Investments and Portfolio Management

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Suggested Solutions

MULTIPLE CHOICE (25 points)

Correct answers in bold.

1. The exchange markets and over the counter markets are considered as two types of
   a) floating markets.
   b) riskier markets.
   c) secondary markets.
   d) primary markets.

2. Consider the situation of a continuous market where the best pending bid is 25 euros and the bid-ask spread is 2 euros. If a market buy order is placed, we can conclude
   a) That order is fully satisfied at 25 euros.
   b) At least part of the order will be filled at 27 euros.
   c) The next market price is higher than 25 but lower than 27
   d) None of the above.

3. Financial Indices are:
   a) Nothing but statistics about financial markets.
   b) Real-life portfolios managed by the entities computing the indices.
   c) Securities any investor can invest in.
   d) None of the above.

4. The portfolio with the smallest VaR (value-at-risk) is also:
   a) The safest portfolio according to Roy.
   b) The safest portfolio according to Kataoka.
   c) The safest portfolio according to Telser.
   d) VaR has no relation to the safety first criteria of Roy, Kataoka or Telser.

5. Investment A stochastically dominates the investment B, considering second-order dominance.
   a) Then A also dominates B when we consider first-order dominance.
   b) The distribution function of returns of B is always higher or equal to that of A.
   c) Investment A is also the preferred investment according to safety first criteria.
   d) None of the above.

GROUP I (30 points)

1. Consider combinations of just (several) risky assets. Explain the similarities and differences between the safety criteria of Roy, Kataoka and Telser. Assuming the necessary conditions hold represent the three criteria in mean-variance space. .... [15p]

Answer:

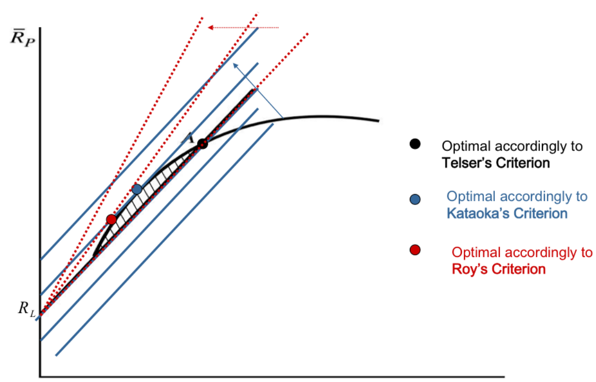
The criteria of Roy, Kataoka and Telser are all used to help choosing portfolios for investors that are mainly worried with safety. However each of the criteria interprets safety in its own way.

Roy criterium is appropriate for investors who are extremely averse to returns below a limit R_L and wish to minimize the probability of occurrence of that event. Kataoka criterium should be used for investors that express their concerns in terms of the worst $\alpha\%$ outcomes/scenarios and choose portfolios that maximize the $\alpha\%$ quantile of the returns distribution. Telser criterium should be applied whenever investors like to say both R_L and $\alpha\%$ requiring one should only consider portfolios that have a probability of returns lower or equal to R_L smaller than $\alpha\%$. In Telser's case if more than one portfolio satisfies the safety constraint one should then pick the one with maximum expected return, since the investor's concern about risk was already considered.

With Gaussian portfolio returns, we have $\Pr\left(R_p \leq R_L\right) = \Pr\left(\frac{R_p - \bar{R}_p}{\sigma_p} \leq \frac{R_L - \bar{R}_p}{\sigma_p}\right) = \Phi\left(\frac{R_L - \bar{R}_p}{\sigma_p}\right)$. So, all safety criteria can be understood in terms of the *safety line*

$$\Pr\left(R_p \leq R_L\right) = \alpha \quad \Leftrightarrow \quad \Phi\left(\frac{R_L - \bar{R}_p}{\sigma_p}\right) = \alpha \quad \Leftrightarrow \quad \bar{R}_p = R_L - \Phi\left(\alpha\right)\sigma_p\ .$$

For a fixed R_L minimising alpha is equivalent to maximising the safety line slope, so the Roy portfolio is a "tangent" portfolio. Maximising R_L for fixed α (Kataoka) keeps the slope fixed but chooses the highest possible y-cross, so Kataoka is also a "tangent" portfolio. Finally, for fixed R_L and α (Telser) all portfolios above the safety line are sufficiently safe, but maximising expected return also guarantees the Telser portfolio is on the envelop hyperbola.



