Foundations of Financial Economics
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Final Exam
3 July 2019
Total time: 1:30 hours. Total points: 20

Instructions:

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. In your answer explain all the steps in your reasoning. Keep answers short; I don't give more credit for long answers, and I can take points off if you add things that are wrong or irrelevant.

Formulas:

If x and y are random variables then

$$cov\left(x,y\right)=E\left(xy\right)-ExEy$$

$$\sigma^{2}\left(x\right)=Ex^{2}-\left(Ex\right)^{2}$$

If x is normal distributed then $\exp(x)$ is lognormal, and

$$E\exp(x)=\exp(Ex+0.5\sigma^{2}\left(x\right))$$

Questions:

(2 pts) 1. Consider a representative agent economy with N assets. The dividend of asset i is $D_{i,t}$ and its price is

$$P_{i,t}=E_{t}\left\{\sum_{s=0}^{\infty}\frac{\beta^{s}u'(C_{t+1+s})}{u'(C_{t})}D_{i,t+1+s}\right\},\ \beta<1,\ \text{for } i=1,...,N, \qquad (1)$$

where $u'(C_t)$ is the period t marginal utility of the representative agent. Use expression (1) to get

$$P_{i,t}=E_{t}\left\{\frac{\beta u'(C_{t+1})}{u'(C_{t})}\left(D_{i,t+1}+P_{i,t+1}\right)\right\},\ \text{for } i=1,...,N. \qquad (2)$$

(2 pts) 2. Use expression (2) to show that

$$1=E_{t}\left\{m_{t+1}R_{i,t+1}\right\}, \qquad (3)$$

where m_{t+1} and $R_{i,t+1}$ denote the stochastic discount factor and the rate of return on asset i, respectively.

(2 pts) 3. Let asset N be a risk free asset. Its return, $R_{N,t+1}$, is known at time t. Show using (3) that

$$E_{t}\left(R_{i,t+1}\right)-R_{N,t+1}=-R_{N,t+1}cov_{t}\left(m_{t+1},R_{i,t+1}\right). \qquad (4)$$

(2 pts) 4. Use equation (4) to show that the return on the riskless asset can be lower than the expected return in a risky asset.

(2 pts) 5. Provide the economic intuition for the result in question (4).

(2 pts) 6. Assume now that there is an additional asset, the "market portfolio", for which condition (4) holds too. This asset pays at $t+1$ dividend, C_{t+1}, but does not pay any dividend in the following periods. At t the price of this asset is $P_{m,t}$ and its return is $R_{m,t+1} = C_{t+1}/P_{m,t}$. Additionally, assume that preferences are quadratic, $u\left(C_t\right) = C_t - \frac{1}{2}C_t^2$, so that

$$m_{t+1} = \beta \frac{1 - C_{t+1}}{1 - C_t}.$$

Use equation (4) to show that the following equation holds

$$E_t\left(R_{i,t+1}\right) - R_{N,t+1} = \frac{cov_t(R_{i,t+1}, R_{m,t+1})}{var_t(R_{m,t+1})}\left[E_t\left(R_{m,t+1}\right) - R_{N,t+1}\right] \quad (5)$$

(2 points) (7) Provide the economic interpretation of equation (5).

(2 points) (8) Now assume the Campbell and Cochrane (1999) preferences

$$u\left(C_t, X_t\right) = \frac{\left(C_t - X_t\right)^{1-\gamma}}{1-\gamma}.$$

The stochastic discount factor for these preferences is

$$m_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma},$$

where $S_t = \frac{C_t - X_t}{C_t}$. The habit, X_t, is determined by the past trend of consumption. Let lower case letters denote logs, $c_t = \ln C_t$ and $s_t = \ln S_t$. Additionally, assume

$$\ln C_t - \ln C_{t-1} \equiv \Delta c_t = g + v_t, \text{ with } v_t \sim \text{iid } N(0, \sigma_v^2)$$

and

$$\Delta s_{t+1} = -\theta\left(s_t - \overline{s}\right) + \lambda\left(s_t\right) v_{t+1}$$

where θ, $\overline{s}$, and g are parameters, all greater than zero, and $\lambda\left(s_t\right)$ a function of s_t. What is the expression for the riskless interest rate?

(2 pts) 9. Can these preferences explain the risk free rate puzzle ?

(2 pts) 10. In the data the equity risk premium is countercyclical. Can these preferences explain this stylized fact? Explain.