1) *Nhame Nhame* is a sashimi gourmet restaurant in Viana da Beira. The table below shows the demand for meals at this restaurant, and its total costs.

Meals per day	Willingness to pay €	Total cost, €
0	-	50
1	100	90
2	90	120
3	80	150
4	70	185
5	62	227
6	55	276
7	45	336

   a) Has this restaurant monopoly power? Explain.
   b) Calculate the marginal revenue. Explain how the difference between marginal revenue and price changes as the number of meals increases.
   c) Find the optimal quantity and price, and the profit.
   d) Find the deadweight loss of the monopoly. Explain.
   e) Suppose the mayor threatens to paralyse the restaurant with frequent inspections and legal harassment if the restaurant does not cap its price at €50. The threat is credible. What is the new marginal revenue? Explain. What is the new optimal quantity and price, and the profit?

2) Derive the marginal revenue as a function of price-elasticity of demand, and show that a monopolist does not maximise profit in the inelastic section of the demand curve.

3) *Quê ò Qubo* is a monopolistic firm facing the demand curve $y(p) = 100 - 2p$. Its cost function is $c(y) = 10y$.
   a) Compare price and marginal revenue when the firm sells 10 units. And if it sells 20 units? And 30? Explain how the difference between marginal revenue and price change as quantity increases.
   b) Without calculating the optimal quantity, explain whether any of the previous quantities is optimal.
   c) What is the price-elasticity of demand when the price is €20? And when it is €25? Without calculating the optimal price explain whether €20 or €25 is the optimal price.
   d) Find the optimal quantity and price.
   e) Find the deadweight loss of the monopoly. What does it mean?
   f) Find the markup at the optimal price.
   g) Derive the general expressions for the markup as a function of price-elasticity of demand. From this expression and the value you found in part g) find the value of the price-elasticity of demand at the optimal quantity.
   h) Calculate the price-elasticity of demand directly from the demand curve, and confirm the value you found in part g).

4) A market with demand curve $p(y) = 30 - 0.2y$ is supplied by a single firm with cost function $c(y) = 10y$. No other firm can enter the market.
   a) Draw the demand, marginal revenue, marginal cost, and average cost, curves.
   b) Find the maximum profit and corresponding quantity and price. Show the area corresponding to profit in the graph in part a).
   c) New safety regulations force the firm to incur a new fixed cost of 200. How does this affect the maximum profit and corresponding quantity and price?
   d) What if the new fixed cost were 600 instead?
   e) Owing to higher energy prices the firm's costs increase 4 per unit produced. Starting from the situation in part b) how does this affect the maximum profit and corresponding quantity and price?

5) Government levies a €10 tax per unit sold by a monopoly. What effect will this tax have if the monopoly:
   a) Faces a linear demand curve and has constant marginal costs?
   b) Faces a linear demand curve and has increasing marginal costs?
   c) Faces a linear demand curve and has decreasing marginal costs?
   d) Faces a demand curve with constant price-elasticity equal to -2 and has constant marginal costs?

6) In which of the following cases would you expect a monopolistic market? Explain.
   a) The cost function is $c(y) = y^2 + 25$ if $y > 0$, and $c(0) = 0$, and the demand curve is $y(p) = 210 - p$.
   b) The cost function is as above and the demand curve is $y(p) = 16 - p$.
   c) The cost function is $c(y) = y + 25$ if $y > 0$, and $c(0) = 0$, and the demand curve is $y(p) = 210 - p$.

7) Suppose government levies a20% tax on a monopolist's economic profit.
   a) What is the effect of this tax on the optimal quantity? And on price?
   b) Would you expect the *IRC* (tax on firms' profits in Portugal) to have the same effect as the tax in part a)? Explain.

