Foundations of Financial Economics
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Special Exam
2 September 2019
Total time: 1:30 hours. Total points: 20

Instructions:

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. In your answer explain all the steps in your reasoning. Keep answers short; I don't give more credit for long answers, and I can take points off if you add things that are wrong or irrelevant.

Formulas:

If x and y are random variables then

$$cov\left(x,y\right)=E\left(xy\right)-ExEy$$

$$\sigma^{2}\left(x\right)=Ex^{2}-\left(Ex\right)^{2}$$

If x is normal distributed then $\exp(x)$ is lognormal, and

$$E\exp(x)=\exp(Ex+0.5\sigma^{2}\left(x\right))$$

Questions:

1. [12 points]

Consider a representative agent economy. The consumer maximizes $E_0\sum_{t=0}^{\infty}\beta^t u(C_t)$ subject to the budget constraints $\sum_{j=1}^{T}Z_{j,t+1}B_{j,t}+\sum_{i=1}^{N}M_{i,t+1}S_{i,t}+C_t=\sum_{j=0}^{T-1}Z_{j,t}B_{j,t}+\sum_{i=1}^{N}M_{i,t}\left(D_{i,t}+S_{i,t}\right)$, for $t=0,1,...$, where C_t is the period t consumption, $S_{i,t}$ is the price of stock i, $D_{i,t}$ is the dividend of stock i, $B_{j,t}$ is the price of a zero coupon bond with maturity at $t+j$, $B_{0,t}=1$, and $Z_{j,t}$ and $M_{i,t}$ are the quantities of the j bond and of the i stock in period t.

(a) What are the first order conditions of the consumer's problem?

(b) Obtain from the first order conditions the equation

$$\frac{1}{R_{j,t+1}}=E_t\left\{\beta^j\frac{u'(C_{t+j})}{u'(C_t)}\right\}$$

where $R_{j,t+1}$ is the return from period t to period $t+j$ on a bond with maturity j.

(c) Obtain the equation

$$\frac{1}{R_{2,t+1}}=E_t\left\{\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{1}{R_{1,t+2}}\right\}=\frac{1}{R_{1,t+1}}E_t\left\{\frac{1}{R_{1,t+2}}\right\}+cov_t\left\{\beta\frac{u'(C_{t+1})}{u'(C_t)},\frac{1}{R_{1,t+2}}\right\}$$

Consider the utility function $u(C_t) = \dfrac{C_t^{1-\gamma}}{1-\gamma}$ and assume that the growth rate of consumption obeys the process $\Delta c_{t+1} = a + bs_t + \varepsilon_{t+1}$; where a and b are positive numbers, $\Delta c_{t+1} = c_{t+1} - c_t$, $c_t = \ln C_t$, $\varepsilon_{t+1} \sim i.i.d.\ N(\mu, \sigma^2)$, where s_t is a stochastic variable that assumes values 0 or 1. Assume that $s_t = 1$ indicates high growth and $s_t = 0$ indicates low growth. The probability of s_{t+1} given s_t is denoted by $\pi(s_{t+1} \mid s_t)$ and is independent of ε_t. In period t the values of s_t, c_t, and ε_t are known.

(d) Determine the expression for the one-period interest rate.

(e) Determine the expression for the two-period interest rate.

(f) Is it possible for the total return on the short-term bond to exceed the total return on the long-term bond? Explain.

2. [8 points]

Consider the economy model of Constantinides and Duffie (1996). The process for the growth rate of agent i's consumption is

$$g_{i,t+1} = g_{t+1} - \frac{1}{2}\sigma^2_{\varepsilon,t+1} + \varepsilon_{i,t+1},$$

where $g_{i,t+1} = \log(C_{i,t+1}/C_{i,t})$, $g_{t+1} = \log(C_{t+1}/C_t)$, C_t is the aggregate consumption and $\varepsilon_{i,t+1} \sim i.i.d.\ N\left(0, \sigma^2_{\varepsilon,t+1}\right)$. Each consumer has preferences

$$\sum_{t=0}^{\infty} e^{-\delta t} \frac{(C_{i,t})^{1-\gamma} - 1}{1-\gamma}$$

There are N stocks in the economy with prices $P_{k,t}$ and dividends $D_{k,t}$, for $k = 1, ..., N$.

(a) State the consumer i's problem.

(b) Obtain the first order conditions for consumer i.

(c) Obtain the pricing equation: $1 = E_t(m_{t+1}R_{k,t+1})$, where m_{t+1} is the discount factor and $R_{k,t+1}$ is the return on asset k. Write the expression for m_{t+1} as a function of the aggregate consumption.

(d) Explain why the risk aversion for the "average household" may be different from γ. Show that when the dispersion in the cross section of the growth rates of consumption increases in economic downturns, then the risk aversion for the "average household" is larger than γ.