6 Decision Analysis

1. Consider the following payoff matrix:

	θ_1	θ_2	θ_3	θ_4
a_1	1	4	7	8
a_2	10	7	15	5
a_3	7	11	5	2
a_4	8	6	10	6

   (a) Determine Laplace action and explain its meaning.

   (b) Determine Maximin/Minimax action and explain its meaning.

   (c) Determine Hurwicz action with $\alpha = 0.5$ and explain its meaning.

2. A company owns a tract of land that may contain oil. A consulting geologist has reported that she believes there is a 1 chance in 4 of oil. Because of this prospect, another oil company offered to purchase the land for \$90,000. However, the company is considering holding the land in order to drill for oil itself. The cost of drilling is \$100,000. If oil is found, the resulting expected revenue will be \$800,000, so the company's expected profit (after deducting the cost of drilling) will be \$700,000. A loss of \$100,000 (the drilling cost) will be incurred if the land is dry (no oil).

   Determine the action that should be selected using the following criterias: Minimax (Maximin); Savage Regret; Laplace; Hurwicz Index (take $\alpha = 0.5$); Maximum Likelihood; Bayes Criteria.

3. A business man wants to decide where to invest 100 000€. With that purpose he needs to choose one of three alternative projects, P1, P2 or P3. The return of the projects depends on the performance of the economy that may stagnate or improve. The following table contains the data.

	stagnates	**improve**
P_1	7%	5%
P_2	-10%	14%
P_3	6%	6%

   Determine the project that should be selected using the following criterias: Minimax (Maximin); Laplace; Savage Regret; Hurwicz Index (take $\alpha = 0.5$); Maximum Likelihood; Bayes Criteria.

4. A farmer must determine whether to plant corn or wheat. If he plants corn, and the weather is warm, he earns \$8 000; if he plants corn, and the weather is cold, he earns \$5 000. If he plants wheat, and the weather is warm, he earns \$7 000; if he plants wheat, and the weather is cold, he earns \$6 500. In the past 40 percent of all years have been cold and 60 percent have been warm. Before planting, the farmer can pay \$600 for an expert weather forecast. If the year is actually cold, there is a 90 percent chance that the forecaster will predict a cold year. If the year is actually warm, there is an 80 percent chance that the forecaster will predict a warm year. How can the farmer maximize his expected profits? Use a decision tree. Compute the EVPI and the EVSI.

5. A farmer must determine whether to plant corn or wheat. If he plants corn, and the weather is warm, he earns $8 000; if he plants corn, and the weather is cold, he earns $5 000. If he plants wheat, and the weather is warm, he earns $7 000; if he plants wheat, and the weather is cold, he earns $6 500. In the past 40 percent of all years have been warm and 60 percent have been cold.

   (a) Build the payoff matrix.

   (b) Determine Laplace action and explain its meaning.

   (c) What action does Bayes' procedure recommend.

   (d) Build a decision tree.

   (e) Compute the expected value of perfect information.

6. Investment advisors have estimated stock market returns for four market segments: computers, financial, manufacturing and pharmaceuticals. The annual return projections vary depending on whether general economic conditions are improving, stable or declining. The expected annual return percentages for each market segment under each economic condition are as follows. A forecast shows

	Economic Condition		
Market Segment	**Improving**	**Stable**	**Declining**
Computers	10	2	-4
Financial	8	5	-3
Manufacturing	6	4	-2
Pharmaceuticals	6	5	-1

   stable to declining economic conditions with the following probabilities: improving (0.2), stable (0.5), and declining (0.3). An individual investor wants to select one market segment for a new investment.

   (a) What is the decision recommended by the Bayes criteria.

   (b) Calculate the EVPI (expected value of perfect information) of the expected return percentage and on the basis of this value say what you advise.

Some solutions

6.1

	θ_1	θ_2	θ_3	θ_4	Laplace	Maxmin	Hurwicz, $\alpha = 0.5$
a_1	1	4	7	8	$\frac{1+4+7+8}{4} = 5$	1	$8\alpha + (1-\alpha) = 7\alpha + 1 = 4.5$
a_2	10	7	15	5	$\frac{10+7+15+5}{4} = 9.25$	5	$15\alpha + 5(1-\alpha) = 10\alpha + 5 = 10$
a_3	7	11	5	2	$\frac{7+11+5+2}{4} = 6.25$	2	$11\alpha + 2(1-\alpha) = 9\alpha + 2 = 6.5$
a_4	8	6	10	6	$\frac{8+6+10+6}{4} = 7.5$	6	$10\alpha + 6(1-\alpha) = 4\alpha + 6 = 8$
					9.25	6	10

