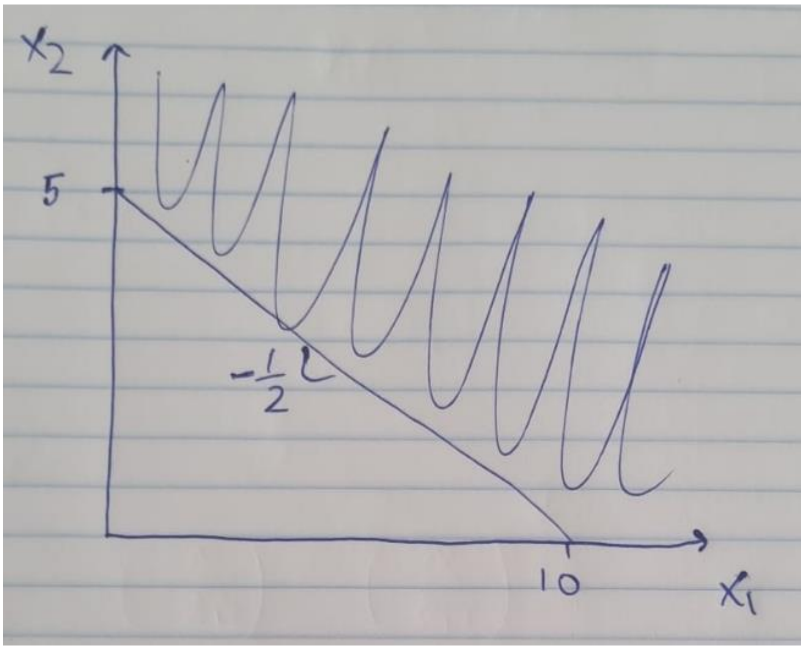
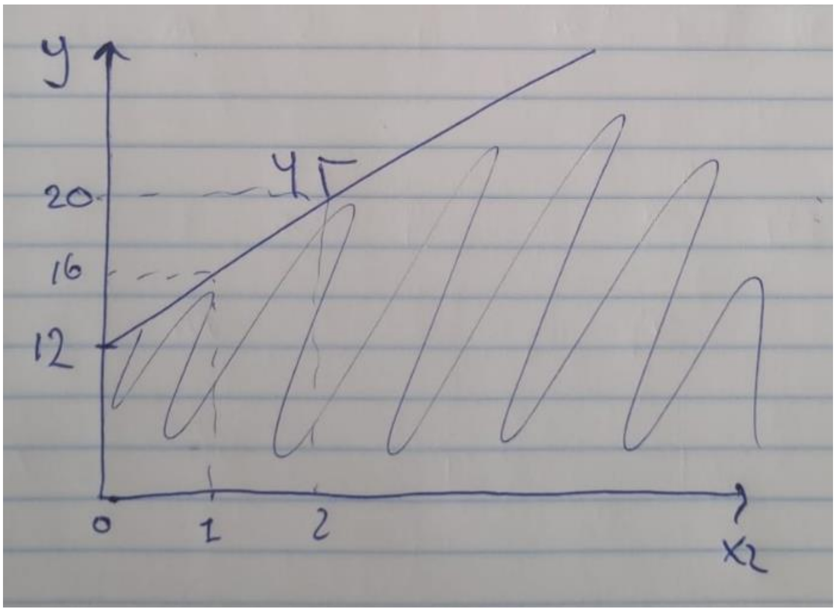
1.1. (1 point)

$$TRS = -\frac{\frac{df}{dx_1}}{\frac{df}{dx_2}} = -\frac{\beta}{\gamma}$$

1.2. (1 point)



1.3. (1 point)



2.1. (2 points)

First, recall that the firm obtains x(p,w) by taking the FOC, setting it to zero,

$$\frac{\partial \pi(x,p,w)}{x} = p\frac{\partial f(x)}{\partial x} - w = 0$$

and solving for x in terms of p and w.

Then substitute x(p,w) into the profit function:

$$\pi(x(p,w),p,w) = pf\big(x(p,w)\big) - wx(p,w)$$

And take the derivative of $\pi(x(p,w),p,w)$ towards p:

$$\frac{\partial \pi(x(p,w),p,w)}{p} = f\big(x(p,w)\big) + p\frac{\partial f(x)}{\partial x}\frac{\partial x(p,w)}{\partial p} - w\frac{\partial x(p,w)}{\partial p}$$

$$\underbrace{= f\big(x(p,w)\big)}_{direct} + \underbrace{\frac{\partial x(p,w)}{\partial p}\left(p\frac{\partial f(x)}{\partial x} - w\right)}_{indirect}$$

Direct effect: if p increases by 1, then profits increase by the amount of units sold, which is y=f().

Indirect effect: if p increases by 1, firm changes usage of x ($\frac{\partial x(p,w)}{\partial p} \neq 0$), which may affect profits.

