
LABORATÓRIO DE ESTATÍSTICA



Matemática Aplicada à Economia e à Gestão
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CONTACTO

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<https://doity.com.br/estatistica-aplicada-a-nutricao>



<https://basiccode.com.br/produto/informatica-basica/>

PROGRAMA



1. Conceitos Fundamentais de Estatística



2. Análise Exploratória de Dados



3. Redução e Sintetização de Dados



4. Associação e Relações entre Variáveis



5. Números Índices

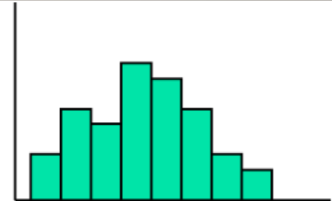
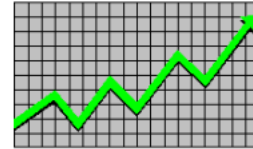


6. Sucessões Cronológicas

AULA 2: ANÁLISE EXPLORATÓRIA DE DADOS

ESTADÍSTICA DESCRIPTIVA

- Present data
 - e.g., Tables and graphs



- Summarize data

- e.g., Sample mean =
$$\frac{\sum X_i}{n}$$

REPRESENTAÇÃO DOS DADOS



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
H	He	Li	Be	B	C	N	O	F	Ne	Na	Mg	Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

- **Tabelas:** distribuições de frequências.

