Discrete random variables:

$$E[X] = \mu = \sum_x xP(x)$$

$$\sigma^2 = E\left[(X-\mu)^2\right] = \sum_x (x-\mu)^2 P(x) \qquad \sigma^2 = E\left[X^2\right] - \mu^2 = \sum_x x^2 P(x) - \mu^2$$

Functions of random variables:

$$Y = a + bX$$

$$\mu_Y = E(a+bX) = a + b\mu_x \qquad \sigma^2{}_Y = Var(a+bX) = b^2\sigma^2 x$$

Bernoulli distribution:

$$\mu_x = P \qquad \sigma^2_x = P(1-P)$$

Binomial distribution:

$$P(x) = \frac{n!}{x!(n-x)!} P^x (1-P)^{n-x} \qquad \mu = E(x) = nP \qquad \sigma^2 = nP(1-P)$$

Poisson distribution:

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!} \; \mu_x = E[X] = \lambda \qquad \sigma^2_x = E\left[(X-\mu_x)^2\right] = \lambda$$

Poisson approximation to the Binomial :

$$P(x) = \frac{e^{-nP}(nP)^x}{x!} \text{ for } x = 0,1,2,\ldots$$

Hypergeometric distribution:

$$P(x) = \frac{C^S_x C^{N-S}_{n-x}}{C^N_n} = \frac{\frac{S!}{x!(S-x)!} \times \frac{(N-S)!}{(n-x)!(N-S-n+x)!}}{\frac{N!}{n!(N-n)!}}$$

Conditional distributions:

$$P(y|x) = \frac{P(x,y)}{P(x)} \qquad \mu_{Y|X} = E[Y|X] = \sum_y (y|x) P(y|x)$$

$$\sigma^2_{Y|X} = E\left[\left(Y - \mu_{Y|X}\right)^2 |X\right] = \sum_y \left[\left(y - \mu_{Y|X}\right)^2 |x\right] P(y|x)$$

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Independence:

$$P(x,y) = P(x)P(y)$$

Covariance and correlation:

$$Cov(X,Y) = E\left[(X-\mu_X)(Y-\mu_Y)\right] = \sum_x \sum_y (x-\mu_x)(y-\mu_y)P(x,y)$$

$$Cov(X,Y) = E(XY) - \mu_x\mu_y = \sum_x \sum_y xyP(x,y) - \mu_x\mu_y$$

$$\rho = Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

Continuous random variables:

$$F(x) = P(X \le x) \qquad P(a < X < b) = F(b) - F(a)$$

$$\mu_X = E[X] \quad \sigma_X^2 = E\left[(X-\mu_X)^2\right]$$

Uniform distribution:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \qquad \mu = \frac{a+b}{2} \qquad \sigma^2 = \frac{(b-a)^2}{12}$$

Normal distribution:

$$Z \sim N(0,1), \quad X = \mu + \sigma Z \sim N(\mu, \sigma^2)$$

Normal distribution approximation to Binomial:

$$Z = \frac{X - E[X]}{\sqrt{Var(X)}} = \frac{X - np}{\sqrt{nP(1-P)}}$$

Exponential distribution:

$$f(t) = \lambda e^{-\lambda t} \text{ for } t > 0 \qquad F(t) = 1 - e^{-\lambda t}$$

Difference between a pair of random variables:

$$E[X-Y] = \mu_X - \mu_Y \qquad Var[X-Y] = \sigma_X^2 + \sigma_Y^2 - 2Cov(X,Y)$$

Linear combination of random variables:

$$W = aX + bY \quad \mu_W = E[W] = E[aX + bY] = a\mu_X + b\mu_Y$$

$$\sigma_W^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2abCov(X,Y)$$

$$\sigma_W^2 = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho(X,Y)\sigma_X\sigma_Y$$

Confidence interval for the population's mean (known and unknown variance respectively), for the population proportion and the population variance respectively:

$$\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \qquad \bar{x} \pm t_{n-1,\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \qquad \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}$$

