Consider the following variant of our stochastic model. Denote the state by $s_t = \left(Z_{t-1}, \bar{M}_{t-1}, \tau_t, g_t\right)$ where the current realization of money growth and productivity growth are stochastic. As usual, the realized productivity level in period t is therefore $Z(s^t) = Z_{t-1}(1+g_t)$ and the realized money transfer at the end of the period is $T(s^t) = \bar{M}_{t-1}\tau_t$, making the end of period money supply $\bar{M}_t = (1+\tau_t)\bar{M}_{t-1}$. For simplicity, we will assume that these growth rates follow a simple i.i.d. process with mean $\bar{\tau}$ and $\bar{g}$ respectively.

The household here has a labor holding cost of money between periods $H(s^t)$. The household's problem can be thought of as choosing a sequence $\{C(s^t), L(s^t), H(s^t), M(s^t), B(s^t)\}_{t=1}^{2}$ so as to

$$\max E\left\{\sum_{t=1,2}\beta^{t-1}\left[u(C(s^t)) - v(L(s^t)+H(s^t))\right] + \beta^2 V(M(s^2), B(s^2), s_3)\right\}$$

subject to

$$M(s^{t-1}(s^t)) \geq P(s^t)C(s^t),$$
$$Z(s^t)H_t(s^t) = \phi M(s^t)/P(s^t) \text{ and}$$

$$\begin{aligned} & P(s^t)Z(s^t)L(s^t) + \left[M(s^{t-1}(s^t)) - P(s^t)C(s^t)\right] + B(s^{t-1}(s^t)) + T(s^t) \\ \geq\; & M(s^t) + q(s^t)B(s^t) \text{ for all } t \leq 2 \text{ and } s^t \in S^t. \end{aligned}$$

A) [10 pts.] Impose the labor cost condition for holding money balances between periods. Write down the Lagrangian for this problem and determine the first-order conditions for the optimal levels of consumption, labor, money and bonds.

B) [10 pts.] To solve out for the equilibrium of the model, we make the following assumptions. Assume that we have log preferences over consumption and that the disutility of labor take our standard form $v(x) = x^{1+\gamma}/(1+\gamma)$. Assume that the cash-in-advance constraint always binds, so that

$$P(s^t) = \frac{\bar{M}(s^{t-1}(s^t))}{Z(s^t)L(s^t)}. \tag{1}$$

Assume also that current labor effort and consumption only depend upon the current realization of the state s_t and not the entire history of realizations s^t. Finally, impose market clearing, so that $C(s^t) = Z(s^t)L(s^t)$. With these assumptions and impositions, construct the revised set of first-order conditions.

C) [10 pts.] The multipliers on the cash-in-advance constraint, which we normally denote by $\lambda(s^t)$ and on the budget constraint which we normally denote by $\mu(s^t)$, will not be stationary. Determine a change-in-variables that will render them stationary, and implement that change in order to derive a closed form solution for labor effort as a function of the current money shock τ_t.

D) [10 pts.] Discuss how you would calibrate model to match the elasticity of real money demand we find in the data.