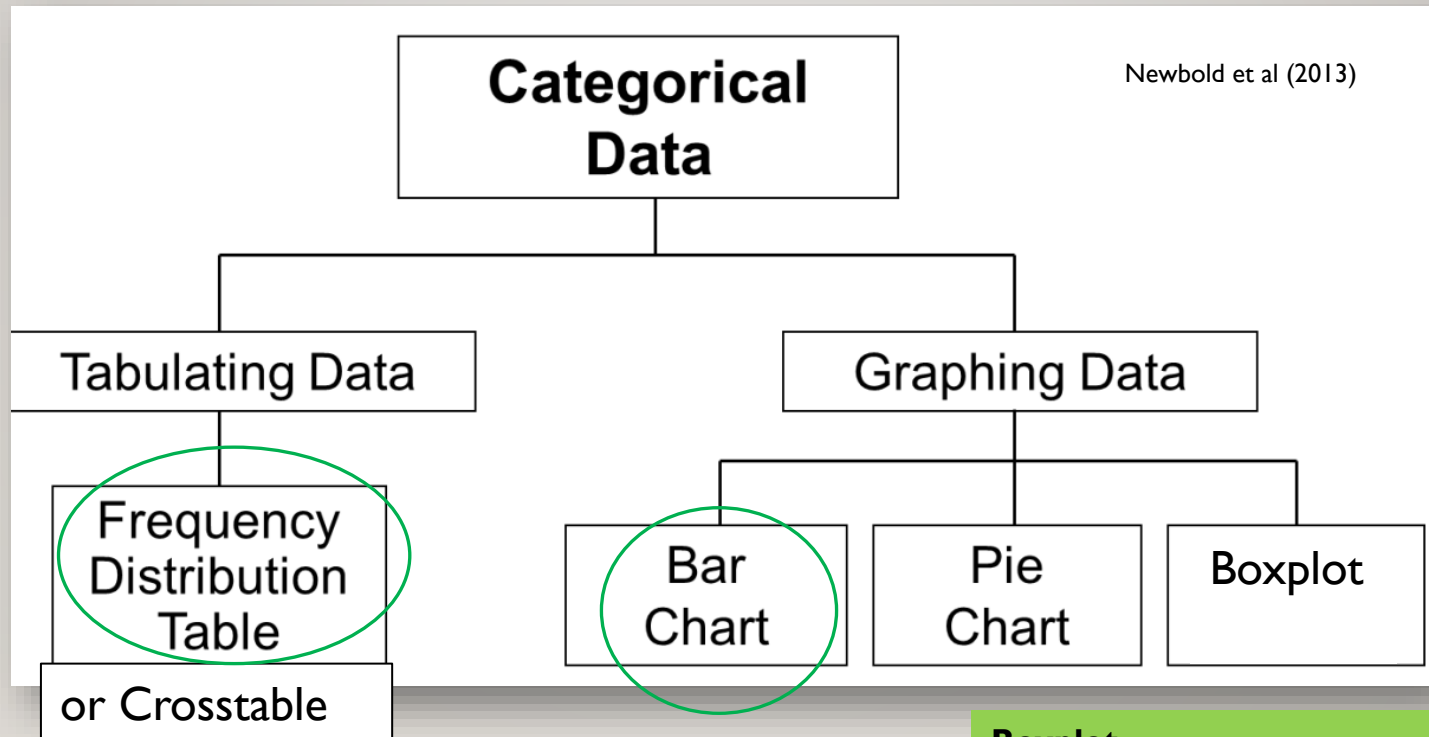


- **Gráficos:** gráfico de barras, gráfico de linhas, diagrama circular, histograma, caixa com bigodes (boxplot), etc.



- **A escolha depende do:** tipo de variável e objetivo da análise.

TABELAS E GRÁFICOS PARA VARIÁVEIS QUALITATIVAS



- **Frequency Table:** shows the distribution of one categorical variable. **Example:** Gender (Male: 40 and Female: 60).
- **Contingency Table / Crosstabulation:** shows the joint distribution of two categorical variables. **Example:** Gender × Education level.

Boxplot

- Not suitable for **nominal variables**, because they **lack order**.
- Can be used for **numerical variables** or **ordinal variables with five or more categories**.

TABELA DE FREQUÊNCIAS

Frequency Distribution

A **frequency distribution** is a table used to organize data. The left column (called classes or groups) includes all possible responses on a variable being studied. The right column is a list of the frequencies, or number of observations, for each class. A **relative frequency distribution** is obtained by dividing each frequency by the number of observations and multiplying the resulting proportion by 100%.

Newbold et al (2013)

TABELA DE FREQUÊNCIAS: EXEMPLO

Frequência Relativa
(porcentagem)

Summarize data by category

Example: Hospital Patients by Unit

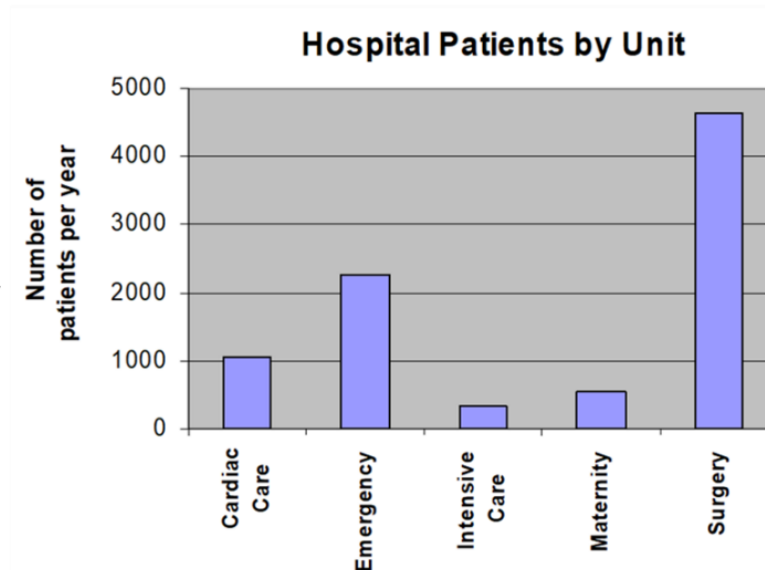
Frequência Absoluta

Hospital Unit	Number of Patients	Percent (rounded)
Cardiac Care	1,052	11.93
Emergency	2,245	25.46
Intensive Care	340	3.86
Maternity	552	6.26
Surgery	4,630	52.50
Total:	8,819	100.0

