Lisbon School of Economics & Management
Universidade de Lisboa

STATISTICS I

Bachelor's degrees in Economics and Finance

2nd Year/2nd Semester

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Pratical Class 1

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https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

1 Random Experiments, Sample Space and Events: Exercises

1. A box contains 5 balls of which 2 are black. The black balls are numbered 1 and 2, the others 3 to 5. Two balls are randomly taken out one after the other without replacement. The numbers on the two balls are observed.

   a) List all the elements of the sample space associated to this random experiment.

   b) From the sample space, define the following events:

   A_1 - The first ball observed is black;

   A_2 - The second ball observed is black;

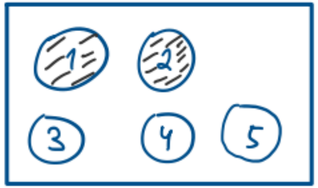
   A_3 - The two balls taken out are black;

   A_4 - At least one of the two balls are black;

   A_5 - Exactly one of the two balls are black;

   A_6 - The sum of the numbers in the two balls is greater than seven.

Exercise 1 a)



5 balls → extraction of 2 balls (no replacement)

a) $S = \{(i,j) : i,j = 1,2,3,4,5 \wedge i \neq j\}$

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Exercise 1 b)

b)

$$A_1 = \{(i,j) \in S: \ i = 1,2 \ \wedge \ j = 1,2,3,4,5\} =$$

$$= \{(1,2),(1,3),(1,4),(1,5),(2,1),(2,3),(2,4),(2,5)\}$$

$$A_2 = \{(i,j) \in S: \ i = 1,2,3,4,5 \ \wedge \ j = 1,2\}$$

$$= \{(2,1),(3,1),(4,1),(5,1),(1,2),(3,2),(4,2),(5,2)\}$$

$$A_3 = A_1 \cap A_2 = \{(i,j) \in S: \ i,j = 1,2\} = \{(1,2),(2,1)\}$$

$$A_4 = A_1 \cup A_2 = \{(i,j) \in S: \ i = 1,2 \ \vee \ j = 1,2\} =$$

1st ball is black

$$= \{\overbrace{(1,2),(1,3),(1,4),(1,5),(2,1),(2,3),(2,4),(2,5)}^{\text{1st ball is black}},$$

$$\underbrace{(3,1),(4,1),(5,1),(3,2),(4,2),(5,2)}_{\text{2nd ball is black}}\}$$

2nd ball is black

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Exercise 1 b)

{At least one ball is black} → A_4

$$A_5 = A_4 - A_3 = \{(i,j) \in S : (i = 1,2 \wedge j = 3,4,5) \vee (i = 3,4,5 \wedge j = 1,2)\} =$$

A_3: Both balls are black $= \{(1,2),(2,1)\}$

only the first ball is black

$$= \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),$$
$$(3,1),(4,1),(5,1),(3,2),(4,2),(5,2)\}$$

only the second ball is black

$$A_6 = \{(i,j) \in S : i + j > 7\} = \{(5,3),(5,4),(4,5),(3,5)\}$$

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5. Let A, B and C be three events in the sample space S. Verify which of the following is true. Justify your answer.

   a) $(A \cap B) \cup C = A \cap (B \cup C)$;

   b) $(A \cap B) \cap C = A \cap (B \cap C)$;

   c) $S \setminus (A \cap B) = \overline{A} \cup \overline{B}$.

Exercise 5

a)

False. Union operation is distributive

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

b)

True. Associative property

c)

True

$S \setminus (A \cap B) = \overline{A \cap B} = \bar{A} \cup \bar{B}$

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6. Let A and B be two events in the sample space S. Show that

a) $P(A \cap B) \geq P(A) + P(B) - 1$;

b) $P(A \cap B) \leq P(A) + P(B)$;

c) $A \subset B \Rightarrow P(A \cap B) = P(A)$;

d) $P(A \cap B) \leq \min(P(A), P(B))$

Exercise 6 a) and b)

a)

$$P(A \cap B) = P(A) + P(B) - \underbrace{P(A \cup B)}_{0 \leqslant P(A \cup B) \leqslant 1} \geqslant P(A) + P(B) - 1$$

b)

$$P(A \cap B) = P(A) + P(B) - \underbrace{P(A \cup B)}_{0 \leqslant P(A \cup B) \leqslant 1} \leqslant P(A) + P(B)$$

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Exercise 6 c) and d)

c)

$A \subset B \Rightarrow A \cap B = A$

Therefore: $P(A \cap B) = P(A)$

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d)

$(A \cap B) \subset A \qquad P(A \cap B) \leq P(A)$

$(A \cap B) \subset B \qquad P(A \cap B) \leq P(B)$

Therefore $P(A \cap B) \leq \min(P(A), P(B))$

2 Probability: Exercises

Concept of Probability, Probability of the Union, Probability of the Intersection, and Probability of the Difference