8) A firm has a monopoly in the domestic market. Imports and exports are banned. The domestic inverse demand curve is $p(y) = 60 - y$. The firm's cost curve is $c(y) = 0{,}5y^2$.
   a) Find the profit-maximising quantity and price, the profit, and the deadweight loss.
   b) A new trade deal allows imports and exports. The price in the world market is 28. There are no transportation costs. How much will the firm produce? What is the quantity sold in the domestic market, and how much are exports or imports? What is the effect on total surplus (producer' and consumers' surplus). How much does profit change?
   c) Now the government bans imports again, but exports are still allowed. How much will the firm sell in the

domestic and foreign markets. What is the domestic price. How much profit does the firm make. Compare the results with those of the previous parts.

Multiple-Choice Questions

1. For any given price, marginal revenue is the higher:
   a) The more elastic the demand.
   b) The more inelastic the demand.
   c) The closer to -1 is the price-elasticity of demand.
   d) None of the other options is correct.

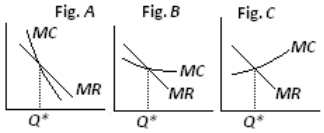
2. If the inverse demand curve is $p = a - by$, the marginal revenue curve is:
   a) $MR = a/2 - by$.
   b) $MR = 2a - by$.
   c) $MR = a - by/2$.
   d) $MR = a - 2by$.

3. If the demand curve is $y = 20 - 0.5p$, the marginal revenue curve is:
   a) $MR = 10 - y$.
   b) $MR = 40 - 4y$.
   c) $MR = 20 - 0.25y$.
   d) $MR = 20 - 2y$.

4. In what circumstances is marginal revenue negative?
   a) Always.
   b) Never.
   c) If demand is elastic.
   d) If demand is inelastic.

5. If price-elasticity of demand is constant marginal revenue:
   a) Is zero.
   b) Is constant, but not necessarily zero.
   c) Decreases as quantity increases.
   d) None of the other options are correct.

6. In which of the figures is Q^* a profit-maximising quantity?
   a) All of them.
   b) *C* only.
   c) *A* and *C* only.
   d) *B* and *C* only.

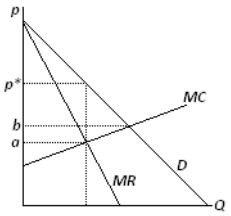


7. A monopolist has constant marginal costs and faces a linear demand curve. The government levies a new specific tax of €6. The profit maximising price will:
   a) Increase by €3.
   b) Increase by €6.
   c) Increase by €12.
   d) None of the other options is correct.

8. A monopolist has constant marginal costs and faces a demand curve featuring constant price-elasticity. The government levies a new €6 specific tax. The profit maximising price will increase?
   a) €3.
   b) €6.
   c) More than €6.
   d) Between €3 and €6.

9. A monopolist maximises profit in the elastic section of the demand curve because:
   a) Otherwise it could reduce costs while maintaining or increasing revenue.
   b) It can charge a higher price in that section.
   c) It can sell a larger quantity in that section.
   d) It has more market power in that section.

10. The figure shows a monopolist currently charging a price p^*. A price ceiling p_c imposes on the monopolist will lead to increased output if and only if:
    a) A price ceiling never leads to increased output.
    b) $b < p_c < p^*$.
    c) $a < p_c < p^*$.
    d) $p_c < p^*$.



Answers

1.a) Yes. The demand curve is negatively sloped, which means the firm can increase prices above those of other restaurants without losing all its customers as would happen it if were a perfectly competitive firm. This means the firm is not a price taker. The likely reason for this is that *Nhame Nhame* is the only restaurant of its kind in the are of Viana da Beira.

1.b) With discrete variables, the marginal revenue can easily be computed calculating first total revenue, $r(y) = p(y)y$, and calculating changes: $r(1) = 100 \times 1 = 100$; $r(2) = 90 \times 2 = 180$. So $MR(2) = r(2) - r(1) = 80$, and so on. Marginal revenue is also $MR(y_i) = \Delta R_i = p_i - \Delta p_i \times y_{i-1}$. As quantity increases p_i falls, but, if Δp_i is constant, marginal revenue falls even more because y_{i-1} rises. See illustration in the chapter on demand.