(Variables are
categorical)

GRÁFICO DE BARRAS: EXEMPLO

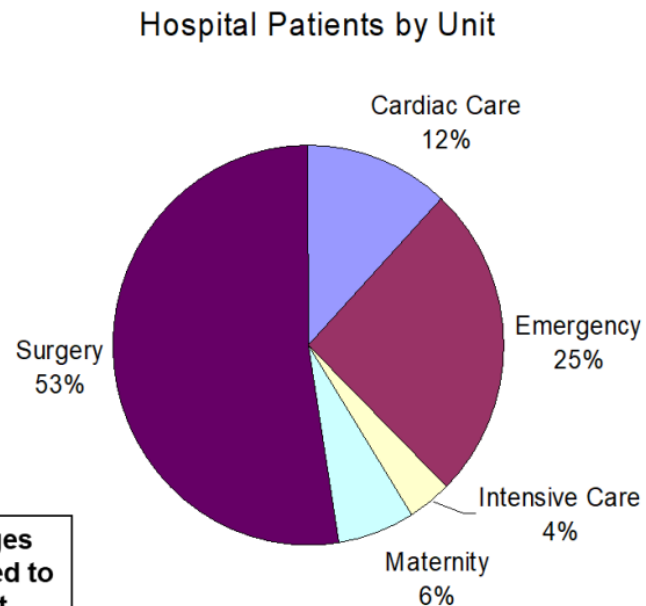
Hospital Unit	Number of Patients
Cardiac Care	1,052
Emergency	2,245
Intensive Care	340
Maternity	552
Surgery	4,630



Newbold et al (2013)

DIAGRAMA CIRCULAR: EXEMPLO

Hospital Unit	Number of Patients	% of Total
Cardiac Care	1,052	11.93
Emergency	2,245	25.46
Intensive Care	340	3.86
Maternity	552	6.26
Surgery	4,630	52.50



(Percentages
are rounded to
the nearest
percent)

Newbold et al (2013)

- **Bar charts and Pie charts** are often used for qualitative (categorical) data.
- Height of bar or size of pie slice shows the frequency or percentage for each category.

EXERCÍCIO: TABELA DE FREQUÊNCIAS & GRÁFICO DE BARRAS

Exercise

A survey was conducted among 30 university students about their preferred mode of transportation to campus. The responses are shown below:

 Walking,  Bicycle,  Bus,  Car,  Metro or underground

Data (sample):

Bus, Car, Walking, Bus, Metro, Car, Car, Bicycle, Walking, Bus,
Bus, Car, Metro, Car, Walking, Walking, Bicycle, Bus, Car, Metro,
Walking, Car, Bicycle, Car, Bus, Bus, Metro, Walking, Walking, Car

Tasks:

1. **Construct a frequency table** for the categorical variable *Mode of Transportation*. Include:
 - Absolute frequency
 - Relative frequency (per
2. **Draw a bar chart** to represent the distribution of transportation modes.



EXERCÍCIO I: SOLUÇÃO

✓ Resposta: Tabela de Frequências

$$\text{Relative Frequency} = \frac{\text{Absolute Frequency}}{n}$$

Mode of Transportation	Absolute Frequency	Relative Frequency
Bus	7	0.233
Car	9	0.300
Walking	7	0.233
Metro/Underground	4	0.133
Bicycle	3	0.100
Total	30	1.000

