Revision 1

Problem 1. Consider the following models:

$$y_t = \alpha_0 + \alpha_1 t + e_t \tag{1}$$

$$y_t = \rho y_{t-1} + v_t \tag{2}$$

where e_t and v_t are i.i.d.(0,1), and $\rho = 1$. A central banker considers the two models above to explain the Gross Domestic Product (GDP).

(a) What is the name of each model?

(b) Derive the unconditional mean and variance of y_t implied by each model. Is any of the two models covariance stationary or/and weak dependent?

(c) Briefly explain what is meant by weak stationarity and weak dependence.

(d) Predicting future GDP is of major importance in decision making regarding investment, spending and hiring (among other things). Hence we are interested in the $h-$step ahead forecast given the last observed information: $E(y_{t+h}|y_t)$. Derive $E(y_{t+h}|y_t)$ from model (1) and (2) assuming $\rho = 1$.

(e) When $|\rho| < 1$, $E(y_{t+h}|y_t) = \rho^h y_t$. What happens with the $h-$step ahead forecast as $h \to \infty$ in model (2) for $|\rho| < 1$ and $\rho = 1$?

(f) y_t in model (1) has trending behaviour, while y_t in model (2) with $\rho = 1$ has highly persistent behaviour. Show that y_t described by the model:

$$y_t = \delta + y_{t-1} + u_t \tag{3}$$

is highly persistent and has a clear linear trend, where u_t is i.i.d.(0,1).