1.c) We need to calculate marginal cost now. Marginal revenue exceeds marginal cost up to the fourth unit, and is lower than marginal cost after the fourth unit. So the monopolist maximises profit with 4 meals and a price of €70. Profit is €95.

1.d) Willingness to pay exceeds marginal cost up to the sixth meal. The net surplus (willingness to pay minus marginal cost) that the fifth and sixth units would generate if they were produced and consumed, €20 + €6 = €26, is the deadweight loss of the monopoly.

1.e) The restaurant cannot charge a price above €50. At this price the quantity demanded is 6. So the marginal revenue is now €50 up to the sixth unit. $r(6) = €300$; $r(7) = €315$. So $MR(7) = €15$. €50 exceeds the marginal cost of the sixth unit but not that of the seventh. So 6 is the optimal number of meals. Profit is €24.

2) See derivation in the texbook: $MR = p[1 - 1/|\varepsilon|] < 0$ if $|\varepsilon| < 1$. That is, marginal revenue is negative if demand is inelastic. So a reduction in quantity increases revenue. As cost falls, profit increases. So the monopolist was not maximising profit.

3.a) The inverse demand curve is $p(y) = 50 - 0.5y$. Total revenue function is $r(y) = 50y - 0.5y^2$. So marginal revenue function is $MR(y) = 50 - y$. $MR(10) = 40$, $MR(20) = 30$, $MR(30) = 20$. Generally, marginal revenue is $\partial r(y)/\partial y = p + y\partial p(y)/\partial y \Leftrightarrow p - \partial r(y)/\partial y = -y\partial p(y)/\partial y$. In this case $\partial p(y)/\partial y$ is constant (-1), So the difference between marginal revenue and price increases as y increases.

3.b) No. Marginal revenue, for those quantities, exceeds marginal cost. So profit increases when quantity increases.

3.c) The price-elasticity of demand is $\varepsilon(p) = \partial y(p)/\partial p \times p/y(p) = -2p/y(p)$. So $\varepsilon(20) = -2 \times 20/60 = -0.667$. As explained in 2), the monopolist is not maximizing profit. If $p = €25$, $\varepsilon(25) = -2 \times 25/50 = -1$. Here quantity falls by the same percentage as the price rises. So revenue does not change when the price rises; but cost falls, so profit increases. Therefore $p = €25$ does not maximize profit.

3.d) $MC(y) = MR(y) \Leftrightarrow 10 = 50 - y \Leftrightarrow y = 40$. $p(40) = 30$. Profit is $\pi = py - c(y) = 30 \times 40 - 10 \times 40 = 800$.

3.e) Total surplus would be maximised with $MC(y) = p(y) \Leftrightarrow 10 = 50 - 0.5y \Leftrightarrow y = 80$. The deadweight loss of the monopoly is the surplus lost relative to the maximum: $(30\text{-}10) \times (80 - 40)/2 = 400$.

3.f) As defined in the textbook *markup* é $p/MC = 3$ (price is 200% higher than marginal cost).

3.g) See derivation in Varian: $MR = p[1 - 1/|\varepsilon|]$. From the optimum condition: $MC = MR \Leftrightarrow p/MC = 1/[1 - 1/|\varepsilon|]$. Making it equal to 3 we get $|\varepsilon| = 1.5$.

3.h) From c): $\varepsilon(p) = 2p/y(p)$. So $\varepsilon(30) = -2 \times 30/40 = -1.5$.

4.a) In the (y, p/costs) space the demand curve goes from (0,30) to (150,0). Revenue function is $r(y) = 30y - 0.2y^2$, so the marginal revenue function is $MR = 30 - 0.4y$, which goes from (0,30) to (75,0); the marginal cost curve is horizontal at $MC = 10$. As the marginal cost is constant and there are no fixed costs the average cost is constant and equal to the marginal cost.