EXERCÍCIO I: SOLUÇÃO

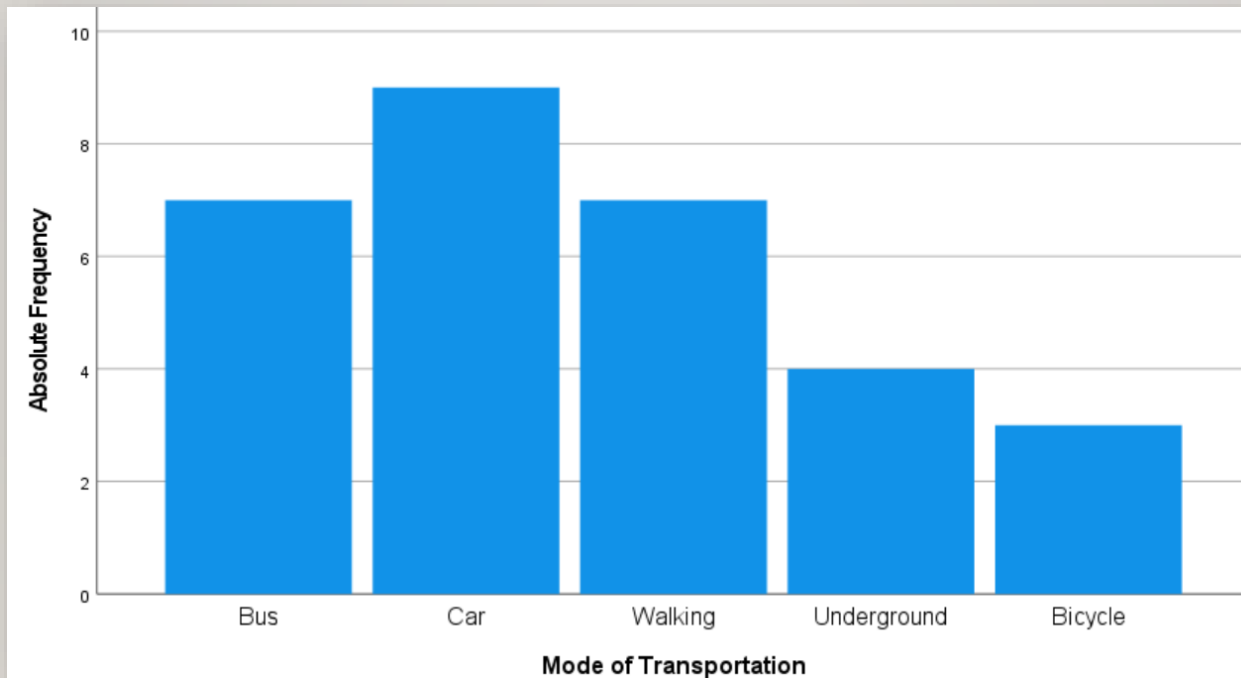
✓ Resposta: Tabela de Frequências com Frequências Absolutas Acumuladas e Frequências Relativas Acumuladas (suplementar)

Mode of Transportation	Absolute Frequency	Relative Frequency	Cumulative Absolute Frequency	Cumulative Relative Frequency
Bus	7	0.233	7	0.233
Car	9	0.300	16	0.533
Walking	7	0.233	23	0.766
Metro/Underground	4	0.133	27	0.899
Bicycle	3	0.100	30	0.999 $\approx$ 1.000
Total	30	1.000	—	—