2. Choose ONE of the following statements and discuss whether it is true or false: .... [15p]

I. *Since the correlation between stocks and bonds' returns is essentially zero, mean-variance theory is useless to determine the way mixed funds should built their portfolios.*
Comment: FALSE.
Even if that correlation would be zero, there would exist a important diversification effect that makes always worth to use mean-variance theory.
Recall that even in a world of just two assets, combinations of independent assets tend to be more efficient that any single asset investment (the exception occurs only for the case of 100% investment in the riskier asset and only if shortselling is forbidden). If that is the case of $N = 2$ it is even more so when considering various stocks and bonds.
The only situation when there is no diversification effect is when all risky assets are positively and perfectly correlated with one another. But in that only one asset in not redundant.

II. *We can rewrite any multi-index model with correlated indices as another equivalent multi-index model with uncorrelated indices. We can rewrite any multi-index model with correlated indices as another equivalent multi-index model with uncorrelated indices.*
Comment: TRUE.
Real life indices tend to be correlated with one another and cannot be used directly in multi-index models computations. One can, nonetheless, always extract from index number 2 the information already contained in index number 1, then from index number 3 the information contained in 1 and 2 and so forward. In doing so we obtain equivalent but uncorrelated indices that can be used in the context of the standard multi-index setup. This process is called orthogonalization of the original indices and it is always possible.

GROUP II (20 points)

Consider a general utility function $U(W)$, where W is the end-of-investment wealth.

Show that using a second order Taylor approximation around the initial investment W_0, the associated risk tolerance function approximation reflect the same preferences as

$$f(\bar{R}, \sigma) = \bar{R} - \frac{1}{2} r_0 \left[\bar{R}^2 + \sigma^2\right]$$

where r_0 stands for the coefficient of relative risk aversion evaluated at W_0 and, as usual, $\bar{R}$ and σ are the expected return and volatility over the investment period.

Proof.
Let us perform a second order Taylor approximation of the utility function $U(W)$ around W_0

$$U(W) \approx U(W_0) + U'(W_0)\,(W - W_0) + \frac{1}{2} U''(W_0)\,(W - W_0)^2 \ .$$

We also know $W = W_0(1 + R)$ and that the risk tolerance function is $\mathbb{E}\,[U(W)]$, so we can derive a risk tolerance function approximation by using the second-order approximation of the utility function

$$\begin{aligned}
\mathbb{E}\,[U(W)] &\approx \mathbb{E}\left[U(W_0) + U'(W_0) W_0 R + \frac{1}{2} U''(W_0) W_0^2 R^2\right] \\
&\approx U(W_0) + U'(W_0) W_0 \mathbb{E}\,[R] + \frac{1}{2} U''(W_0) W_0^2 \mathbb{E}\left[R^2\right] \\
&\approx \underbrace{U(W_0) + U'(W_0) W_0 \bar{R} + \frac{1}{2} U''(W_0) W_0^2 \left(\bar{R}^2 + \sigma^2\right)}_{g(\sigma,\bar{R})}
\end{aligned}$$

where we have used the notation $\bar{R} = \mathbb{E}\,[R]$, $\sigma^2 = \mathrm{Var}(R)$ and the fact $\mathrm{Var}(R) = \mathbb{E}\left[R^2\right] - \bar{R}^2$. It remains to show the expression that resulted from this derivation

$$g(\sigma, \bar{R}) = U(W_0) + U'(W_0) W_0 \bar{R} + \frac{1}{2} U''(W_0) W_0^2 \left(\bar{R}^2 + \sigma^2\right)$$

reflect the same preferences as $f(\sigma, \bar{R}) = \bar{R} - \dfrac{1}{2} r_0 \left[\bar{R}^2 + \sigma^2\right]$.