Probability: Rules

- $0 \leq P(A) \leq 1, \ \forall A \in$ Espaço dos acontecimentos

- $P(\Omega) = 1$

- $P(\{\}) = 0$

Probability of the complement of A

- $P(\overline{A}) = 1 - P(A)$ e $P(A) + P(\overline{A}) = 1$

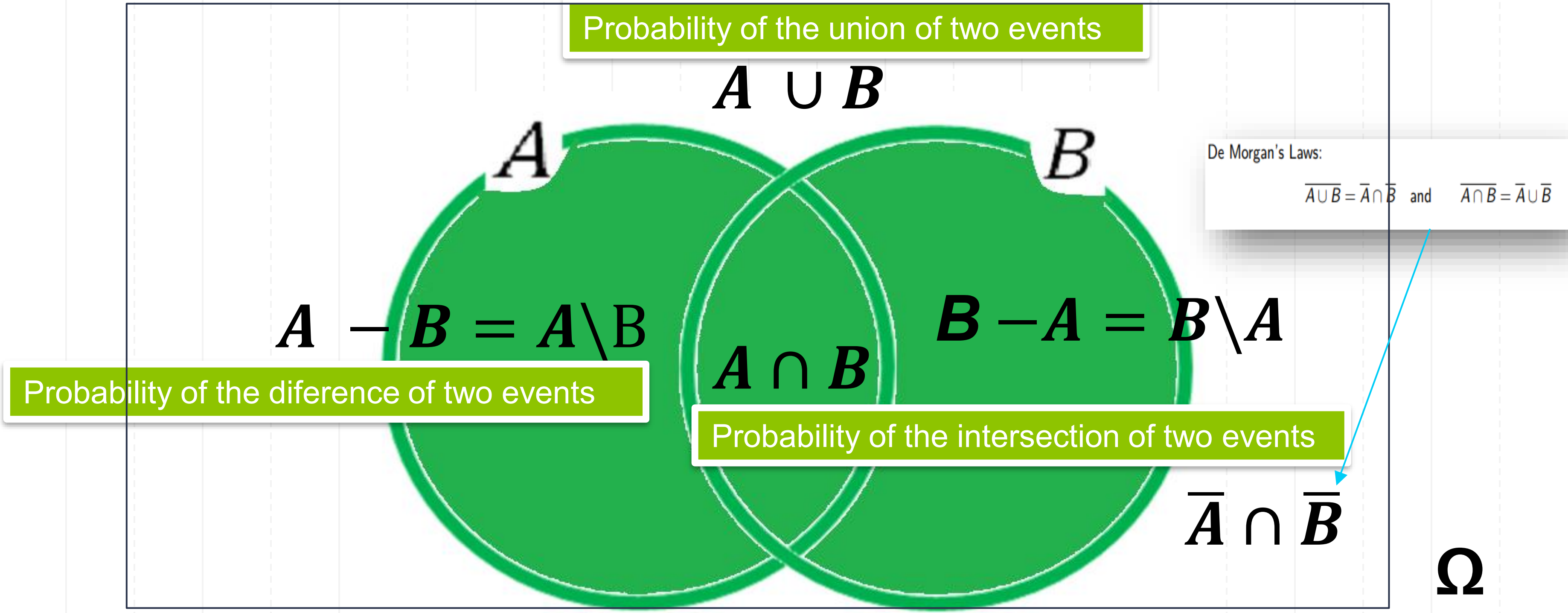
- Se $A \subseteq B$ (Se a realização do acontecimento B implica a realização do acontecimento A), então $P(A) \leq P(B)$

Probabilidade da diferença: P(A-B) = P(A\B) e P(B-A) = P(B\A)