EXERCÍCIO 2: SOLUÇÃO

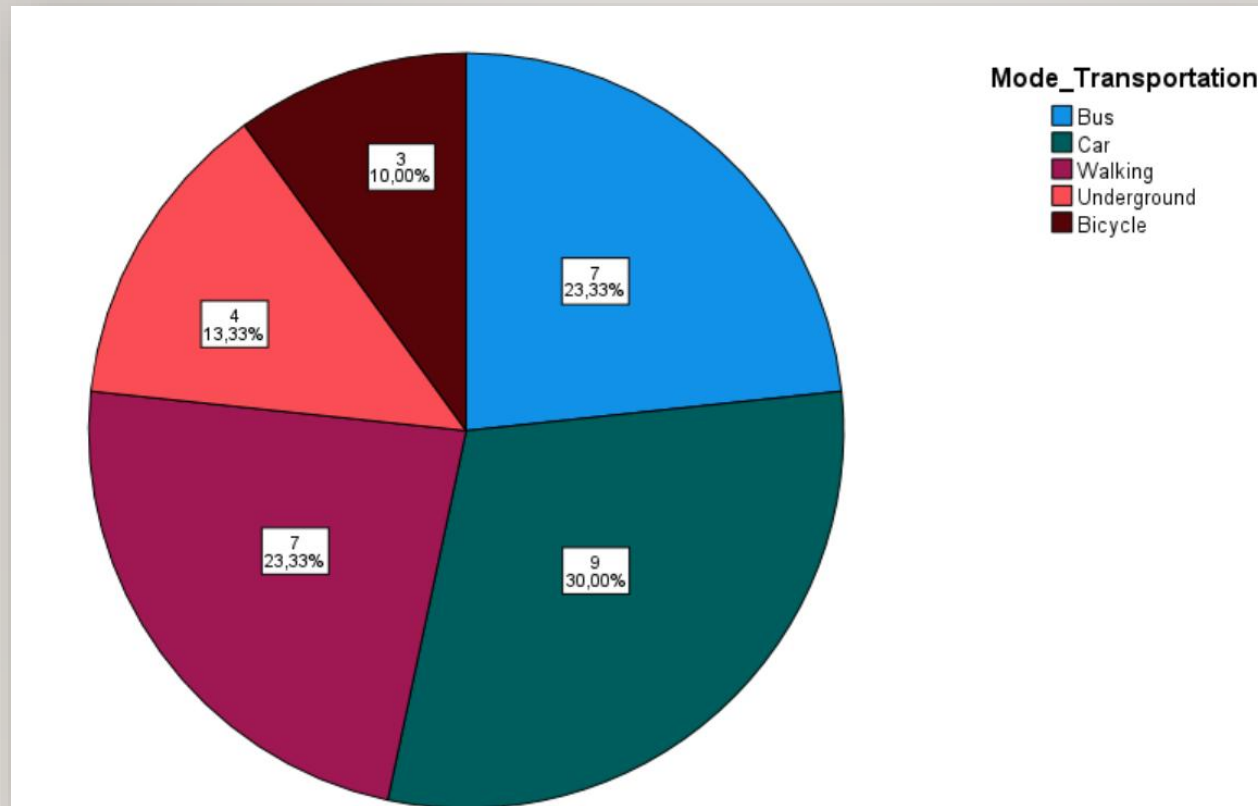


Resposta: Gráfico de Barras (Frequência Absoluta)



