Submission date: December 6^{th}

Combinatorial Optimization Applied to Urban Waste Collection and Recycling

The objective of this project is to apply Combinatorial Optimization concepts and techniques to the planning and management of urban waste collection and recycling operations.

The project consists of studying two related decision-making problems: the location of waste transfer stations and the routing of waste collection vehicles. The aim is to develop solutions that minimize the operational costs and travelling distances while satisfying the operational requirements of the waste management system.

The problems should be formulated, implemented, solved, and analysed.

A municipal waste management company is responsible for collecting waste from several areas of a city and transporting it to waste transfer stations, where it is temporarily stored before being sent to recycling or treatment facilities.

The company is considering reorganizing its logistics network to improve the efficiency of its operations. In particular, it has identified several candidate locations for establishing waste transfer stations and needs to determine which locations should be selected.

The company must also organize the collection routes. A fleet of vehicles is available to collect waste from the different areas of the city and transport it to the corresponding transfer stations.

The decisions involve two interconnected problems:

- determining the location of waste transfer stations and assigning collection areas to the selected stations;
- determining the collection routes for the vehicles operating from each selected station.

The overall objective is to design an efficient waste collection network while taking into account the investment costs, transportation distances, and operational capacities.

Consider:

- a set of candidate locations for waste transfer stations;
- a set of urban areas where waste must be collected;
- the amount of waste generated in each urban area;
- the installation cost and processing capacity of each candidate transfer station;
- the distances between all candidate locations and urban areas; and
- a fleet of waste collection vehicles, each with a limited capacity.

The company must decide which transfer stations to open and assign each urban area to exactly one selected station. The total amount of waste assigned to a station cannot exceed either its processing capacity or the capacity of the collection vehicle operating from that station.

After determining the locations of the transfer stations and the assignment of the urban areas, the company must organize the collection routes. Each selected station will have exactly one collection vehicle operating from it. Each vehicle starts and ends its route at its corresponding transfer station and must visit all the urban areas assigned to that station.

The routing decisions should minimize the total travelling distance.

For simplicity, assume that the company has a sufficient number of vehicles available, that all vehicles have the same capacity, and that each selected transfer station is served by exactly one vehicle.

Each group must construct its own problem instance. The instance must contain, at a minimum:

- 4 candidate locations for waste transfer stations;
- 8 urban areas where waste must be collected;
- the amount of waste generated in each urban area;
- the installation cost and processing capacity of each candidate station;
- a complete distance matrix between all candidate locations and urban areas; and
- the capacity of the collection vehicles.

The groups may use:

- real-world data;
- data inspired by urban waste collection operations;
- data generated by the students; or
- use data from Project 1.

All data used must be properly documented and referenced.

The data should be selected so that the resulting problem admits feasible solutions and is sufficiently challenging to allow a meaningful analysis.

Tasks

For simplicity, the location and routing problems will be considered independently.

1. **Waste Transfer Station Location**

   The company must determine which candidate locations should be selected for establishing waste transfer stations and how the urban areas should be assigned to the selected stations.

   1.1 Formulate the problem as an Integer Linear Programming (ILP) model. Define all the sets, parameters, and decision variables required for the formulation.

   The model should include, at a minimum, constraints that ensure:

   - each urban area is assigned to exactly one transfer station;
   - an urban area can only be assigned to a station if that station is selected;
   - the total amount of waste assigned to each station does not exceed its processing capacity; and
   - the total amount of waste assigned to each station does not exceed the capacity of the corresponding collection vehicle.

   The objective function should minimize the total cost, including the installation costs of the selected stations and the transportation costs associated with the assignments.

   1.2 Implement the model using Excel Solver and obtain an optimal solution.

   1.3 Analyse the solution obtained. In particular, discuss:

   - the number and location of the selected transfer stations;
   - the assignment of the urban areas to the selected stations;
   - the utilization of the processing capacity of each station and the capacity of the corresponding vehicle; and
   - the contribution of installation and transportation costs to the total cost.

   1.4 Discuss the advantages and limitations of the proposed solution. Explain how changes in the data, such as installation costs or station capacities, could affect the selected locations and assignments.

   1.5 Use Excel Solver to solve its Linear Programming (LP) relaxation.

   1.6 Use Excel Solver (Evolutionary mode) to obtain a feasible solution to the original Integer Linear Programming (ILP) problem.

   1.7 Analyse and compare the bounds obtained. Calculate the relative optimality gap between the lower and upper bounds and discuss the quality of the feasible solution obtained.

   The relative optimality gap can be calculated as follows:

$$\text{Gap}(\%) = \frac{UB - LB}{UB} \times 100.$$

   where LB and UB denote, respectively, a lower bound and an upper bound on the optimal objective function value.

2. **Waste Collection Vehicle Routing**

   Assume that the solution obtained in the previous problem will be implemented.

   The company must now determine the collection routes for the vehicles operating from the selected transfer stations.

   2.1 Identify the combinatorial optimization problem that can be formulated to determine the collection routes. Describe its main characteristics and the decisions involved.

   2.2 Use Excel Solver (Evolutionary mode) to obtain a feasible routing solution that minimizes the total travelling distance.

   The solution must ensure that:

   - each urban area is visited exactly once;
   - each vehicle starts and ends its route at its corresponding transfer station;
   - each vehicle visits only the urban areas assigned to its transfer station; and
   - the total amount of waste collected along each route does not exceed the vehicle capacity.

   2.3 Present and analyse the routes obtained. In particular, discuss:

   - the sequence of visits for each vehicle;
   - the total distance travelled by each vehicle and by the entire fleet;
   - the amount of waste collected by each vehicle; and
   - the utilization of the vehicle capacities.

   2.4 Discuss the advantages and limitations of the proposed routing solution. Suggest possible improvements to reduce the total travelling distance or improve the utilization of the vehicles.

The computational implementation and resolution of both problems can be carried out using Microsoft Excel Solver. The use of other optimization software or programming languages, such as Python, PuLP, Gurobi, or FICO Xpress, may be used but it is not required.

Write a report of no more than eight pages. The report should be accurate, clear, and concise.
Submit the following two files:

- **Report**: a PDF file containing the report.
- **Computational files**: an Excel workbook or a ZIP file containing all the files used to implement the models and obtain the solutions. This may include Excel workbooks, data files, code files, or files associated with other solvers or computational tools, whenever applicable.

  The submitted computational files must include all the data, models, code, and other necessary files required to reproduce the results presented in the report.

Good work!