5 Heuristics

1. Consider the Binary Knapsack Problem and the following instance with $n = 7$ objects, capacity $C = 9$, utilities $u = (7, 2, 4, 3, 5, 8, 3)$ and volumes $v = (3, 6, 2, 3, 2, 2, 1)$. Propose a lower and an upper bound for the optimal value of the problem by using the critical index. Explain how you arrived at the proposed bounds.

Obtain solutions to the following problems using heuristics.

2. The following binary Knapsack problem.

$$\begin{aligned} \max\ & z = 2x_1 + 5x_2 + 3x_3 + 4x_4 + 5x_5 + 4x_6 \\ s.t.\ & 6x_1 + 8x_2 + 4x_3 + 6x_4 + 7x_5 + 2x_6 \leq 12 \\ & x_j \in \{0, 1\}, j = 1, \ldots, 6 \end{aligned}$$

3. Jobco uses a single machine to process three jobs. Both the processing time and the due date (in days) for each job are given in the following table. The due dates are measured from zero, the assumed start time of the first job. The objective of the problem is to determine the job sequence that minimizes the late penalty for processing all three jobs.

Job	Processing time (day)	Due date (day)	Late penalty ($/day)
1	5	25	19
2	20	22	12
3	15	35	34

4. The problem of sequencing jobs on a single machine so as the total cost is minimized and the following set of jobs.

Job j	Processing time in days, t_j	Due date d_j	Holding cost h_j ($/day)	Penalty cost p_j ($/day)
1	10	15	3	10
2	8	20	2	22
3	6	10	5	10
4	7	30	4	8
5	4	12	6	15

5. The daily production schedule at the Rainbow Company includes batches of white (W), yellow (Y), red (R), and black (B) paints. The production facility must be cleaned between successive batches.

Inter-batch Cleanup Times (in minutes)				
Paint	White	Yellow	Black	Red
White	∞	10	17	15
Yellow	20	∞	19	18
Black	50	44	∞	22
Red	45	40	20	∞

The objective is to determine the sequencing of colors that minimizes the total cleanup time.

6. The Traveling Salesman Problem and the following distance matrix between five locations.

$$[c_{ij}] = \begin{bmatrix} - & 10 & 22 & 12 & 10 \\ & - & 12 & 8 & 13 \\ & & - & 15 & 15 \\ & & & - & 9 \\ & & & & - \end{bmatrix}$$

Some solutions

5.1 critical index $k = 5$; $LB = 23$; $UB = 25$