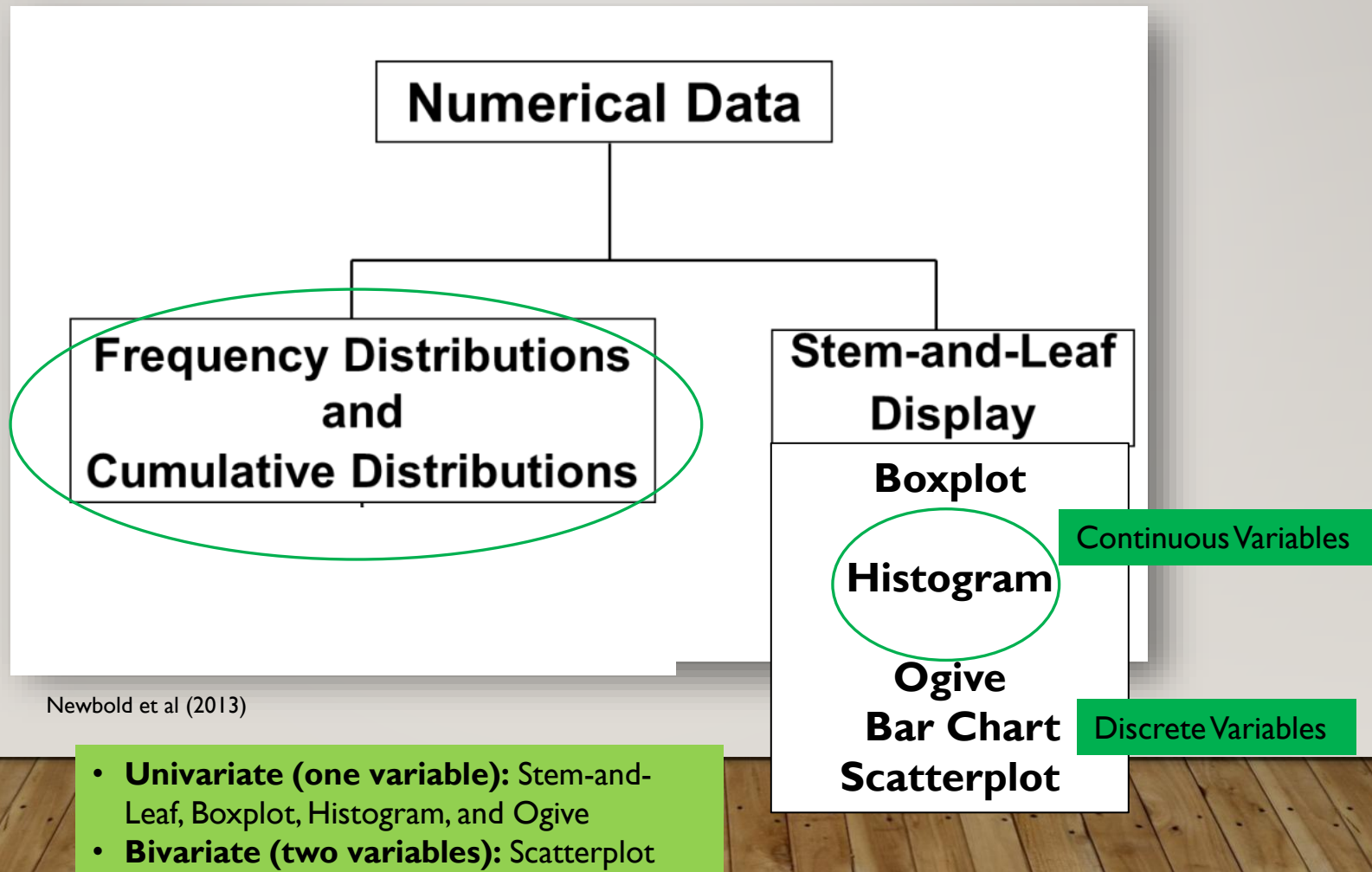
SPSS output

EXERCÍCIO 2: SOLUÇÃO



SPSS output

GRÁFICOS PARA VARIÁVEIS QUANTITATIVAS



REGRAS PARA CONSTRUIR AS CLASSES DE UMA TABELA DE FREQUÊNCIAS & HISTOGRAMA



1. Number of classes (k):

Sturges' Rule:

The number of classes k for a histogram is the smallest integer such that:

$$2^k \geq n$$

where n is the sample size.

or

"Square root rule for the number of classes: The number of classes (k) can be estimated as the square root of the number of observations (n):

$$k \approx \sqrt{n}$$

where n is the total number of data points."



2. Class width (h):

$$h = \frac{\max - \min}{k}$$

- Always round class width, h , upward.
- Classes must be inclusive and nonoverlapping.



3. Where to Start the First Class? (options)

- At the minimum observed value
 - e.g. data from 12 to 87, $k = 8$, $h \approx 9.4$
 - Classes: [12, 21.4), [21.4, 30.8), ...
- Round down to a "nice" number (e.g. multiple of 5 or 10)
 - Easier to read → start at 10, width = 10
 - Classes: [10, 20), [20, 30), ...
- General rule
 - Lower limit $\leq$ minimum
 - Upper limit $\geq$ maximum

CLASSES

- Each class grouping has the same width
- Determine the width of each interval by
$$w = \text{interval width} = \frac{\text{largest number} - \text{smallest number}}{\text{number of desired intervals}}$$
- Use at least 5 but no more than 15-20 intervals
- Intervals never overlap
- Round up the interval width to get desirable interval endpoints

