PRACTICE MIDT.I 23/24

1.1 TRS: if the firm changes (eg increases) x_1, how much does x_2 need to change (eg decrease) ~~xxx~~ to keep producing the same y.

let $x_2(x_1)$ be the isoquant: for each x_1 it gives the x_2 to produce constant y.

$$\boxed{TRS = \frac{\partial x_2(x_1)}{\partial x_1}}$$

↳ slope of the isoquant

$f(x_1, x_2) = y$ } by def of isoquant

$f(x_1, x_2(x_1)) = y$

$$\frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial x_1} = \frac{\partial y}{\partial x_1} = 0$$

$$\boxed{TRS = \frac{\partial x_2}{\partial x_1} = -\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}}}$$

$y = 2x_1 + 4x_2$

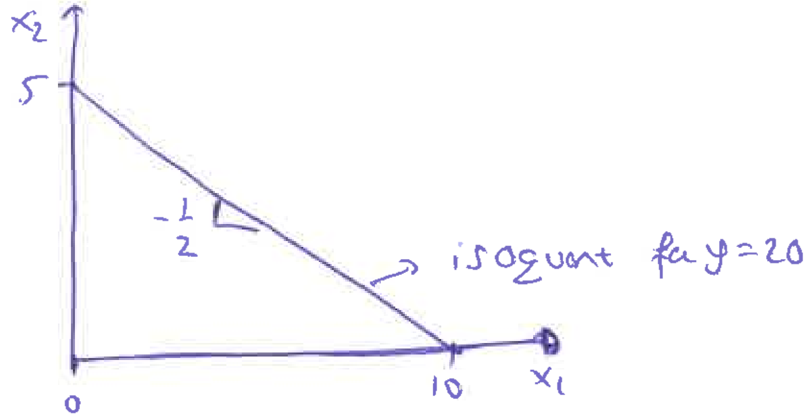
$4x_2 = y - 2x_1$

$x_2 = \frac{y}{4} - \frac{2}{4}x_1$

$$TRS = \frac{\partial x_2}{\partial x_1} = -\frac{2}{4} = -\frac{1}{2};$$

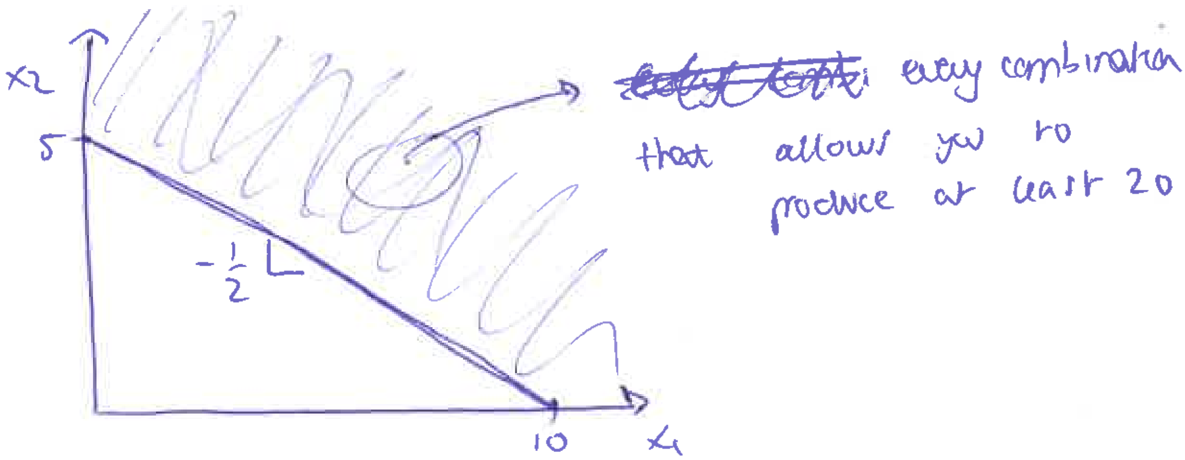
eg if $y = 20$

the isoquant:



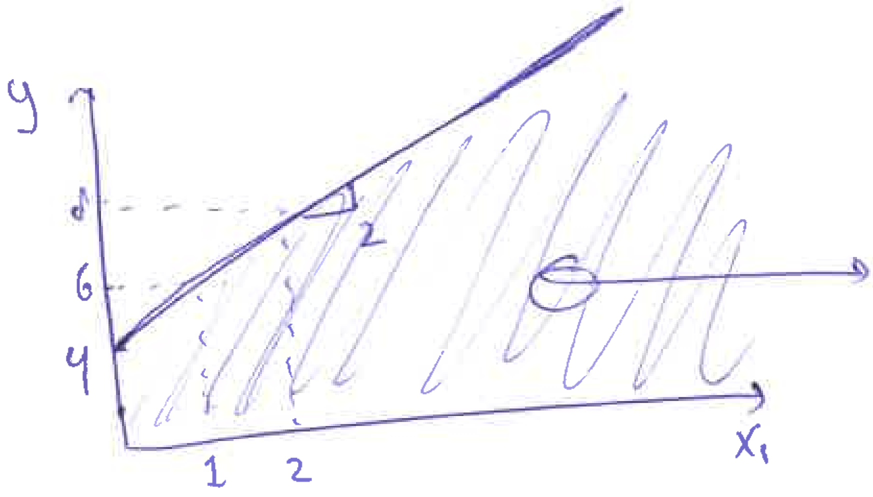
$$TRS = -\frac{\frac{\partial f}{\partial x_1}}{\frac{\partial f}{\partial x_2}} = -\frac{2}{4} = -\frac{1}{2}.$$

1.2.



input requirement set: the set of all input bundles x that produce at least y units of output

1.3 (sr) production possibilities set: all production plans (y x) that are feasible (consistent with the sr constraint).