To see this we use the fact that r_0 is the coefficient of relative risk aversion evaluated at W_0, so

$$r_0 = -\frac{U''(W_0) W_0}{U(W_0)} \qquad \text{and} \qquad f(\sigma, \bar{R}) = \bar{R} + \frac{1}{2} \frac{U''(W_0) W_0}{U(W_0)} \left[\bar{R}^2 + \sigma^2\right] \ .$$

Comparing now f and g we see that

$$f(\sigma, \bar{R}) = \underbrace{\frac{1}{W_0 U'(W_0)}}_{b}\, g(\sigma, \bar{R}) - \underbrace{\left(\frac{U(W_0)}{W_0 U'(W_0)}\right)}_{a}$$

i.e. f is a linear combination of g since a and b are constants. Moreover, since $b > 0$ for all investors who prefer more to less (since $U'(W_0) > 0$), indeed, f an g represent the same preferences.

GROUP III (125 points)

Problem 1 (65 points)

In a given market, all *efficient portfolios* are well described by the expression

$$\bar{R}_p = 3.5\% + 0.3436\sigma_p \ .$$

1. Based upon the expression of the efficient frontier, what can you conclude about (a) the existence (or not) of the risk free asset; and (b) the Sharpe ratio of the tangent portfolio T? Explain your answer. . . . . [10p]
Solution:
Since the expression for the efficient frontier is a straight line we know

$$\bar{R}_p = R_F + \frac{\bar{R}_T - R_F}{\sigma_T}\sigma_p \ ,$$

which tells us that: (a) in this market there is a risk-free asset and that borrowing and lending is possible at the exact same rate $R_F = 3.5\%$, also (b) since the slope of the efficient frontier equals the Sharpe ratio of the tangent portfolio we have $SR_T = \dfrac{\bar{R}_T - R_F}{\sigma_T} = 0.3436$

2. We know Mr. Silva preferences are well represented by $U(W) = 50W - 0.01W^2$ and that he wishes to invest 1 000 euros in this market.

   (a) What are the absolute and relative risk aversion coefficients of Mr.Silva before investment? How can you interpret the his absolute and relative risk aversion functions? . . . . [10p]
   Solution:
   Mr. Silva has a quadratic utility function. For his particular function we have:

   - $U'(W) = 50 - 2(0.01)W > 0$ for wealth levels that satisfy $W < \dfrac{50}{0.02} = 2500$. So, for an interval big enough around his initial wealth he prefers more to less.
   - $U''(W) = -0.02 < 0$. From this we conclude Mr. Silva is risk averse.
   - $A(W) = -\dfrac{U''(W)}{U'(W)} = \dfrac{0.02}{50 - 0.02W}$. Evaluating this function at the initial wealth $W_0 = 1000$ we get his absolute risk aversion coefficient before investment $A(1000) = \frac{0.02}{50-0.02\times1000} = \frac{0.02}{30}$. Taking the first derivative of the absolute risk aversion function we get $A'(W) = \frac{0.0004}{(50-0.02W)^2} > 0$ and we can conclude Mr. Silva has increasing absolute risk aversion, i.e. when his wealth increases he will decrease the amount of euros invested in risky assets.
   - $R(W) = A(W)W = \dfrac{0.02W}{50 - 0.02W}$. Evaluating this function at the initial wealth $W_0 = 1000$ we get his relative risk aversion coefficient before investment $R(1000) = \frac{0.02\times1000}{50-0.02\times1000} = \frac{20}{30}$. Taking the first derivative of the relative risk aversion function we get $R'(W) = \frac{50}{(50-0.02W)^2} > 0$. Not surprisingly (given his increasing absolute risk aversion) Mr.Silva also has increasing relative risk aversion, i.e. when his wealth increases he keeps a smaller portion of his wealth in risky assets.