- $P(A-B) = P(A) - P(A \cap B)$

Probability of the diference of two events

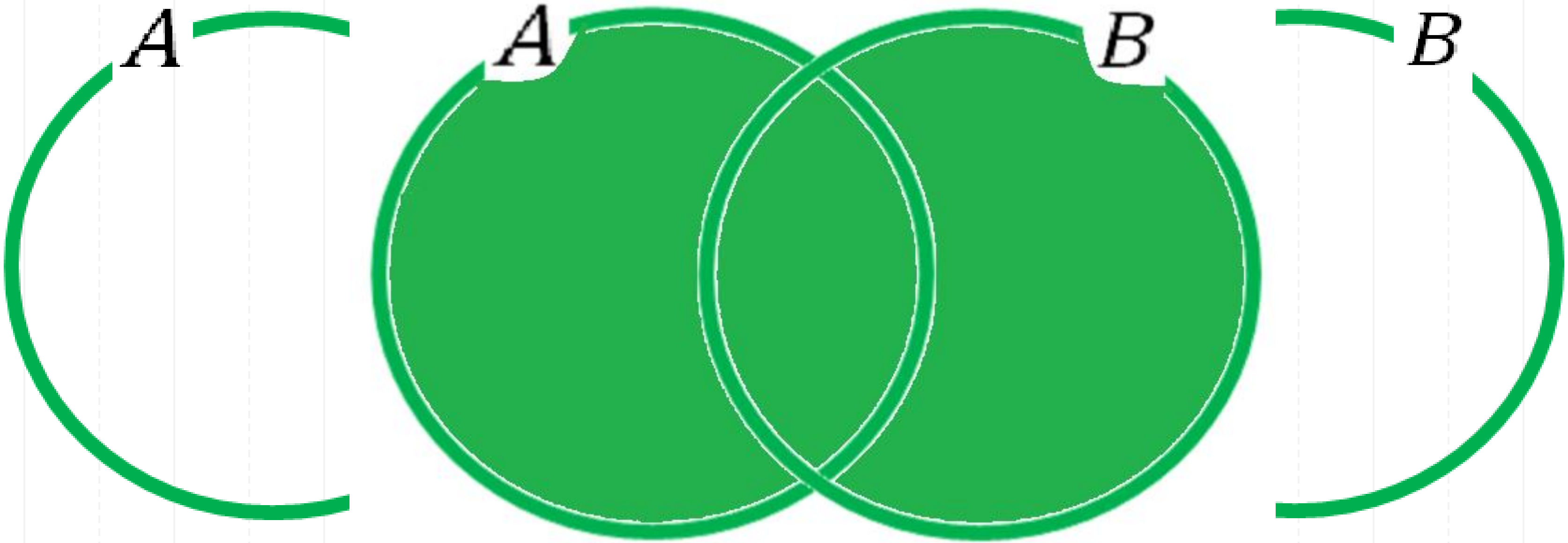
Venn Diagram / Operations with events



Baseado em https://docente.ifrn.edu.br/julianaschivani/disciplinas/matematica-iii/probabilidade/slides-4-probabilidade

Probability of the Union of Events

$$A \cup B$$



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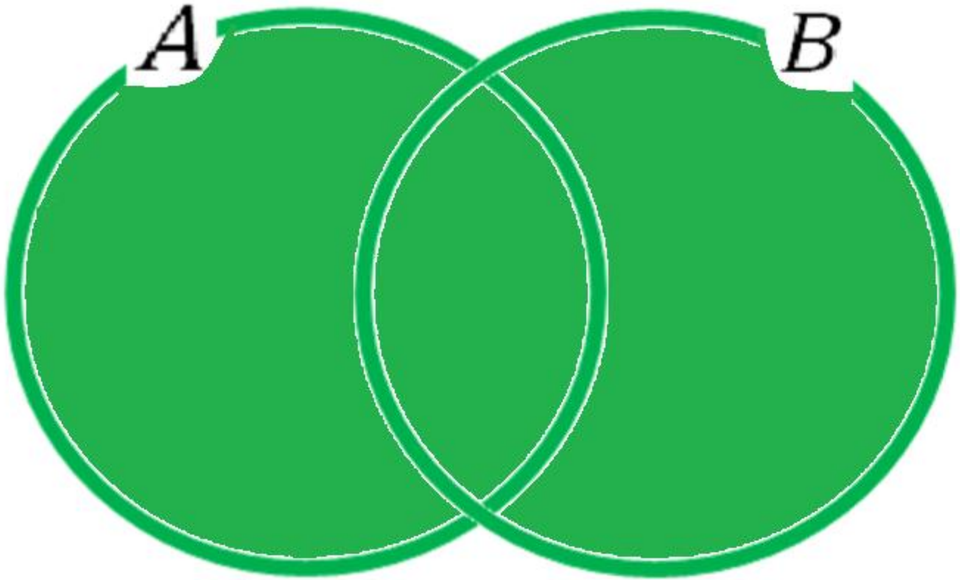
Probability of the Union of Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



