## General Guidelines

- You may use a calculator;
- You may not use a programmable calculator;
- You may not use notes or books;
- You may have some food and beverages on your desk;
- All other belongings, including phones, must be on the floor;
- You can only leave the room after 30 minutes into the exam and up unto 15 minutes before the exam ends;
- Write all your answers on the blank answer sheets brought by you;
- Write your name and student number on every answer sheet;
- Number all your answer sheets and hand them in in chronological order;
- If a question does not ask for an explanation, there is no need to give one;
- This exam is to be handed in together with your answer sheets;
- Any form of fraud will, at least, imply an invalid grade for this course.


## 1. Production (3 points)

Let $y=\beta x_{1}+\gamma x_{2}$ be a production function, where $y$ is the output and $x_{1}$ and $x_{2}$ are the two inputs.
1.1 Find the Technical Rate of Substitution (TRS) for the production above.

Consider for the following two questions that $\beta=2$ and $\gamma=4$.
1.2. Carefully sketch the input requirement set for producing at least 20 units of output:

$$
\left\{\left(x_{1}, x_{2}\right) \text { in } R_{+}^{2} \mid 2 x_{1}+4 x_{2} \geq 20\right\}
$$

1.3. Consider that in the short run $x_{1}$ is fixed at a value of 6 . Carefully sketch the short-run production possibilities set:

$$
\left\{\left(y, x_{2}\right) \text { in } R_{+}^{2} \mid 2 x_{1}+4 x_{2} \geq y, x_{1}=6\right\}
$$

## 2. Profit and costs (4 points)

Consider a firm that uses one input $x$ against price $w$. With that input it produces one output $y$ via the production function $y=f(x)$ that it sells at price $p$. Hence, the profit of the firm is:

$$
\pi(x, p, w)=p f(x)-w x
$$

The firms' factor demand function is $x(p, w)$, which is the optimal choice of the input $x$ as a function of the prices.
2.1. Substitute $x=x(p, w)$ into $\pi(x, p, w)$ and subsequently take the derivative of $\pi(x(p, w), p, w)$ towards $p$. Rewrite this derivative into a "direct" and "indirect" effect of $p$. Briefly interpret this direct and indirect effect.
2.2. Briefly explain why the indirect effect is equal to zero.
2.3. The CEO of this firm wants to test the Weak Axiom of Profit Maximization (WAPM). Which data does the CEO need to collect? Write down the formula that the CEO should calculate with those data.

## 3. Consumer choice (5 points)

Consider a consumer with a utility function equal to $u=x_{1}^{1 / 3} x_{2}^{2 / 3}$. The consumer has income $m$ and the prices for good $x_{1}$ and $x_{2}$ are denoted by $p_{1}$ and $p_{2}$ respectively.
3.1. Find the Marshallian demand functions for both good 1 and 2 .
3.2. Take the derivative of the Marshallian demand functions for good 1 and 2 towards $p_{1}$ and $p_{2}$, respectively. Use these derivatives to draw a conclusion about what type of goods $x_{1}$ and $x_{2}$ are.
3.3. Briefly explain what the Marginal Rate of Substitution (MRS) is. Find the MRS for the utility function above.
3.4. Find the indirect utility function as a function of the exogenous variables for the utility maximization problem above. Briefly discuss what the indirect utility function represents.
3.5. To find the Lagrange multiplier lambda one can take the derivative of the indirect utility function towards an exogenous variable. Which variable is this? Find lambda via this route. Provide a brief economic interpretation for lambda when $p_{1}=1 / 3, p_{2}=2 / 3$, and $m=20$.

## 4. Welfare (4 points)

Consider a consumer with a utility function equal to $u=2 \sqrt{x_{1}}+x_{2}$. The consumer has income $m=$ 10 , and the price for good $x_{1}$ and $x_{2}$ are $p_{1}=1$ and $p_{2}=2$ respectively.
4.1. Carefully sketch three indifference curves with varying levels of utility for the utility function above. Briefly explain the special feature of these indifference curves. Also briefly explain what this special feature implies for the income effect.
4.2. Consider that $p_{1}$ changes from 1 to 2 . Find the compensating variation for this change in the price for good 1.
4.3. Explain why for this utility function the compensating variation must be equal to the change in consumer surplus.

## 5. Perfect competition (2 points)

Consider a perfect competitive market. Let the total cost function of a single firm be equal to

$$
c(y)=0.5 y^{2}+8
$$

Let the market demand be given by

$$
X(p)=60-5 p
$$

And suppose that in the long run there is free entry into and exit out of this market, where all potential firms have the same cost function $c(y)$ as above.
5.1. How many firms will there be active in this perfect competitive market in the long run?

## 6. Monopoly ( 2 points)

In class we discussed the profit-maximizing behavior of a monopolist that charges the same price to each consumer. Now consider a monopolist that can charge a different price to each consumer. This is called perfect price discrimination.
6.1. Use the figure below to indicate what would be the profit maximizing quantity and pricing strategy of the monopolist that can perfectly price discriminate. MC stands for marginal costs, MR for marginal revenue, and $D(p)$ for demand.