CONSTRUÇÃO DAS CLASSES DE TABELA DE FREQUÊNCIAS PARA VARIÁVEIS QUANTITATIVAS: EXEMPLO

12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Classe	Ponto Médio	Frequência Absoluta	Frequência Relativa
[10, 20]	15	3	0.150
]20, 30]	25	7	0.350
]30, 40]	35	4	0.200
]40, 50]	45	4	0.200
]50, 60]	55	2	0.100
Total	—	20	1.000

Dimensão amostral: $n = 20$

Número de classes: $k = 5$ (Sturges' Rule)

Largura de cada classe = $(\text{Max-Min})/k = (58 - 12) / 5 = 9.2 \sim 10$

Sturges' Rule:

The number of classes k for a histogram is the smallest integer such that:

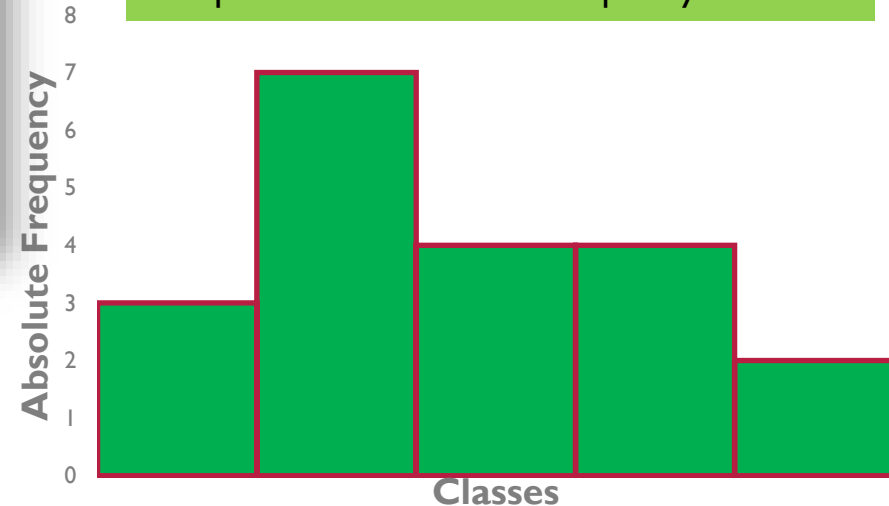
$$2^k \geq n$$

where n is the sample size.

HISTOGRAMA: EXEMPLO

Class Interval	Class Midpoint	Absolute Frequency	Relative Frequency
[10, 20]	15	3	0.150
]20, 30]	25	7	0.350
]30, 40]	35	4	0.200
]40, 50]	45	4	0.200
]50, 60]	55	2	0.100
Total	—	20	1.000

- If all class widths are equal, histogram heights can simply represent absolute or relative frequencies.
If class widths differ, using absolute frequency $\div$ class width as the bar height ensures that the area of each bar correctly represents the absolute frequency.



(No gaps
between
bars)

Temperature in Degrees

EXERCÍCIO 1.32

1.32 Consider the following data:

17	62	15	65
28	51	24	65
39	41	35	15
39	32	36	37
40	21	44	37
59	13	44	56
12	54	64	59

- Construct a frequency distribution.
- Construct a histogram.

Newbold et al (2013)



EXERCÍCIO 1.32 A): SOLUÇÃO



Resposta:

Class	Midpoint	Absolute Frequency	Relative Frequency
[10,21]	15.5	6	$6/28 \approx 0.214$
]21,32]	27	2	$2/28 \approx 0.071$
]32,43]	38	9	$9/28 \approx 0.321$
]43,54]	49	4	$4/28 \approx 0.143$
]54,65]	60	7	$7/28 \approx 0.250$

Sample size: $n = 28$

Number of classes: $k = 5$ (Sturges's Rule)

