{\frac{1-\alpha}{\alpha}}}{B} \quad (2.62b) \end{cases}$$

and

$$\frac{d\gamma}{dt} = r\gamma\left(\frac{2}{K}B - 1\right) + Pe^{-\delta t}\left[r\left(1 - \frac{2}{K}B\right) - \delta\right] + q\gamma\left(\frac{\alpha q}{b}e^{\delta t}\gamma B\right)^{\frac{\alpha}{1-\alpha}} \tag{2.63}$$

$$= \gamma \left[ q \left( \frac{\alpha q}{b} e^{\delta t} \gamma B \right)^{\frac{\alpha}{1-\alpha}} - r \left( 1 - \frac{2}{K} B \right) \right] + P e^{-\delta t} \left[ r \left( 1 - \frac{2}{K} B \right) - \delta \right]. \quad (2.64)$$

The optimal trajectory can thus be characterized in general terms. A more complete description could be obtained using numerical methods.

## 2.2 Designing the financial instrument

Having a characterization of the optimal effort strategy that will maximize long-term revenue, we now propose an impact instrument for investing in fisheries, using private capital to move a fishery into a sustainable system of use, thereby increasing revenues and social benefits for all stakeholders.

Through the purchase of the instrument, impact investors would fund the implementation of a rights-based management system and also stabilize fishers' income during the transition to sustainability. Our formulation relies on a local or federal government, other centralized authority or community agency. This entity would have all the necessary information and empowerment to perpetuate the exploitation of the fishery at sustainable levels. It would be responsible for the management of existing fishing licenses, setting annual catch limits and the application of the funds initially received from the investors.

Existing licensed fishers would be allocated shares of the fishery under the new management system. The literature finds that a possible result, and even objective, is that least efficient fishers often sell their share (Grimm et al., 2012). Defining criteria for eligibility for holding shares, such as requiring shareholders to be owner-operators, or defining limits of the shares held, could avoid shares being bought up in bulk by larger industrial operations and "over-consolidating" the fishery (Bonzon et al., 2010).

The amount initially raised would finance improvements in management, monitoring and surveillance, and the needed research backing the management decisions. Subsequent maintenance costs of the system will be covered by taxation revenues.

The design of the instrument is intended to compensate investors for initial losses with subsequent gains, and end the duration of the instrument leaving the fishery in a state of maximum sustainable profits. Investors will pay an initial investment and will be entitled to returns in excess of the stable level for a pre-determined period from when the fishery reaches a state where the profits are positive and sustainable. In the meantime, investors absorb possible losses resulting from the adoption of a sustainable path of use, compensating fishers for income they would have lost during periods when effort must be substantially reduced to recover the fishery.

More precisely, under the instrument fishers contract to apply the optimally determined levels, $\hat{u}_t$, and in return receive a baseline income level, $R_0$, based on recent historical levels or other pre-agreed levels, which is covered by investors. Investors also cover costs and receive revenues from the fishery. Covering costs is intended to compensate for the possible decrease in revenue in the case of overexploited stocks, as effort will need to be reduced and catches will decrease accordingly. Note that although catches may decrease leading to lower revenue, costs may also decrease due to lower effort, although newly introduced taxes are an added cost.

The cash-flows associated with this proposed financial instrument are as follows:

- In the initial moment, a payment of $m$ is made by investors to set up the rights-based management system. This is an initial one-off fixed cost that covers planning and designing, as per Mangin et al., 2018.

- In addition, under the condition that fishing effort follows the optimal effort, $\hat{u}_t$, investors pay the fishery costs, including the new taxes, and an agreed income level to the fishers, and receive the revenues of the fishery

$(Py_t - C_t - R_0 = \Pi_t - R_0)$ up until $\theta$ years past the point in time when the fishery reaches its target state of dynamic $B_{MEY}$ (designated $t_{MEY}$), so up until $t = t_{MEY} + \theta = T$.

The payoff for investors, is represented as

$$\int_0^T e^{-\delta t} (\Pi_t - R_0)\, dt - m \tag{2.65}$$

After time $t_{MEY} + \theta$, investors would exit the fishery and fishers should now receive an expected average profit of $\hat{\Pi}$ as long as $f_{MEY}$ is the long-term strategy. Revenue is held and costs are paid from an escrow account from the start of the instrument.

Throughout the duration of the instrument, whenever investors are paying costs and receiving revenues, they are on paper receiving the fish but in practice acting as intermediaries to the sales. In this way, established commercial relationships between fishers and suppliers are unchanged throughout the duration of the instrument.

## 2.3 Pricing the financial instrument

The pricing of the instrument considers the payoff from eq. 2.65. A dynamic stochastic model is proposed, considering the main source of uncertainty in the model to be represented by the biomass dynamics. Modifying the dynamics to consider a stochastic term, we obtain

$$dB(t) = \left[ rB_t \left( 1 - \frac{B_t}{K} \right) - qB_t u_t \right] dt + \sigma B_t dW_t \tag{2.66}$$

where $\sigma$ is the standard deviation of the biomass and $W_t$ is a standard Brownian motion. The uncertainty impacts the payoff solely through the yield term $y_t = qB_t u_t$, as the fishing effort $u_t$ is foreseen in the contract, and can be considered to be applied in a deterministic way. One possible approach to pricing the risk would be to consider risk-neutral valuation. The catch rate $y(t)$ could be seen as the underlying or the measure. Any contingent claim based on $y(t)$ can be replicated as there is a liquid market for fish, as a soft commodity, so completeness can be assumed. However, volatility of $y(t)$ will not be identical in the liquid market, so the absence of arbitrage opportunities cannot be shown. An alternative would be to penalize risk via an increasing and concave utility function $U : \mathbb{R} \mapsto \mathbb{R}$ , with $U(0) = 0$. The instrument could then be priced using expected value

$$\mathbb{E} \int_0^T e^{-\delta t} U \left( \Pi_t - R_0 \right) dt - m. \tag{2.67}$$

A simple choice for the utility function could be

$$U(x) = \frac{1 - e^{-\rho x}}{\rho} \tag{2.68}$$

where $\rho$ is a positive constant which has the advantage of allowing for its interpretation as a coefficient of risk aversion. This function has the disadvantage of being upper bounded, but for the current purposes that is unlikely to be a disadvantage in practice.

