



Investments and Portfolio Management

Raquel M. Gaspar

February 4 2022

Closed-book exam. You can only use the official formulas' sheet.

Duration: 2.5h

.....
Name:

.....
Number:

MULTIPLE CHOICE (25 points)

There is only one correct answer per question. Mark your choices with circles. Each correct answer is worth 5 points and for each incorrect answer there is a deduction of 1.5 points.

1. The exchange markets and over the counter markets are considered as two types of
 - a) floating markets.
 - b) riskier markets.
 - c) secondary markets.
 - d) primary markets.

2. Consider the situation of a continuous market where the best pending bid is 25 euros and the bid-ask spread is 2 euros. If a market buy order is placed, we can conclude
 - a) That order is fully satisfied at 25 euros.
 - b) At least part of the order will be filled at 27 euros.
 - c) The next market price is higher than 25 but lower than 27
 - d) None of the above.

3. Financial Indices are:
 - a) Nothing but statistics about financial markets.
 - b) Real-life portfolios managed by the entities computing the indices.
 - c) Securities any investor can invest in.
 - d) None of the above.

4. The portfolio with the smallest VaR (value-at-risk) is also:
 - a) The safest portfolio according to Roy.
 - b) The safest portfolio according to Kataoka.
 - c) The safest portfolio according to Telser.
 - d) VaR has no relation to the safety first criteria of Roy, Kataoka or Telser.

5. Investment A stochastically dominates the investment B, considering second-order dominance.
 - a) Then A also dominates B when we consider first-order dominance.
 - b) The distribution function of returns of B is always higher or equal to that of A.
 - c) Investment A is also the preferred investment according to safety first criteria.
 - d) None of the above.

GROUP I (30 points)

1. Consider combinations of just (several) risky assets. Explain the similarities and differences between the safety criteria of Roy, Kataoka and Telser. Assuming the necessary conditions hold represent the three criteria in mean-variance space. [15p]

Answer:

2. Choose ONE of the following statements and discuss whether it is true or false: [15p]
- I. *Since the correlation between stocks and bonds' returns is essentially zero, mean-variance theory is useless to determine the way mixed funds should built their portfolios.*
 - II. *We can rewrite any multi-index model with correlated indices as another equivalent multi-index model with uncorrelated indices.*

GROUP II (20 points)

Consider a general utility function $U(W)$, where W is the end-of-investment wealth.

Show that using a second order Taylor approximation around the initial investment W_0 , the associated risk tolerance function approximation reflect the same preferences as

$$f(\bar{R}, \sigma) = \bar{R} - \frac{1}{2}r_0 [\bar{R}^2 + \sigma^2]$$

where r_0 stands for the coefficient of relative risk aversion evaluated at W_0 and, as usual, \bar{R} and σ are the expected return and volatility over the investment period.

GROUP III (125 points)

Problem 1 (65 points)

In a given market, all *efficient portfolios* are well described by the expression

$$\bar{R}_p = 3.5\% + 0.3436\sigma_p .$$

1. Based upon the expression of the efficient frontier, what can you conclude about (a) the existence (or not) of the risk free asset; and (b) the Sharpe ratio of the tangent portfolio T ? Explain your answer. [10p]
2. We know Mr. Silva preferences are well represented by $U(W) = 50W - 0.01W^2$ and that he wishes to invest 1 000 euros in this market.
 - (a) What are the absolute and relative risk aversion coefficients of Mr.Silva before investment? How can you interpret the his absolute and relative risk aversion functions? [10p]
 - (b) What is the optimal risk level for Mr. Silva? [15p]
3. Knowing that the tangent portfolio has only two assets with return distributions as in the following table

Scenarios	Probability	Asset 1	Asset 2
Bad	0.25	-5%	10%
Average	0.5	0%	-5%
Good	0.25	50%	35%

- (a) Determine the composition and risk of the tangent portfolio T ? [15p]
- (b) What is the composition and expected return of Mr.Silva's optimal portfolio O ?..... [10p]
- (c) Would the optimal portfolio of Mr. Silva change if the active risk free rate would increase to 10%? Explain without computations.. [5p]

Solution to Problem 1:

Solution to Problem 1 (cont.):

Solution to Problem 1 (cont.):

Problem 2 (60 points)

Consider two Gaussian risky assets with $\bar{R}_1 = 12\%$, $\bar{R}_2 = 6\%$, $\sigma_1 = 20\%$, $\sigma_2 = 15\%$ and $\rho = +0.5$. Shortselling is allowed without bound but it is not possible to get a loan to invest in risky assets. Still, there exist a riskless rate $R_f = 3\%$ for deposits.

1. Sketch the investment opportunity set (IOS) and the efficient frontier (EF) in the mean-variance plan. Explain..... [7.5p]

2. Show that the equations of the efficient frontier are given by

$$EF : \begin{cases} \bar{R}_p = 0.03 + 0.451 \sigma_p & \text{for } \sigma_p \leq 21.36\% \\ \sigma_p^2 = 9.0278 \bar{R}_p^2 - 1.3333 \bar{R}_p + 0.07 & \text{for } \sigma_p > 21.36\% \end{cases}$$

..... [12.5p]

3. Consider that Mr. Exact wants a portfolio E with $\bar{R}_E = 15\%$ and $\sigma_E = 28\%$.

- (a) What can you conclude about the efficiency of E ? Explain. [5p]
- (b) Find out the composition of portfolio E . Motivate all steps. [10p]
- (c) Suggest an alternative to portfolio E that Mr. Exact would always accept, no matter his risk profile. Explain. [7.5p]

4. Suppose now a new financial institution, Safety Bank, appears in this market. The Safety Bank is willing to give credit to investments in financial markets at a 3% interest rate, provided the probability of not getting paid (capital plus interest) is not higher than 10%.

- (a) Write down the Safety Bank credit condition and represent it in mean-variance space. [recall that for $z \sim N(0, 1)$ $Pr(z \leq -1.2816) \leq 10\%$] [7.5p]
- (b) Find the Telser portfolio satisfying the Safety Bank restriction..... [5p]
- (c) What type of investors will use the credit services of Safety Bank? Explain. [5p]

Solution to Problem 2:

Solution to Problem 2:

Extra Space (if needed):