1 Multicriteria Analysis

1. For the following problems find the set of efficient solutions, the set of weak efficient solutions, the set of non-dominated points.

a)

$$\begin{aligned} \max \quad & z_1 = 25x_1 + 20x_2, \\ \max \quad & z_2 = x_1 + 8x_2, \\ \text{s. to:} \quad & x_1 + x_2 \leq 50, \\ & 2x_1 + x_2 \leq 80, \\ & 2x_1 + 5x_2 \leq 220, \\ & x_1, x_2 \geq 0. \end{aligned}$$

b)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \max \quad & z_2 = 3x_1 + 2x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

c)

$$\begin{aligned} \min \quad & z_1 = x_1, \\ \min \quad & z_2 = 3x_1 - x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

d)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \min \quad & z_2 = 3x_1 - x_2, \\ \text{s. to:} \quad & 1.5x_1 + 3x_2 \leq 13.5, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \leq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

e)

$$\begin{aligned} \max \quad & z_1 = x_1, \\ \max \quad & z_2 = x_2, \\ \text{s. to:} \quad & x_2 \leq 4, \\ & x_1 + x_2 \leq 10, \\ & 2x_1 + x_2 \geq 10, \\ & x_1, x_2 \geq 0. \end{aligned}$$

f)

$$\begin{aligned} \max \quad & z_1 = 3x_1 + 2x_2, \\ \max \quad & z_2 = x_1, \\ \text{s. to:} \quad & x_1 + x_2 \leq 6, \\ & x_1 \leq 4, \\ & 2x_1 + x_2 \geq 9, \\ & x_1, x_2 \geq 0. \end{aligned}$$

g)

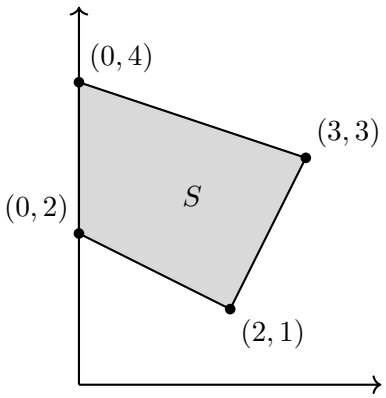
$$\begin{aligned} \min \quad & z_1 = 3x_1 + x_2, \\ \max \quad & z_2 = x_1 + x_2, \\ \text{s. to:} \quad & x_2 \leq 3, \\ & 3x_1 - x_2 \leq 6, \\ & x_1, x_2 \geq 0. \end{aligned}$$

h)

$$\begin{aligned} \max \quad & z_1 = -x_1 + 3x_2, \\ \max \quad & z_2 = 3x_1 + 3x_2, \\ \max \quad & z_3 = x_1 + 2x_2, \\ \text{s. to:} \quad & x_2 \leq 4, \\ & x_1 + 2x_2 \leq 10, \\ & 2x_1 + x_2 \leq 10, \\ & x_1, x_2 \geq 0. \end{aligned}$$

2. Find the efficient solutions to the following multicriteria linear program, with S as shown below.

$$\begin{aligned} \min \quad & x_1 - x_2 \\ \max \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & (x_1, x_2) \in S \end{aligned}$$



Present all your calculations and drawings.

Some solutions

1.a) $x^* = (20, 60),\ z^* = 180,$

1.2 efficient solutions are the points $x = \alpha(0, 4) + (1 - \alpha)(3, 3)$, for $\alpha \in [0, 1]$