Regarding $\theta$, its value is to be set so as to ensure an expected rate of return for investors. The expected rate of return can be selected based on historical returns from impact investments.

# Conclusions

A bioeconomic model of a fishery allows for the analysis of the system that produces maximum sustainable profit. This system ensures continuous sustained use in a deterministic model. This can guide the fishing effort that should be applied in order to obtain the largest continuous financial benefits.

Static equilibrium biomass can be defined in terms of fishing effort. Profits at equilibrium can be modelled and maximized to determine that a solution to optimal fishing effort exists and is unique. For certain values of the effort scaling parameter $\alpha$ an explicit function can be obtained, while for other cases, a solution could be approximated by numerical methods. Although in general it cannot be expressed explicitly, bounds can be obtained in terms of $\alpha$. The sensitivity of optimal effort to changes in the model parameters can be studied through evaluation of elasticities. These reveal a positive elasticity to price changes, limited by the value of $\alpha$. Elasticity with changes to the catchability coefficient $q$ (or efficiency of the fishing gear) can be positive or negative, depending on the value of $\alpha$. Elasticity with changes to the intrinsic growth rate $r$ is positive but inelastic, at values less than 1. Optimal effort's elasticity to changes in carrying capacity $K$ reveal the same relationship as to changes in price. The elasticity to changes in the variable cost parameter $b$ is negative, with optimal effort tending to decrease as costs increase. Elasticity with respect to $\alpha$ is negative or positive for efforts on either side of $e^{-\alpha}$. In general, $\alpha$ determines the range of variation of the elasticity of the optimal effort to changes in many of the parameters.

A time-variable trajectory of optimal effort could be obtained using a dynamic optimization model. The characterization of the trajectory is not trivial but can be approximated for ranges of the system parameters and costate variable, and an optimal strategy can be shown to exist with a unique solution. Numerical methods could be employed to more fully describe the optimal trajectory. Furthermore, a stochastic element could have been introduced to better approximate reality.

The proposed instrument funds improvements to the management system and a transition to a system where the fishery does not rely on public funding of services. Fisheries in a state of overfishing will be restored to the biomass level that generates maximum economic yield, maximizing profits for fishers and ensuring sustainability. Cash flows from the instrument are described and a method for pricing suggested.

The model developed in this work can be used to better understand key considerations in this research field. One of these considerations is the payment of subsidies. Using this model we can elaborate on the need for subsidies to reach a sustainable trajectory for fisheries. Another consideration is scaleability. The results can still be valid in a wider setting provided some assumptions are revisited. A final consideration should be made on other elements of sustainability, several of which could be handled by adaptations to our model.

**Subsidies**

There is a risk that the instrument could be seen as a subsidy to fishers. Subsidies are often shown to have unintended negative effects, such as generating inequalities in international markets pricing and leading to changes in fishing practices that may threaten the sustainability of fish stocks (Schrank, 2003). In this case however, consequences would be expected to have the opposite effect. The instrument may instead lead to less competitive pricing as a higher price may be required by the fishers if true costs are built in. An analysis would be merited into whether the increased yield would compensate for this effect. Alternatively, marketing the product as sustainable and empowering to fishers may appeal to impact-minded consumers who may be willing to pay a sustainability premium. The critique forecasting increasingly unsustainable fishing practices is also expected to be directly opposed to the outcomes of the instrument.

Also countering the subsidy argument is the proposed limited duration and conditional nature of the payments to the fishers, whereas another risk to subsidies is that they become ingrained and difficult to remove (Schrank, 2003).

In fact, assessment and management costs that are generally publicly-covered could be considered to be subsidies, and this instrument proposes to modify that situation. However, the difficulties of a one-sided economic correction should not be understated, and widespread internalization of costs is the intended aim.

**Scaleability**

This project focused on investment in a single fishery, so any potential benefits are limited in scale. In biological terms, each population of fishery is a relatively closed system, which must be modelled independently. For this reason, a bioeconomic model must also be performed at the scale of an individual fishery. An investment venture such as a Benefit Corporation (B-corporation) or intermediary could be envisioned which develops instruments for fisheries globally, working with local agencies.

Considering the profile of potential investors, this project may appeal to smaller-scale investors who have a local interest in a particular fishery, whereas the involvement of a larger venture and the possibility of investment across a diversity of fisheries could appeal to larger impact investors.

The modelling of the price parameter is assumed not to be impacted by the instrument but for a larger scale project that assumption would need to be revisited.

**Other elements of sustainability**

Assuring the health of fish stocks is just one element of fisheries' sustainability; other aspects that should also be considered are the impacts of fisheries on other elements of ecosystems such as bycatch, habitat impacts, trophic and other ecosystem impacts including on endangered and threatened species, carbon emissions by fishing vessels, ghost fishing and reduction of use and loss of plastic-based fishing gear. Measures to address other impacts could be incorporated into future iterations of the instrument.

Minimizing environmental impacts is critical to sustainability, but it should be underscored that the value of fisheries is essentially as social resources, and is closely tied to communities and livelihoods in many parts of the world. This work proposes an impact investment instrument that produces social benefits from a natural resource.

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