Macroeconometrics I - Exercises

1. Why is it important to test whether the time series used in a modelling exercise are stationary?

2. Show that the AR(1) process,

$$y_t = \alpha y_{t-1} + u_t, \quad \textit{with } u_t \sim iid(0, \sigma^2) \textit{ and } y_0 \text{ constant},$$

is weakly stationary when $|\alpha| < 1$ and nonstationary when $\alpha = 1$.

3. Show that the process,

$$\begin{aligned} y_t &= \mu + \beta t + u_t \\ u_t &= \alpha u_{t-1} + v_t \end{aligned}$$

can be rewritten as,

$$\Delta y_t = (\alpha - 1)y_{t-1} + (\mu(1-\alpha) + \alpha\beta) + (1-\alpha)\beta t + v_t.$$

4. A macroeconomist considered the following model for log US GDP,

$$A(L)(y_t - \delta_0 - \delta_1 t) = \epsilon_t$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2$ and $\epsilon_t \sim iid(0, \sigma^2)$. OLS estimation of this model gives,

$$y_t = -0.321 + 0.003t + 1.335y_{t-1} - 0.401y_{t-2} + u_t$$

a) Determine the values of $\alpha_1, \alpha_2, \delta_0$ and δ_1.

b) What is the value of $A(1)$ and what may this indicate?

5. Consider the two processes,

$$y_t = \alpha_0 + \alpha_1 t + \varepsilon_t$$

and

$$x_t = \delta_0 + x_{t-1} + u_t$$

where $\varepsilon_t \backsim iid(0, \sigma_\varepsilon^2)$ and $u_t \backsim iid(0, \sigma_u^2)$. Indicate whether y_t and x_t are stationary and justify your answer.

6. Consider the process,

$$y_t = \delta + y_{t-1} + \varepsilon_t \tag{1}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$ and y_0 constant. Indicate what type of DF test regression you would use if you suspected that your data had been generated by (1) and state the corresponding null and alternative hypotheses.

7. Consider the AR(3),

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \varepsilon_t \tag{2}$$

with $y_0 \neq 0$ and $\varepsilon_t \sim iid(0, \sigma^2)$.

   a) Assuming that this process may have at most one unit root, what DF type test regression would you use and why?
   b) In case there is a unit root in this process what restriction would it impose on the autoregressive parameters?

8. Consider the following estimated models,

$$y_t = \underset{(0.198)}{0.33} + \underset{(0.085)}{0.01}\, t + \underset{(0.019)}{0.81}\, y_{t-1} + \underset{(0.290)}{1.81}\, \Delta y_{t-1} - \underset{(0.381)}{1.61}\, \Delta y_{t-2}$$

and

$$\Delta x_t = \underset{(0.29)}{0.21} - \underset{(0.90)}{0.73} x_{t-1} + \underset{(0.324)}{1.55}\, \Delta x_{t-1} - \underset{(0.610)}{1.71}\, \Delta x_{t-2} + \underset{(0.101)}{0.71}\, \Delta x_{t-3}.$$

The numbers in brackets are the corresponding standard errors.

   a) Indicate the objective of estimating these regressions.
   b) What can you conclude from their analysis?

9. Assuming that you want to evaluate the behaviour of an economic time series with 250 observations, indicate which of the following models you would choose and why? What do you conclude with respect to the stationarity properties of the series under analysis?

$$i)\widehat{\Delta c_t} = \underset{(3.05)}{4.30} + \underset{(2.05)}{0.25t} + \underset{(-2.55)}{0.52}\ c_{t-1} + \underset{(3.05)}{2.22}\Delta c_{t-1} + \underset{(0.98)}{0.96}\Delta c_{t-2} + \underset{(1.05)}{1.32}\Delta c_{t-3} \tag{3}$$

$LM_1 = 2.35; \quad LM_2 = 4.35$

$$ii)\widehat{\Delta c_t} = \underset{(3.23)}{4.15} + \underset{(2.15)}{0.28t} + \underset{(-2.58)}{0.61}\ c_{t-1} + \underset{(3.15)}{2.62}\Delta c_{t-1} + \underset{(1.8)}{1.56}\Delta c_{t-2} \tag{4}$$

$LM_1 = 2.75; \quad LM_2 = 4.95$

$$iii)\widehat{\Delta c_t} = \underset{(3.5)}{4.50} + \underset{(2.5)}{0.5}t + \underset{(-2.60)}{0.70}\ c_{t-1} + \underset{(3.50)}{2.91}\Delta c_{t-1} \tag{5}$$

$LM_1 = 3.90; \quad LM_2 = 6.20$

Note: LM_1 and LM_2 are first and second order autocorrelation tests.

10. Consider the following two estimated models for US GDP for the period 1950-1970,

$$\widehat{\Delta y_t} = \underset{(0.837)}{1.654} + \underset{(0.007)}{0.0134t} - \underset{(0.196)}{0.374} y_{t-1} + \underset{(0.243)}{0.052}\Delta y_{t-1}$$

and

$$\widehat{\Delta y_t} = \underset{(0.705)}{1.574} + \underset{(0.006)}{0.0127t} - \underset{(0.165)}{0.355} y_{t-1}.$$

Note: The values in brackets correspond to the standard errors.

a) Indicate which of the two DF test regressions you would use.

b) What can you conclude with respect to the stationarity of GDP?

11. Consider the following model,

$$y_t = a + bx_t + e_t \tag{6}$$

where y_t and x_t are economic time series. Assuming that the following regression was estimated,

$$\Delta\widehat{e}_t = \underset{(-2.46)}{-0.421}\widehat{e}_{t-1} - \underset{(-1.87)}{0.277}\Delta\widehat{e}_{t-1} \tag{7}$$

using the residuals obtained from (6), indicate:

a) What is the objective of using a test regression as in (7)?

b) Given that the values in brackets in (7) correspond to the t-tests and that the critical value for the $t - test$ of the estimated parameter of $\widehat{e}_{t-1}$ is (-3.46) what can you conclude.