Foundations of Financial Economics
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Final Exam
Total time: 1:30 hours. Total points: 20
Instructions:
Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. In your answer explain all the steps in your reasoning. Keep answers short; I don't give more credit for long answers, and I can take points off if you add things that are wrong or irrelevant.
Formulas:
If x and y are random variables then

$$E(xy) = ExEy + cov(x, y)$$

$$\sigma^2(x) = Ex^2 - (Ex)^2$$

If x is normal distributed then $\exp(x)$ is lognormal, and

$$E\exp(x) = \exp(Ex + 0.5\sigma^2(x))$$

Brownian motion

$$z_{t+\Delta} - z_t \sim N(0, \Delta)$$

Differential

$$dz_t = lim_{\Delta\searrow 0}(z_{t+\Delta} - z_t)$$

$$dz_t^2 = dt,\ dz_t dt = 0,\ dt^\alpha = 0,\ \text{if } \alpha > 1$$

$$E_t(dz_t) = 0, var_t(dz_t) = E_t(dz_t^2) = dz_t^2 = dt$$

Ito's Lemma

$$\begin{aligned} df(x,t) &= f_t dt + f_x dx + \frac{1}{2} f_{xx} dx^2 \\ &= \left(f_t + f_x\mu_x + \frac{1}{2} f_{xx}\sigma_x^2\right) dt + f_x\sigma_x dz \end{aligned}$$

1. (4 pts). (4 pts) 1. What is a complete market? Explain how we can complete the market with options.

2. Consider a model with power utility, $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, and $\beta = 1$. Assume that $\ln\left(\frac{C_{t+1}}{C_t}\right) \equiv \Delta c_{t+1}$ is normal distributed. Let m_{t+1} denote the stochastic discount factor.

(2 pts) a. Show that the ratio

$$\frac{\sigma(m_{t+1})}{E(m_{t+1})}$$

is approximately equal to $\gamma\sigma(\Delta c_{t+1})$.

(2 pts) b. Consider the market Sharpe ratio

$$\left|\frac{ER_m - R_f}{\sigma(R_m)}\right| \leq \frac{\sigma(m)}{E(m)}$$

where R_m is the return on the market portfolio. The postwar NYSE index excess return is around 8% per year, with standard deviation around 16%. The standard deviation of log consumption growth is about 1%. Is this a challenge to the model? Explain.

(2 pts) c. Assume that $E(\Delta c_{t+1}) = 2$. What do the data and the model imply for the riskless interest rate? Discuss.

3. Consider the Epstein and Zin utility model

$$V_t \equiv \left((1-\beta)C_t^{1-\gamma} + \beta\left(H_t\left(V_{t+1}\right)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$$

where, $H_t(V_{t+1}) = \left(E_t V_{t+1}^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$.

(2 pts) a. Show that when $\alpha = 0$ we have the usual standard time-separable expected discounted utility with discount factor β and risk aversion γ.

(2 pts) b. The stochastic discount factor with the Epstein and Zin utility is

$$m_{t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma\theta} R_{m,t+1}^{\theta-1}, \ \theta = \frac{1-\alpha}{1-\gamma}.$$

Assume that $\log\left(\frac{C_{t+1}}{C_t}\right) \equiv \Delta c_{t+1}$ is normal distributed, $\log(R_{m,t+1}) = r_{m,t+1}$ is normal distributed and they are independently distributed. What is the expression for the riskless interest rate?

(2 pts) c. Can these preferences explain the risk free rate puzzle?

(4 pts) 4. Let there be N assets with payoffs over S states of nature given by $\mathbf{R}_n = (R_{n1}, R_{n2}, ..., R_{nS}) \in \mathcal{R}^S$ for $n = 1, ..., N$. Let asset prices $\mathbf{P} = (P_1, P_2, ..., P_N) \in \mathcal{R}^N$ be given by $P_n = \sum_{s=1}^{S} \lambda_s R_{ns}$, for some $\lambda_s > 0$, $s = 1, ..., S$ and $n = 1, ..., N$. Show that $\mathbf{P}$ is arbitrage free. What is the relationship between the λ_s and the contingent claims?