(b) What is the optimal risk level for Mr. Silva? .... [15p]
Solution:
To find Mr.Silva's optimal risk level we have to maximize his risk tolerance function, subject to the efficient frontier.

$$\max_{\sigma,\bar{R}} f(\sigma,\bar{R}) \quad s.t. \quad \bar{R} = 3.5\% + 0.3436\sigma$$

Including the restriction in the objective function we get

$$f(\sigma,\bar{R})|_{R=3.5\%+0.3436\sigma} = 40000+30000(3.5\%+0.3436\sigma)-10000\sigma^2-10000(3.5\%+0.3436\sigma)^2$$

This new restricted f function, depends only on σ. So to get its maximum we need to take its first derivative w.r.t. σ and set it to zero

$$\begin{aligned}
\frac{\partial f}{\partial \sigma^*} &= 0\\
30000\times 0.03436 - 20000\sigma^* - 20000(0.035+0.03436\sigma^*)0.3436 &= 0\\
3\times 0.3436 - 2\sigma^* - 2\times 0.3436\,[0.035+0.3436\sigma^*] &= 0\\
\sigma^* &= 23.13\%
\end{aligned}$$

3. Knowing that the tangent portfolio has only two assets with return distributions as in the following table

Scenarios	Probability	Asset 1	Asset 2
Bad	0.25	-5%	10%
Average	0.5	0%	-5%
Good	0.25	50%	35%

(a) Determine the composition and risk of the tangent portfolio T? .... [15p]
Solution
We start by computing the inputs to mean-variance theory

$$\begin{aligned}
\bar{R}_1 &= 0.25(-5\%)+0.5(0\%)+0.25(50\%) = 11.25\%\\
\bar{R}_2 &= 0.25(10\%)+0.5(-5\%)+0.25(35\%) = 8.75\%\\
\sigma_1^2 &= 0.25(-5\%-11.25\%)^2+0.5(0\%-11.25\%)^2+0.25(50\%-11.25\%)^2 = 0.05047\\
&\quad \Rightarrow \quad \sigma_1 22.46\%\\
\sigma_2^2 &= 0.25(10\%-8.75\%)^2+0.5(-5\%-8.75\%)^2+0.25(35\%-8.75\%)^2 = 0.02672\\
&\quad \Rightarrow \quad \sigma_2 = 16.35\%\\
\sigma_{12} &= 0.25(-5\%-11.25\%)(-5\%-11.25\%)+0.5(0\%-11.25\%)(-5\%-8.75\%)+\\
&\quad +0.25(50\%-11.25\%)(35\%-8.75\%) = 0.03265
\end{aligned}$$

From before we also know there is a risk-free asset with $R_F = 3.5\%$. The tangent portfolio is the one that maximizes the Sharpe ratio which is the same as solving a linear system of equations in z_1, z_2 which are proportional to the optimal weights

$$\begin{cases}\bar{R}_1 - R_f = \sigma_1^2 z_1 + \sigma_{12} z_2\\ \bar{R}_2 - R_f = \sigma_{12} z_1 + \sigma_2^2 z_2\end{cases} \Rightarrow \begin{cases}11.25\% - 3.5\% = 0.05047 z_1 + 0.03265 z_2\\ \bar{R}_2 - R_f = 0.03265 z_1 + 0.02672 z_2\end{cases} \Leftrightarrow \begin{cases} z_1 = 1.263158\\ z_2 = 0.421053\end{cases}$$

Since z_1, z_2 are proportional to the tangent portfolio weights we can easily find them

$$x_1^T = \frac{z_1}{z_1 + z_2} = \frac{1.263158}{1.263158 + 0.421053} = 75\% \qquad x_2^T = \frac{z_2}{z_1 + z_2}\frac{0.421053}{1.263158 + 0.421053} = 25\%$$

The expected return as risk of the tangent portfolio are as follows

$$\begin{aligned}
\bar{R}_T &= 0.75 \times 11.25\% + 0.25 \times 8.75\% = 10.625\% \\
\sigma_T^2 &= 0.75^2 \times 0.05047 + 0.25^2 \times 0.02672 + 2 \times 0.75 \times 0.25 \times 0.03265 = 0.0423 \\
\sigma_T &= 20.57\%
\end{aligned}$$