Width of each interval $= (\text{Max-Min})/k = (65 - 12) / 5 = 10.6 \sim 11$

EXERCÍCIO 1.32 B): SOLUÇÃO

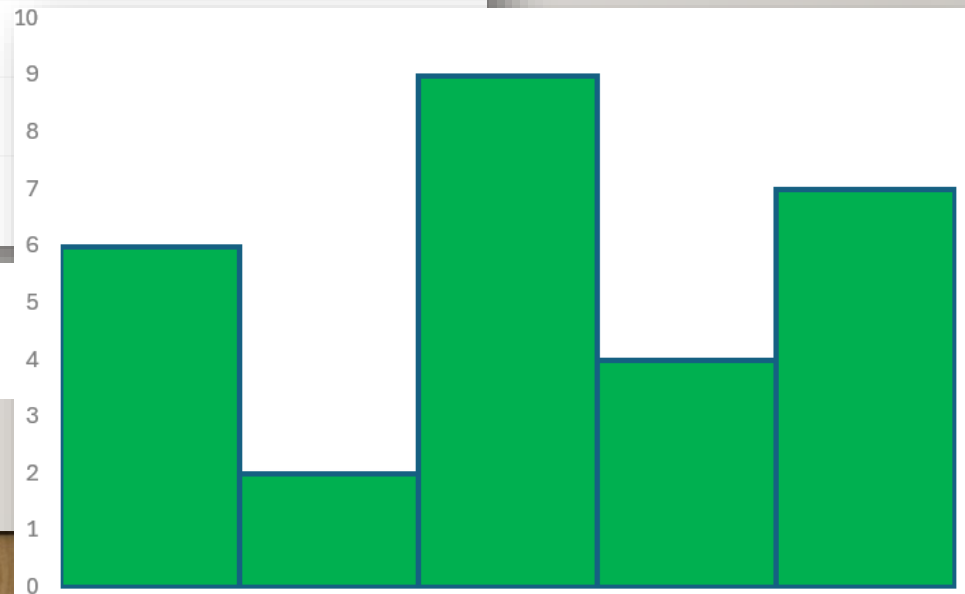


Resposta:

Class	Midpoint	Absolute Frequency	Relative Frequency
[10,21]	15.5	6	$6/28 \approx 0.214$
]21,32]	27	2	$2/28 \approx 0.071$
]32,43]	38	9	
]43,54]	49	4	
]54,65]	60	7	

Excel output

Absolute
Frequency



Classes

EXERCÍCIO 1.36

1.36 The following table shows the ages of competitors in a charity tennis event in Rome:

Age	Percent
18–24	18.26
25–34	16.25
35–44	25.88
45–54	19.26
55+	20.35

- Construct a relative cumulative frequency distribution.
- What percent of competitors were under the age of 35?
- What percent of competitors were 45 or older?

Newbold et al (2013)



EXERCISE 1.36 A): SOLUTION



Answer:

a) Relative cumulative frequency distribution

To get the cumulative percentage, we add the percentages successively:

Age	Percent	Cumulative Percent
18–24	18.26	18.26
25–34	16.25	$18.26 + 16.25 = 34.51$
35–44	25.88	$34.51 + 25.88 = 60.39$
45–54	19.26	$60.39 + 19.26 = 79.65$
55+	20.35	$79.65 + 20.35 = 100.00$

The **cumulative percentage** column is the **relative cumulative frequency**.

EXERCÍCIO 1.36 B): SOLUÇÃO



Resposta:

b) Percent of competitors under the age of 35

- Competitors under 35 include 18–24 and 25–34.
- Add the percentages:

$$18.26 + 16.25 = 34.51\%$$

34.51% of competitors were under 35.

EXERCÍCIO 1.36 C): SOLUÇÃO



Resposta:

c) Percent of competitors 45 or older

- Competitors 45 or older include 45–54 and 55+.
- Add the percentages:

$$19.26 + 20.35 = 39.61\%$$

39.61% of competitors were 45 or older.

OBRIGADA!

Questões?