2.2. (1 point)

Indirect effect is zero by FOC above: $\left(p\frac{\partial f(x)}{\partial x} - w\right) = 0$

The intuition for this result is that the firm already choose x as to maximize profits. Hence if p increases, the firm may slightly change her choice of x, but this does not affect profits because x was chosen as to maximize profits.

2.3. (1 point)

As stated in the question, the firm produces one output y with one input x, that have price p and w respectively. To test WAPM, the CEO needs to collect data across time on:

Prices: $\boldsymbol{p}^t = (p^t, w^t)$ for each time t

Output and input: $\boldsymbol{y}^t = (y^t, -x^t)$ for each time t

Note that actual profits in period t are equal to:

$\pi^t = \boldsymbol{p}^t\boldsymbol{y}^t = p^t y^t - w^t x^t$

The formula for WAPM is:

$\boldsymbol{p}^t\boldsymbol{y}^t \geq \boldsymbol{p}^t\boldsymbol{y}^s$ for all t and s=/t.

3.1. (1 point)

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 to reach:

$$x_1(p_1, m) = \frac{m\left(\frac{1}{3}\right)}{p_1}$$

$$x_2(p_2, m) = \frac{m\left(\frac{2}{3}\right)}{p_2}$$

3.2. (1 point)

$$\frac{\partial x_1(p_1, m)}{\partial p_1} = -\frac{m\left(\frac{1}{3}\right)}{p_1^2} < 0$$

$$\frac{\partial x_2(p_2, m)}{\partial p_2} = -\frac{m\left(\frac{2}{3}\right)}{p_2^2} < 0$$

For positive prices and income, these derivatives are smaller than zero. This implies that both goods are ordinary goods: if the price increases, the demand decreases.

3.3. (1 point)

MRS: If a consumer increases consumption for good 1, how much does the consumer need to decrease her consumption for good 2 to keep the same utility level. This describes the slope of the indifference curve.

$$MRS = \frac{\partial x_2(x_1)}{\partial x_1} = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{\frac{1}{3}x_1^{-\frac{2}{3}}x_2^{\frac{2}{3}}}{\frac{2}{3}x_1^{\frac{1}{3}}x_2^{-\frac{1}{3}}} = -\left(\frac{1}{2}\right)\left(\frac{x_2}{x_1}\right)$$

3.4. (1 point)

1. fill the Marshallian demand functions into the utility function

2. simplify

$$v(\boldsymbol{p}, m) = \left(\frac{m\left(\frac{1}{3}\right)}{p_1}\right)^{\frac{1}{3}}\left(\frac{m\left(\frac{2}{3}\right)}{p_2}\right)^{\frac{2}{3}} = m\left(\frac{1}{3}\right)^{\frac{1}{3}}\left(\frac{2}{3}\right)^{\frac{2}{3}}p_1^{-\frac{1}{3}}p_2^{-\frac{2}{3}}$$

The indirect utility function gives the maximum utility achievable given prices and income.

3.5. (1 point)

To obtain lambda one needs to take the derivative of the indirect utility function towards m:

$$\frac{\partial v(\boldsymbol{p}, m)}{\partial m} = \lambda(\boldsymbol{p}) = \left(\frac{1}{3}\right)^{\frac{1}{3}} \left(\frac{2}{3}\right)^{\frac{2}{3}} p_1^{-\frac{1}{3}} p_2^{-\frac{2}{3}}$$

Plugging in for $p_1 = \frac{1}{3}$ and $p_2 = \frac{2}{3}$ gives us:

$$\lambda(p_1, p_2, \bar{u}) = \left(\frac{1}{3}\right)^{\frac{1}{3}} \left(\frac{2}{3}\right)^{\frac{2}{3}} \left(\frac{1}{3}\right)^{-\frac{1}{3}} \left(\frac{2}{3}\right)^{-\frac{2}{3}} = 1$$