An alternative to compute the tangent portfolio's volatility would be to use its expected return $\bar{R}_T$ and the equation for the efficient frontier

$$10.625\% = 3.5\% + 0.3436\sigma_T \Leftrightarrow \sigma_T = 20.57\% \ .$$

(b) What is the composition and expected return of Mr.Silva's optimal portfolio O? . . . . . [10p]
Solution:
From before we know the optimal risk level of Mr. Silva is 23.13%. This is a point in the efficient frontier, so the optimal portfolio expected return is

$$\bar{R}^* = 3.5\% + 0.3436 \times 23.13\% = 11.51\% \ .$$

The optimal portfolio is a particular combination of the risk-free asset and the tangent portfolio. We find out the exact composition by solving

$$11.51\% = 3.5\% x_F + (1 - x_F)10.625\% \quad \Leftrightarrow \quad x_F = -12.45\% \quad \Rightarrow x_T = 112.45\% \ .$$

The optimal for Mr.Silva is to take a loan (of about 12.45% of his initial investment) to leverage a bit his position and invest 112.45% in the tangent portfolio.

(c) Would the optimal portfolio of Mr. Silva change if the active risk free rate would increase to 10%? Explain without computations.. . . . . [5p]
Solution:
Yes it would change since the current optimal portfolio involves taking a loan. Possibly at the new active rate he is no longer interested in taking a loan. His new optimum is most likely a combination of the tangent portfolio with a second portfolio belonging to the hyperbola that is the frontier of the investment opportunity set of risky assets.

Problem 2 (60 points)

Consider two Gaussian risky assets with $\bar{R}_1 = 12\%, \bar{R}_2 = 6\%$, $\sigma_1 = 20\%, \sigma_2 = 15\%$ and $\rho = +0.5$. Shortselling is allowed without bound but it is not possible to get a loan to invest in risky assets. Still, there exist a riskless rate $R_f = 3\%$ for deposits.

1. Sketch the investment opportunity set (IOS) and the efficient frontier (EF) in the mean-variance plan. Explain. . . . . [7.5p]

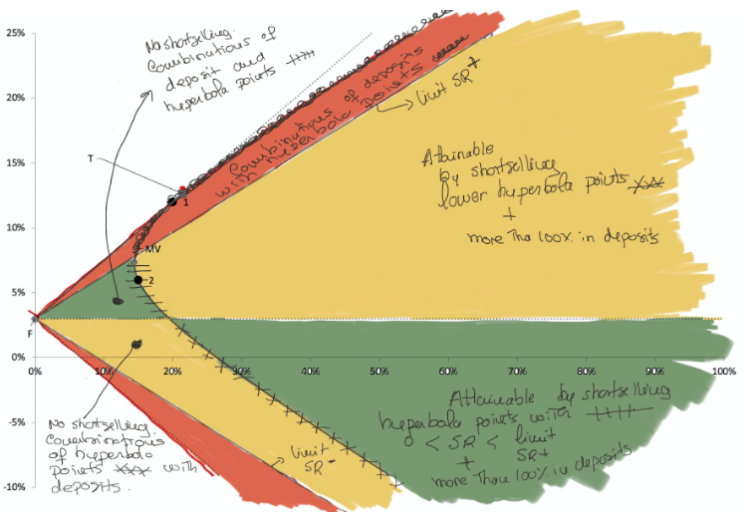
   Solution:

   Since we have two risky assets plus a deposit (3 assets), and shortselling is allowed *without limits*, the IOS will be an open set (area), even if it is not possible to ask for a loan to invest in risky assets.

   The IOS is limited from below by a straight with slope equal to the symmetric of the tangent portfolio's Sharpe ratio.

   It is limited from above by the segment of the straight line connecting the deposit to the tangent portfolio (T) for volatilitiies lower than σ_T, and by a segment of the envelop hyperbola upper part, for volatilities higher than σ_T. This limit from above is the EF.

   All points inside the lower and upper limits of the IOS are attainable by combining hyperbola risky points with deposit, although some require extreme shortselling of hyperbola points. See explanation inside Figure below.

