## General Guidelines

- You may use a calculator;
- You may not use a programmable calculator;
- You may not use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up unto 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.


## 1. Production (3 points)

Let $y=\theta x_{1}^{\alpha} x_{2}^{\beta}$ be a production function, where $y$ is the output, $\theta$ is an unknown constant, and $x_{1}$ and $x_{2}$ are the two inputs.
1.1. Discuss under what values of $\alpha$ and $\beta$ this production function is Decreasing, Constant, and Increasing Returns to Scale (DRTS, CRTS, and IRTS).
1.2. Consider that $\alpha=\beta$. What is the Technical Rate of Substitution (TRS) of the production function above?
1.3. Use your answer to question 1.2 to discuss how the TRS depends upon the level of $x_{1}$ and $x_{2}$. What is the economic intuition for this relationship?

## 2. Profit and costs (4 points)

Consider a firm that produces output $y$ with inputs $x_{1}$ and $x_{2}$. The prices for $x_{1}$ and $x_{2}$ are denoted by $w_{1}$ and $w_{2}$ respectively. Imagine that you observe the following data for the firm:

| Month | $\boldsymbol{x}_{\mathbf{1}}$ | $\boldsymbol{x}_{\mathbf{2}}$ | $\boldsymbol{w}_{\mathbf{1}}$ | $\boldsymbol{w}_{\mathbf{2}}$ | $\boldsymbol{y}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 20 | 4 | 2 | 30 |
| 2 | 20 | 10 | 2 | 4 | 30 |

2.1. Can you test the Weak Axiom of Cost Minimization (WACM) for this firm? If yes, test if WACM can be rejected.

Consider for the following three questions that the firm uses the two inputs $x_{1}$ and $x_{2}$ to produce output $y$ via the following production function: $y=x_{1}+x_{2}$. Also consider that $w_{1}=1$ and $w_{2}=2$.
2.2. Carefully sketch the isoquant that allows the firm to produce exactly 10 units of output:

$$
\left\{\left(x_{1}, x_{2}\right) \text { in } R_{+}^{2} \mid x_{1}+x_{2}=10\right\}
$$

Carefully sketch into the same graph the isocost line that corresponds to the minimum costs $\underline{c}$ that allows the firm to produce those 10 units of output:

$$
\left\{\left(x_{1}, x_{2}\right) \text { in } R_{+}^{2} \mid x_{1}+2 x_{2}=\underline{c}\right\}
$$

Conclude from your graph what are the conditional factor demands for $x_{1}$ and $x_{2}$ and the minimum costs $\underline{C}$ to produce 10 units of output.
2.3. Consider that the price for $x_{1}$ increases to 3 , so that $w_{1}=3$ and $w_{2}=2$. What are the new conditional factor demands for $x_{1}$ and $x_{2}$ and the new minimum costs $\underline{c}$ to produce 10 units of output?
2.4. For which value(s) of $w_{1}$ and $w_{2}$ will the firm be indifferent when it comes to the conditional factor demands for $x_{1}$ and $x_{2}$ ? Note that $w_{1}$ and $w_{2}$ cannot be zero.

## 3. Consumer choice (5 points)

Consider a consumer with a utility function equal to $u=\frac{1}{\alpha} x_{1}^{\alpha}+x_{2}$. The consumer has income $m$, and the prices for good $x_{1}$ and $x_{2}$ are denoted by $p_{1}$ and $p_{2}$ respectively.
3.1. What are the Marshallian demand functions for both $x_{1}$ and $x_{2}$ ?
3.2. Consider that $p_{2}=1$. What is the elasticity of demand for $x_{1}$ ?

To answer the following three questions, consider a consumer that maximizes utility given prices and income. There is no need to consider the specific utility function above.
3.3. If the Marshallian demand curve is upwards sloping, what can you conclude about the income effect? Draw a graph with indifference curves and budget lines to explain your answer.
3.4. Which assumption on preferences guarantees that the Hicksian demand curve is downwards sloping? Draw a graph with an indifference curve and budget lines to explain your answer.
3.5. Which assumption guarantees that the Marshallian demand curve and the Hicksian demand curve are equal at every price? Use the Slutsky equation to explain your answer.

## 4. Welfare (4 points)

Consider a consumer with a utility function equal to $u=3 x_{1}^{2 / 3}+x_{2}$. Consider that, in addition, the consumer has income $m$, and the prices for good $x_{1}$ and $x_{2}$ are $p_{1}=2$ and $p_{2}=2$ respectively.
4.1. Consider that $p_{1}$ changes from 2 to 4 . Find the change in the consumer surplus for this change in the price of $x_{1}$.
4.2. With the utility function above the change in the consumer surplus is an exact measure of welfare change. Carefully explain the reason for this.

## 5. Perfect competition (2 points)

Consider a perfect competitive market. Let the total cost function of a single firm be equal to:

$$
c(y)=4 y^{2}+16
$$

Where $y$ is the output. Let the market demand be given by:

$$
X(p)=50-2 p
$$

Where $p$ is the price. Suppose that in the long run there is free entry into, and exit out of, this market, and that all potential firms have the same cost function $c(y)$ as above.
5.1. How many firms will there be active in this perfect competitive market in the long run?

## 6. Monopoly ( 2 points)

6.1. A monopolist faces a demand curve of $D(p)=100-2 p$, where $p$ denotes the price, and has a total cost function of $c(y)=50+20 y+y^{2}$. What is the profit-maximizing level of output?
6.2. A monopolist will never produce at a point where the elasticity of demand is inelastic (that is, between -1 and 0 ). Explain the economic intuition for this behavior of a monopolist.