~~everything~~ ~~every~~ ~~constraint~~ every production plans (y x) that are feasible

2.1 How does the firm get $x(p,w)$? via FOC.

$$\frac{\partial \pi}{\partial x} = p\frac{\partial f}{\partial x} - w = 0$$

↳ solve for x in terms of (p,w) to find $x(p,w)$

Then substitute $x = x(p,w)$ into π

$$\pi^* = \pi(x(p,w),p,w) = p\,f(x(p,w)) - w\,x(p,w)$$

Take derivative towards w.

$$\frac{\partial \pi^*}{\partial w} = p \frac{\partial f}{\partial x} \frac{\partial x}{\partial w} - x(p,w) - w \frac{\partial x}{\partial w}$$

$$= \underbrace{\left\{ p \frac{\partial f}{\partial x} - w \right\} \frac{\partial x}{\partial w}}_{\text{indirect effect}} - \underbrace{x(p,w)}_{\text{direct effect}}$$

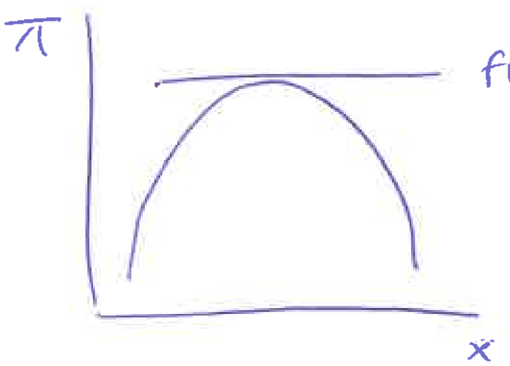
direct effect: $w \uparrow$ by 1 $\rightarrow$ each input x becomes 1 more expensive $\rightarrow$ $\pi \downarrow$ by $-x$

indirect effect: $w \uparrow$ by 1 $\rightarrow$ firm decreases usage of x: $\frac{\partial x}{\partial w} < 0 \rightarrow \pi \downarrow$ by $\left\{ p \frac{\partial f}{\partial x} - w \right\} \frac{\partial x}{\partial w}$

2.2 Indirect is zero by FOC.

FOC : $p \frac{\partial f}{\partial x} - w = 0.$

Intuition: since firm choose x optimally, if it slightly changes x because w changes it should not affect profits



FOC $\rightarrow \frac{\partial \pi}{\partial x} = p \frac{\partial f}{\partial x} - w = 0,$

slightly changing x does not change profits.

$$\frac{\partial \pi^*}{\partial w} = - x(p, w)$$

⇒ this is Hotelling's lemma

⇒ Hotelling's lemma is an application of the Env. theorem:

if you want to know how an optimized function (e.g. π^*) changes when an exogenous variable ~~changes~~ (e.g. w) changes, only the direct effects ~~...~~ of the exogenous var need to be considered and the indirect effects that enter via the solution of the endogenous choice var (e.g. $x(p, w)$) is zero.

<u>2.3</u> prices $p^t = (p^t, w^t)$ $\forall t$

"net" output $y^t = (y^t, -x^t)$

$$\pi^t = p^{tT} \cdot y^t = p^t y^t - w^t x^t$$

<u>WAPM</u>: $\underbrace{p^{tT} y^t}_{\pi^t} \geq p^{tT} y^s$

if the firm max profits, then the observed net output choice at price p^t must have a level of profit at least as great as all other net outputs that the firm could have chosen, otherwise the firm did not choose the net output to max profits.

3.1 cost minimization:

① $\mathcal{L} = 1x_1 + 4x_2 - \lambda\{x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} - 10\}$

② $\frac{\partial \mathcal{L}}{\partial x_1} = 1 - \lambda \frac{1}{4} x_1^{-\frac{3}{4}} x_2^{\frac{1}{4}} = 0$

$\frac{\partial \mathcal{L}}{\partial x_2} = 4 - \lambda \frac{1}{4} x_1^{\frac{1}{4}} x_2^{-\frac{3}{4}} = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda} = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} - 10 = 0$

③ divide first two FOCs: $\frac{\frac{\partial \mathcal{L}}{\partial x_1}}{\frac{\partial \mathcal{L}}{\partial x_2}} = \frac{1}{4} = \frac{x_2}{x_1}$

$x_1 = 4x_2$

use third FOC: $x_1^{\frac{1}{4}} x_2^{\frac{1}{4}} = 10$

$(4x_2)^{\frac{1}{4}} x_2^{\frac{1}{4}} = 10$

$x_2^{\frac{1}{2}} = \frac{10}{4^{\frac{1}{4}}}$

$$\boxed{x_2 = \left(\frac{10}{4^{\frac{1}{4}}}\right)^2 = 50 \qquad x_1 = \left(\frac{10}{50^{\frac{1}{4}}}\right)^4 = 200}$$

④ $\boxed{\text{minimum costs} = 1 \cdot 200 + 4 \cdot 50 = \underline{\underline{400}}}$