2. Show that the equations of the efficient frontier are given by

$$EF: \quad \begin{cases} \bar{R}_p = 0.03 + 0.451\ \sigma_p & \text{for } \sigma_p \leq 21.36\% \\ \sigma_p^2 = 9.0278\ \bar{R}_p^2 - 1.3333\ \bar{R}_p + 0.07 & \text{for } \sigma_p > 21.36\% \end{cases}$$

   . . . . [12.5p]

   Solution:

   As explained in Question 1, the EF is given by the segment of a straight line connecting the deposit with the tangent portfolio T and by the upper part of the envelop hyperbola for higher risk levels. So we start by determining the portfolio T

$$Z = V^{-1}\left(\bar{R} - R_f \mathbb{1}\right) = \begin{pmatrix} 2.3333 \\ -0.2222 \end{pmatrix} \qquad \Rightarrow \qquad X_T = \begin{pmatrix} 1.1053 \\ -0.1053 \end{pmatrix},$$

and conclude it requires shortselling of -20.53% of asset 2 to invest 110.53% in asset 1. Portfolio T has expected return, volatility and Sharpe ratio given by

$$\bar{R}_T = X'\bar{R} = \begin{pmatrix} 1.1053 & -0.1053 \end{pmatrix} \begin{pmatrix} 0.12 \\ 0.06 \end{pmatrix} = 12.63\%$$

$$\sigma_T^2 = X'VX = \begin{pmatrix} 1.1053 & -0.1053 \end{pmatrix} \begin{pmatrix} 0.04 & 0.015 \\ 0.015 & 0.0225 \end{pmatrix} \begin{pmatrix} 1.1053 \\ -0.1053 \end{pmatrix} = 0.04562 \quad \Rightarrow \quad \sigma_T = 21.36\%$$

$$SR_T = \frac{\bar{R}_T - R_f}{\sigma_T} = \frac{0.1263 - 0.03}{0.2136} = 0.451 \ .$$

Then we use the envelop hyperbola equation $\sigma_p^2 = \frac{A\bar{R}_p^2 - 2B\bar{R}_p + C}{AC - B^2}$, where for this case we have

$$\begin{aligned} A &= \mathbb{1}'V^{-1}\mathbb{1} = 48.1481 \\ B &= \mathbb{1}'V^{-1}\bar{R} = \ 3.5556 \qquad \Rightarrow \qquad \sigma_p^2 = 9.0278\ \bar{R}_p^2 - 1.3333\ \bar{R}_p + 0.07 \\ C &= \bar{R}'V^{-1}\bar{R} = \ 0.3733 \ . \end{aligned}$$

Finally, the exact expression for the EF is given by,

$$EF: \quad \begin{cases} \bar{R}_p = 0.03 + 0.0451\ \sigma_p & \text{for } \sigma_p \leq 21.36\% \\ \sigma_p^2 = 9.0278\ \bar{R}_p^2 - 1.3333\ \bar{R}_p + 0.07 & \text{for } \sigma_p > 21.36\% \end{cases} .$$

3. Consider that Mr. Exact wants a portfolio E with $\bar{R}_E = 15\%$ and $\sigma_E = 28\%$.

   (a) What can you conclude about the efficiency of E? Explain. .... [5p]
   Solution:
   For that level of risk efficient portfolios lie on the envelop hyperbola. For $\bar{R}_E = 15\%$ we know the efficient risk is given by

   $$\sigma_p^2 = 9.0278 \times (15\%)^2 - 1.3333 \times (15\%) + 0.07 = 0.0731 \qquad \Rightarrow \sigma_p = 27.04\% \ ,$$

   so, we can conclude portfolio E is in the interior of the envelope hyperbola.

   (b) Find out the composition of portfolio E. Motivate all steps. .... [10p]
   Solution:
   To find the exact composition of portfolio E we need to remember that any point inside the hyperbola is a combination of an hyperbola point with deposit.
   It is possible to find the exact hyperbola point(s) by finding first the Sharpe Ratio of portfolio E, $SR_E = \frac{\bar{R}_E - R_f}{\sigma_E} = \frac{0.15-0.03}{0.28} = 0.4286$ and then finding the hyperbola points with the same Sharpe Ratio (crossing points between the straight line that gives use all combinations of E with deposit).
   In this case we have,

   $$\begin{cases} \bar{R}_p = 0.03 + 0.4286\sigma_p \\ \sigma_p^2 = 9.0278\ \bar{R}_p^2 - 1.3333\ \bar{R}_p + 0.07 \end{cases} \qquad \Rightarrow \qquad \begin{cases} \bar{R}_{S1} = 0.101 \\ \bar{R}_{S2} = 0.18 \end{cases} .$$