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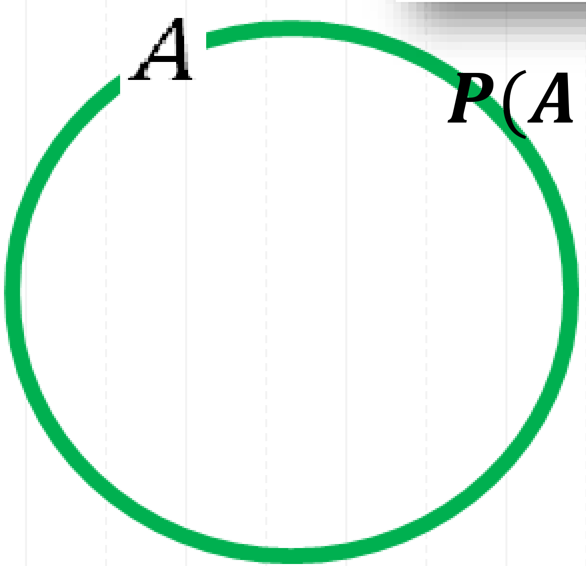
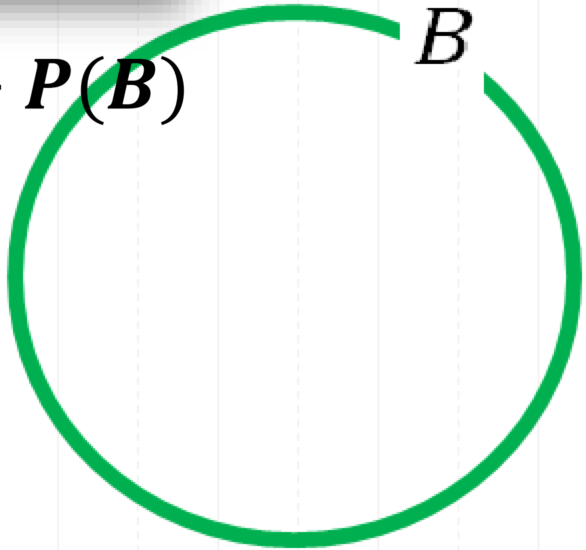
Probability of the Union of Events: Mutually Exclusive Events

Mutually Exclusive Events: Two events having no elements in common are said to be mutually exclusive (or **Disjoint** or **Incompatible**). In other words, the events A and B, with $A, B \subset S$, are disjoint if

$$A \cap B = \emptyset.$$ Empty set



$$P(A \cup B) = P(A) + P(B)$$



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8. In the manufacturing of a product, two errors may occur. The error A occurs 10% of the time and the error B occurs 5% of the time. Additionally, in 12% of the time, either A or B occur. Compute the probability that a random unit of the product has

a) both errors;

b) only A;

c) only B;

d) no error.

Exercise 8

$P(A) = 0.1$ $\quad P(A \cup B) = 0.12$

$P(B) = 0.05$

a)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) =$$

$$= 0.1 + 0.05 - 0.12 = 0.03$$

b) $P(A \setminus B) = P(A) - P(A \cap B) = 0.1 - 0.03 = 0.07$

c) $P(B \setminus A) = P(B) - P(A \cap B) = 0.05 - 0.03 = 0.02$

d) $P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.12 = 0.88$

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9. An electronic system is composed of two sub-systems, A and B. From previous experiments it is known that : the probability of A failing is 0.2, the probability that only B fails is 0.15 and the probability that A and B fail simultaneously is 0.15. Evaluate the probability that:

a) B fails.

b) Only A fails.

c) Either A or B fails.

d) Neither A nor B fails.

e) A and B don't fail simultaneously.

Exercise 9 a)

A ≡ Sub-system A fails

B ≡ " " B "

$P(A) = 0.2$

$P(B \setminus A) = 0.15$

$P(A \cap B) = 0.15$

a)

$P(B \setminus A) = P(B) - P(A \cap B) \Leftrightarrow$

$\Leftrightarrow 0.15 = P(B) - 0.15 \Leftrightarrow$

$\Leftrightarrow P(B) = 0.3$

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Exercise 9 b) and c)

b)

$$P(A \setminus B) = P(A) - P(A \cap B) =$$

$$= 0.2 - 0.15 = 0.05$$

c)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) =$$

$$= 0.2 + 0.3 - 0.15 = 0.35$$

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Exercise 9 d) and e)

d)

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.35 = 0.65$$

e)

$$P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.15 = 0.85$$

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10. In a College, 70% of the students have a desktop computer at home, 40% have a portable computer, and 30% have both. If a student is randomly chosen, evaluate the probability of the student:

  a) Has at least one of the two types of computers;
  b) Has no computer;
  c) Has one of the two types of computers.

Exercise 10 a)

HC ≡ Student has computer at home

PC ≡ " " portable computer

$P(HC) = 0.7$ $P(HC \cap PC) = 0.3$

$P(PC) = 0.4$

a)

$$P(HC \cup PC) = P(HC) + P(PC) - P(HC \cap PC) =$$

$$= 0.7 + 0.4 - 0.3 = 0.8$$

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Exercise 10 b) and c)

b) $P(\overline{HC} \cap \overline{PC}) = P(\overline{HC \cup PC}) = 1 - P(HC \cup PC) =$

$= 1 - 0.8 = 0.2$

c)

$P((HC \cap \overline{PC}) \cup (\overline{HC} \cap PC)) = P(HC \cup PC) - P(HC \cap PC) =$

$= 0.8 - 0.3 = 0.5$

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Thanks!

Questions?