With finite sample corrections:

$$\hat{\sigma}^2_{\bar{x}} = \frac{s^2}{n}\left(\frac{N-n}{N-1}\right) \qquad \hat{\sigma}^2_{\hat{p}} = \frac{\hat{p}(1-\hat{p})}{n}\left(\frac{N-n}{N-1}\right)$$

Confidence intervals for the mean difference (dependent samples):

$$\bar{d} \pm t_{n-1,\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}} \qquad s_d = \sqrt{\frac{\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2}{n-1}} \qquad \bar{d} = \frac{\sum_{i=1}^{n} d_i}{n} \qquad d_i = x_i - y_i$$

Confidence intervals for the mean difference (independent samples, known variance):

$$\left(\bar{x} - \bar{y}\right) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$$

Confidence interval for the mean difference (independent samples, unknown and equal variances):

$$(\bar{x} - \bar{y}) \pm t_{n_x+n_y-2,\frac{\alpha}{2}} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}} \qquad s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x + n_y - 2}$$

Confidence interval for the mean difference (independent samples, unknown and unequal variances):

$$(\bar{x} - \bar{y}) \pm t_{\nu,\frac{\alpha}{2}} \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}} \qquad \nu = \frac{\left[\left(\frac{s_x^2}{n_x}\right) + \left(\frac{s_y^2}{n_y}\right)\right]^2}{\frac{\left(\frac{s_x^2}{n_x}\right)^2}{(n_x-1)} + \frac{\left(\frac{s_y^2}{n_y}\right)^2}{(n_y-1)}}$$

Confidence interval for two population proportions:

$$\left(\hat{p}_x - \hat{p}_y\right) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_x\left(1-\hat{p}_x\right)}{n_x} + \frac{\hat{p}_y\left(1-\hat{p}_y\right)}{n_y}}$$

Tests about the population mean: $z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \sim N(0,1)$, $t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$

ANOVA: $\mathrm{SST} = \mathrm{SSW} + \mathrm{SSG}$, SSW=sum of squares within groups, SSG=sum of squares between groups; MSW=mean-square within, MSG=mean-square between

$$\text{SST} = \sum_{i=1}^{K}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{\overline{x}}\right)^2 \qquad \text{SSW} = \sum_{i=1}^{K}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{x}_i\right)^2 \qquad \text{SSG} = \sum_{i=1}^{K} n_i\left(\overline{x}_i - \overline{\overline{x}}\right)^2$$

$$\text{MST} = \frac{\text{SST}}{n-1} \qquad \text{MSW} = \frac{\text{SSW}}{n-K} \qquad \text{MSG} = \frac{\text{SSG}}{K-1}$$

$$F = \frac{\text{MSG}}{\text{MSW}} \sim F_{K-1,\, n-K}$$

Test for association (contingency table): $\sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}} \sim \chi^2_{(r-1),(c-1)}$, where $E_{ij} = \frac{R_i C_j}{n}$, R_i and C_j are row and column totals, r and c are number of categories.

Regression. Coefficient of determination:

$$R^2 = \frac{\text{SSR}}{\text{SST}} = \frac{\text{regression sum of squares}}{\text{total sum of squares}}$$

Unbiased estimate of the variance of the residuals:

$$s_e^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-K-1} = \frac{\text{SSE}}{n-K-1}$$

Adjusted coefficient of determination:

$$\bar{R}^2 = 1 - \frac{\text{SSE}/(n-K-1)}{\text{SST}/(n-1)}$$

Confidence interval limits for the population slope β_j:

$$b_j \pm t_{n-K-1,\frac{\alpha}{2}} S_{b_j}$$

F-statistic:

$$F = \frac{(\text{SSR}_{\text{restricted model}} - \text{SSR}_{\text{unrestricted model}})/\text{R}}{\text{SSR}_{\text{unrestricted model}}/(\text{N} - \text{K} - 1)}$$

where R is the number of restriction, N is the sample size, K is the number of regressors in the unrestricted model (which is assumed to have a constant, hence we also subtract 1).