A major tax that we have not considered is a consumption tax. Consumption taxes are those paid by consumers when they purchase goods. So, instead of paying the price per unit, p, they pay $(1+\tau_c)p$. Then the seller keeps p while the amount $\tau_c p$ is turned over to the government. We are going to try and extend our basic model to allow for these kinds of taxes. To keep things simple we will drop labor and capital taxes. As a result, we can go back to the original backyard model of production in which every household simply produces output using their own labor and capital.

Now we want to augment our basic model to include consumption taxes. Here is a basic layout of the model. The equation of motion for the capital stock is

$$K_{t+1} = (1-\delta)K_t + X_t.$$

The resource constraint for the economy is given by

$$[Z_t L_t]^{1-\alpha} K_t^\alpha = C_t + X_t.$$

Assume that both productivity and the money supply grow at constant rates given by

$$Z_t = (1+g)Z_{t-1}, \text{ and } M_t = (1+\tau)M_{t-1}.$$

respectively. Make the normalization that $Z_1 = 1$, then $Z_t = (1+g)^{t-1}$, so it is just the accumulated growth factor. This will turn out to be notationally convenient.

We can write the household's choice problem as choosing a sequence of quantities $\{C_t, L_t, M_{t+1}, B_{t+1}, K_{t+1}\}_{t=1}^2$ so as to maximize

$$\max \sum_{t=1,2} \beta^{t-1} [u(C_t) - v(L_t)] + \beta^2 V(M_3, B_3, K_3)$$

subject to

$$M_t \geq (1+\tau_c)P_t C_t \text{ and}$$

$$\begin{aligned} & P_t \left[[Z_t L_t]^{1-\alpha} K_t^\alpha - \delta K_t \right] + [M_t - (1+\tau_c)P_t C_t] + B_{t-1} + T_t \\ \geq \quad & M_{t+1} + q_t B_{t+1} + P_t [K_{t+1} - K_t] \text{ for all } t \leq 2. \end{aligned}$$

We will assume that the revenue collected from the consumption taxes are lump-sum rebated to the households. The key assumption here is that you receive the per capita level collected as a transfer along with the money injection. So, your transfers are independent of your individual tax payments.

1. Discuss consumption taxes. Give some examples of them and some of the positive and negative things that are said about them.

2. Set-up the Lagrangian and derive the first-order conditions.

3. Assume that we have a balanced growth path in which

- $L_t = L$, i.e., labor is constant.
- The marginal product of capital is constant so $K_t = Z_t K$.

Given this, show that (i) $Y_t = Z_t Y$, (ii) $X_t = Z_t X = Z_t K_t (g+\delta)$, and hence that (iii) $C_t = Z_t C$. Finally assume that the cash-in-advance constraint binds and use that to pin down the price level.

4. Plug these results into your first-order conditions, and come up with a change-in-variables to render things stationary. Use this change to generate some nice stationary conditions which we can use to solve out for the balanced growth path.

5. Have you verified the two assumptions we made about the evolution of labor and capital?

6. Discuss how you put this model on a computer and solve things out.

7. How do consumption taxes differ from labor and capital taxes? In the context of a representative agent model, which form of taxation is likely to be superior?