6.5 (a) Let the set of **alternative actions** be $A = \{C, W\}$ with $C \leftarrow$ plant corn, and $W \leftarrow$ plant wheat. Let the set of $n = 2$ **nature states** be $\Theta = \{\theta_w, \theta_c\}$ with $\theta_w \leftarrow$ the weather is warm, and $\theta_c \leftarrow$ the weather is cold. The payoff matrix with values $p(a, \theta)$ is

	θ_w	θ_c
C	8000	5000
W	7000	6500

(b) The Laplace action is such that $\max_{a \in A} \left\{ \frac{1}{n} \sum_{\theta \in \Theta} p(a, \theta) \right\}$.

	θ_w	θ_c	$\frac{1}{n} \sum_{\theta \in \Theta} p(a, \theta)$
C	8000	5000	$\frac{1}{2}(8000 + 5000) = 6500$
W	7000	6500	$\frac{1}{2}(7000 + 6500) = 6750$

Thus $\max_{a \in A} \left\{ \frac{1}{n} \sum_{\theta \in \Theta} p(a, \theta) \right\} = \max_{a \in A} \{6500, 6750\} = 6750$, therefore the Laplace action should be W, plant wheat.

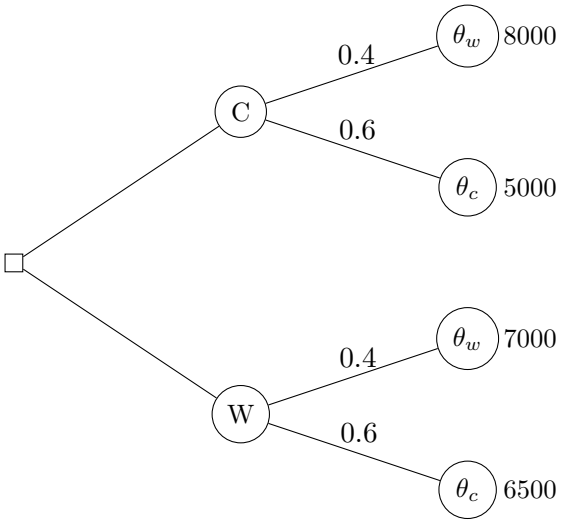
(c) The *a priori* probability $h_\theta(\bar{\theta})$ of each nature state is $h_\theta(\theta_w) = P(\theta = \theta_w) = 0.4$, and $h_\theta(\theta_c) = P(\theta = \theta_c) = 0.6$.

The Bayes' action is such that $\max_{a \in A} \left\{ \sum_{\bar{\theta} \in \Theta} h_\theta(\bar{\theta})\ p(a, \bar{\theta}) \right\}$.

	θ_w	θ_c	$\sum_{\bar{\theta} \in \Theta} h_\theta(\bar{\theta})\ p(a, \bar{\theta})$
C	8000	5000	$0.4 \times 8000 + 0.6 \times 5000 = 6200$
W	7000	6500	$0.4 \times 7000 + 0.6 \times 6500 = 6700$
$h_\theta(\bar{\theta})$	0.4	0.6	

Thus $\max_{a \in A} \left\{ \sum_{\bar{\theta} \in \Theta} h_\theta(\bar{\theta})\ p(a, \bar{\theta}) \right\} = \max_{a \in A} \{6200, 6700\} = 6700$, therefore the Bayes's procedure recomends action W, plant wheat.

(d)



(e)

	θ_w	θ_c	$\sum_{\bar{\theta}\in\Theta} h_\theta(\bar{\theta})\ p(a,\bar{\theta})$
C	8000	5000	$0.4 \times 8000 + 0.6 \times 5000 = 6200$
W	7000	6500	$0.4 \times 7000 + 0.6 \times 6500 = 6700$
$h_\theta(\bar{\theta})$	0.4	0.6	

The expected return without any additional information (corresponds to the Bayes's criteria value) is 6700.

The expected return using a "perfect" prediction and weighting by the *a priory* probability is $0.4 \times 8000 + 0.6 \times 6500 = 7100$.

Hence EVPI $= 7100 - 6700 = 400$.

6.6 (a) Recommends pharmaceuticals, $EVwoPI = 3.4$.

(b) $EVPI = |EVwPI - EVwoPI| = 4.2 - 3.4 = 0.8$