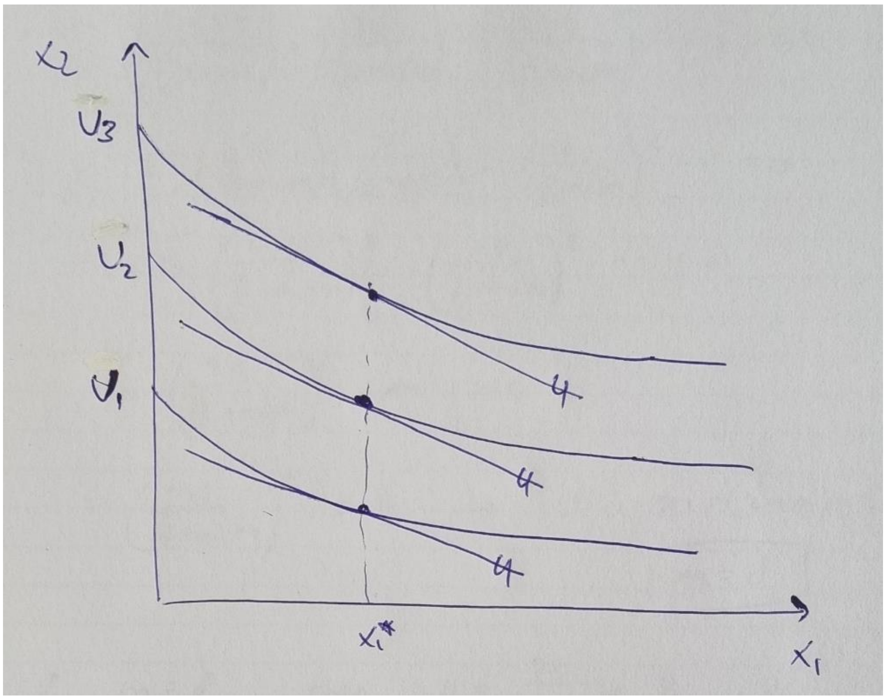
This implies that if the consumer's income increases by 1 (so we relax the constraint by 1), then the consumer can obtain one additional util.

4.1. (1 point)

The formula for the indifference curve is:

$x_2 = u - 2\sqrt{x_1}$

And we need to draw this for three levels of u:



Explanation of special feature: The varying levels of u just change the intercept of this indifference curve, where the slope of this indifference curve does not depend upon the levels of u. Hence, the indifference curves are parallel: they are simply vertical shifts from one to the other.

Explanation of consequence for income effect: since the slope of this indifference curve does not depend upon the level of u, the indifference curves related to the varying levels of u will be tangent to the budget line for varying income levels at the same level of x1. This implies that the income effect for x1 is zero. This is reflected by x_1^* in the figure.

4.2. (2 points)

To find the compensating variation you can follow three steps:

1. Solve the UMP at original prices and income to find indirect utility

$v(p^0, m) = 7$

2. Solve the EMP at new prices and v^0 to find minimum expenditure

$e(p^1, v^0) = 12$

3. Calculate CV by subtracting income in step 1 from minimum expenditure in step 2.

$CV = e(p^1, v^0) - m = 12 - 10 = 2$

4.3. (1 point)

Since the income effect is zero, the Marshallian and Hicksian demand coincide.

The compensating variation is the area to the left of the Hicksian demand curve. The change in consumer surplus is the area to the left of the Marshallian demand curve. Since the Hicksian and Marshalian demand coincide, the compensating variation and change in consumer surplus are the same.

5.1. (2 points)

There are two conditions for long run equilibrium:

$Y(p) = X(p)$

$\pi_i = 0 \quad for\ each\ firm\ i$

1. We derive the firms' supply function $y_i(p)$ for each firm i.

$mc_i(y) = \frac{dc_i(y)}{dy} = y,$

and since supply curve is $mc_i(y) = p$

we have that $y_i(p) = p$.

2. We derive market supply, which is the sum over all firms m.

$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p = mp.$

3. We use the first condition to find equilibrium price and firm supply in terms of number of firms m.

$$mp = 60 - 5p$$

$$p = \frac{60}{m+5}$$

$$y_i(p) = p = \frac{60}{m+5}$$

4. We use the second condition to find the number of firms m so that profits are zero.

$$\pi_i = py_i(p) - c_i(y) = 0$$

$$\pi_i = \left(\frac{60}{m+5}\right)^2 - 0.5\left(\frac{60}{m+5}\right)^2 - 8 = 0$$

$$0.5\left(\frac{60}{m+5}\right)^2 = 8$$

$$\left(\frac{60}{m+5}\right)^2 = 16$$

$$\frac{60}{m+5} = 4$$

$$m = 10$$

Hence, in the long run there will be 10 active firms in this perfect competitive market.

6.1. (2 points)

The monopolist that can perfectly price discriminate will ask the willingness to pay of each consumer. That is, for each consumer it will ask an epsilon below the price that makes the consumer indifferent between buying and not buying the product. This implies that each consumer buys the product while the monopolist can extract maximum rent. The monopolist does this until the willingness to pay of the consumer is equal to the marginal costs. When a consumer's willingness to pay is below marginal cost, the monopolist must ask a price below marginal cost to that consumer as to sell the product. This means that profits will decrease by selling to that consumer. In turn, when a consumer's willingness to pay is above marginal cost, the monopolist can ask a price above marginal costs to that consumer, and profits will increase by making that sell.

For the figure below, this implies that the monopolist produces until the demand curve hits the marginal cost curve, indicated by Q*.

Moreover, the pricing strategy implies that the producer surplus (or "profits" if fixed costs are zero) is indicated by the green area in the figure below. The price just follows the demand curve (each consumer pays their willingness to pay, which is different for each consumer) and (P-MC) for each sell reflects the surplus for the monopolist.