   Since we cannot borrow, portfolio E cannot be attained by using the first solution $S1$, so we need to use solution $S2$, and see portfolio E as a combination of $S2$ and deposit. See Figure at the end of the exercise

Using $\bar{R}_{S2} = 18\%$ and the hyperbola equation we can find $\sigma_{S2} = 35\%$. So, point E can be reached by investing $x_{S2} = \frac{0.18}{0.35} = 0.8$ in $S2$ and $x_f = 0.2$ in deposit.

Finally we can find the exact composition of $S2$, because it is the combination of the basic assets 1 and 2 that has 18% expected return. From the Figure it is also clear $S2$ requires shortselling of asset 2. Concretely, $X'_{S2} = \begin{pmatrix} 2 & , & -1 \end{pmatrix}$.

Portfolio E is, thus, attainable by investing 20% in deposit, 160% in asset 1 and -80% in asset 2!

(c) Suggest an alternative to portfolio E that Mr. Exact would always accept, no matter his risk profile. Explain. .... [7.5p]

Solution:

Since all investors prefect more to less, all investors (also Mr.Exact) would prefer a portfolio with the exact same risk as portfolio E,$\sigma_E = 28\%$,but on the efficient frontier. Using once again the hyperbola equation we get

$$(0.28)^2 = 9.0278\ \bar{R}_p^2 - 1.3333\ \bar{R}_p + 0.07 \Leftrightarrow \begin{cases} \bar{R}_{P1} = 0.1537 \\ \bar{R}_{P2} = -0.61 \end{cases} .$$

Only the efficient solution matters so, we can propose a portfolio $P1$ with $\bar{R}_{P1} = 15.37\%$ and we can attain it by investing $x_1 = 156,24\%$ and $x_2 = -56.24\%$. See Figure at the end of the exercise.

4. Suppose now a new financial institution, Safety Bank, appears in this market. The Safety Bank is willing to give credit to investments in financial markets at a 3% interest rate, provided the probability of not getting paid (capital plus interest) is not higher than 10%.

(a) Write down the Safety Bank credit condition and represent it in mean-variance space. [recall that for $z \sim N(0,1)$ $Pr(z \leq -1.2816) \leq 10\%$] .... [7.5p]

Solution: The Safety bank requires full payment of capital plus interest. That is only possible if the portfolio one invests in has at least 3% return,

$$\begin{aligned}
&\mathbb{P}(R_p \leq 0.03) \leq 0.1 \\
&\mathbb{P}\left(\frac{R_p - \bar{R}_p}{\sigma_p} \leq \frac{0.03 - \bar{R}_p}{\sigma_p}\right) \leq 0.1 \\
&\mathbb{P}\left(z \leq \frac{0.03 - \bar{R}_p}{\sigma_p}\right) \leq 0.1 \\
&\Phi\left(\frac{0.03 - \bar{R}_p}{\sigma_p}\right) \leq 0.1 \\
&\frac{0.03 - \bar{R}_p}{\sigma_p} \leq \underbrace{\Phi^{-1}(0.1)}_{-1.2816} \\
&\bar{R}_p \geq 0.03 + 1.2816\sigma_p
\end{aligned}$$

Notice that since the Telser restriction has higher slope than the EF.

(b) Find the Telser portfolio satisfying the Safety Bank restriction. .... [5p]

Solution: Since the Telser restriction has higher slope than the EF, the only point satisfying the condition would be 100% investment in the riskless asset! See figure below.

(c) What type of investors will use the credit services of Safety Bank? Explain. . . . . [5p]
Solution:
Since the tangent portfolio is also the Roy portfolio for $R_L = R_f$ (which is the case), it is the portfolio with the lowest probability of returns lower than 3% and its Sharpe ratio is much lower than 1.2816 (slope of the Telser restriction), its has a probability of returns lower than 3% higher than 10%. That is, in this case, the Telser restriction does not touch the IOS and therefore the Safety bank is useless in this market!