



# Investments and Portfolio Management

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*Closed-book exam. You can only use the official formulas' sheet.*

**Duration: 2.5h**

.....  
**Name:**

.....  
**Number:**

**GROUP I (35 points)**

1. Explain how individual investor bias such as *overconfidence*, *anchoring* and *inertia* may affect portfolio decision making. In which way may mandatory investment policy statements (IPS) help dealing with them? ..... [20p]

**Answer:**

2. Choose ONE of the following statements and discuss whether it is true or false: ..... [15p]

- I. *Any investor worried with safety is indifferent between the optimal portfolios according to Roy, Kataoka or Telser.*
- II. *The possibility of obtaining abnormal returns based upon technical analysis is inconsistent with all forms of market efficiency.*

**Comment:** .....

**GROUP II (30 points)**

Any second order Taylor approximation of a generic utility function  $U(W)$  around the initial wealth  $W_0$ , is quadratic in  $W$ , i.e. we always get  $U(W) \approx aW^2 + bW + c$ .

(i) Derive the parameters  $a, b$  and  $c$  in terms of initial wealth, utility function and its derivatives. [10p]

(ii) Show the quadratic approximation of  $U$  is *equivalent* to  $V(W) = (1 + RRA_0)W - \frac{1}{2}ARA_0W^2$ , where  $RRA_0$  and  $ARA_0$  stand for the  $U$  relative and absolute risk aversions evaluated at  $W_0$ ... [10p]

(iii) Consider investments  $A, B$  and  $C$  represented by the points  $A = (0\%, 5\%)$ ,  $B = (10\%, 10\%)$  and  $C = (15\%, 25\%)$  in the space  $(\sigma, \bar{R})$ . to rank  $A, B$  and  $C$  according to his preferences. Analyze his risk profile and find the certainty return equivalent to his preferred investment. .... [10p]

**Solution:**

**GROUP III (135 points)**

**Problem 1 (75 points)**

Consider as valid all standard CAPM assumptions and consider the equilibrium relationship

$$\bar{R}_i^e = 3.5\% + 12\%\beta_i .$$

1. Criticise the key assumptions of the standard CAPM and interpret the parameters values in the equilibrium relationship above. ....[10p]
2. Find out the *market portfolio* equilibrium expected return? .....[5p]
3. Consider now the volatility of the *market portfolio* to be  $\sigma_m = 25\%$ . Write down the Capital Market Line (CML) equation and explain what it represents. .... [7.5p]
4. Mr. X risk tolerance function is  $f(\sigma, \bar{R}) = 7\bar{R} - 7(\sigma^2 + \bar{R}^2)$ .
  - (a) From his risk tolerance function what can you conclude about Mr. X risk profile? ... [5p]
  - (b) Show the certainty equivalent of Mr. X of an investment in the market portfolio  $M$  is  $R_c \approx 7,39\%$  and compute the associated risk premium. What can you conclude about his risk profile? .....[10p]
  - (c) Suppose now Mr. X considers not only the market portfolio but all efficient portfolios. What is the optimal investment for him? Explain<sup>1</sup>. .... [12.5p]
  - (d) His wife, Ms. X, would prefer to maximize the probability of returns' above 3.5%.
    - (i) Identify the criteria underlying Ms X concerns? Explain. ....[5p]
    - (ii) What would you recommend her? Any chance her husband's portfolio would make her happy? ..... [7.5p]
5. In this world there is a risky asset,  $A$ , with  $\beta_A = 0.7$ .
  - (a) Determine its equilibrium expected return. ....[5p]
  - (b) A market model has been fit to the observed returns of  $A$ . The parameters are  $\alpha_A = 2\%$ ,  $\beta_A = 0.7$ ,  $\sigma_{\epsilon A} = 5\%$ . What is your investment recommendation on asset  $A$ ? .....[7.5p]

**Solution to Problem 1:**

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<sup>1</sup>If you did not answer Question 3, consider  $\bar{R}_p = 0.035 + 0.48\sigma_p$  as the CML for Question 4.

**Solution to Problem 1 (cont.):**

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**Problem 2 (60 points)**

Consider the following returns associated with two risky investments  $X$  and  $Y$ .

Prob	X Return	Y Return
0.25	0%	20%
0.50	5%	8%
0.25	30.65%	-1.1%

1. Identify possible first, second and third order dominances. Interpret. .... [15p]
2. Mr. Logarithm is an investor whose preferences are well described by  $U(W) = \ln(W)$ .
  - (a) Evaluate Mr. Logarithm's risk profile in terms of absolute and relative risk aversion. Financially interpret all results. .... [10p]
  - (b) When seen as alternative investments, what is the best choice for Mr. Logarithm?  $X$  or  $Y$ ? Explain. .... [5p]
  - (c) Show that, for an initial investment of  $W_0$ , the certainty equivalent and risk premium of Mr. Logarithm associated with investment  $X$  are given by

$$c = W_0 e^{0.09123} \quad \text{and} \quad \pi = W_0 (1.101625 - e^{0.09123}) .$$

Explain all steps. .... [10p]

3. Consider now combinations of  $X$  and  $Y$  with risk-free rate for deposits  $R_f = 5\%$ .
  - (a) Determine the correlation between the returns of  $X$  and  $Y$ . Based upon that information should one also consider combinations of  $X$  and  $Y$ ? Why or why not?..... [5p]
  - (b) Derive and sketch the efficient frontier in this market. .... [7.5p]
  - (c) What is the optimal portfolio for an investor who wishes to maximize long-term growth? Explain. .... [7.5p]

**Solution to Problem 2**

**Solution to Problem 2 (cont.)**



**Solution to Problem 2 (cont.)**

Extra space if needed (clearly identify your answers)