4.b) Profit is maximal when $MC = MR \Leftrightarrow 10 = 30 - 0.4y \Leftrightarrow y = 50$; $p(50) = 20$. Profit: $\pi = py - c(y) = 20 \times 50 - 10 \times 50 = 1000 - 500 = 500$. Profit also equals quantity multiplied by the average profit (difference between price and average cost); so the profit equals the area od the rectangle between the average (and marginal) cost curve and the line corresponding to the price and with width equal to quantity.

4.c) The condition for profit maximization is marginal cost equals marginal revenue. The fixed cost has no impact on these variables, so optimal quantity and price do no change. So profit falls by 200 to 300.

4.d) Now the firm would incur a loss of 100, so it would exit the market.

4.e) The marginal cost increases by 10 to 14. $MC = MR \Leftrightarrow 14 = 30 - 0.4y \Leftrightarrow y = 40$; $p(42.5) = 22$. We could also notice that as the marginal cost curve shifts up by 4, so marginal revenue must increase by 4 too to maximise profit; as the marginal revenue curve is twice as steep as the inverse demand curve, price increases only half as much as marginal revenue (check the graph). The profit is $\pi = py - c(y) = 22 \times 40 - 14 \times 40 = 320$.

5.a) The marginal cost with tax increases by €10. So profit maximisation requires that marginal revenue rise by €10 too. Price increases by half as much as marginal revenue, €5 (with linear demand curves the slope of the marginal revenue curve is twice that of the inverse demand curve).

5.b) As quantity falls, marginal cost without tax falls, so with tax it rises less than €10. So marginal revenue rises less than €10, and price rises less than €5.

5.c) It's the opposite of 4.b). Marginal cost without tax rises, so with tax it rises more than €10. So does marginal revenue. Hence the price rises by more than €5.

5.d) As we saw in 3.g), $MR = MC \Leftrightarrow p = MC/[1 - 1/|\varepsilon|]$. With $|\varepsilon| = 2$ we have $p = 2MC$. That is, price rises by €20.

6.a) The minimum efficient scale (quantity that minimises average cost) is $y = 5$, and the corresponding average

and marginal cost is 10. At this price quantity demanded would be 200, which allow 40 firms operating at the minimum efficient scale in the market. So the market would probably be a perfectly competitive one.

6.b) With such a small market, there would be room for only one firm, which would be a monopoly.

6.c) Average cost is $1 + 25/y$. The more the firm produces the lower the average cost. The firm is therefore a natural monopoly.

7.a) The monopolist's after tax profit is $\pi = 0.8 \times [r(y) - c(y)]$. To maximise profit: $\partial\pi/\partial y = 0 \Leftrightarrow 0.8(MR - MC) = 0 \Leftrightarrow MR - MC = 0$. This condition is the same as in the no-tax situation. That is, to maximise 80% of profit you have to maximise 100% of profit.

7.b) *IRC* is levied on accounting profit, not economic profit. One important difference between the two is that the remuneration of own capital (its opportunity cost) is included in the costs to compute economic profit (so not taxed in 6.a)) but not to compute accounting profit. So *IRC* is in effect levied on economic profit plus some of the costs (opportunity cost of own capital). Therefore *IRC* increases costs, namely marginal cost, which decreases optimal quantity and increases price.

8.a) $r(y) = 60y - y^2$. $MC(y) = MR(y) <=> y = 60 - 2y \Leftrightarrow y = 20$. $p(20) = 40$. $\pi = 40 \times 20 - 0.5 \times 20^2 = 600$. Total surplus is maximised with $MC(y) = p(y) \Leftrightarrow y = 30$. The deadweight loss is $(p(20) - MC(20)) \times (30 - 20)/2 = (40 - 20) \times 10/2 = 100$ (draw a graph for better understanding).

8.b) The firm must now content itself with the international price. Its marginal revenue is now equal to the international price. So to maximise profit: $MC(y) = 28 \Leftrightarrow y = 28$. This is the firm's output. The domestic demand curve is $y(p) = 60 - p$. So que quantity demanded domestically is $y(28) = 32$. Imports are 4. If you draw a graph you'll see that the extra surplus is the former deadweight loss plus a little triangle, the area of which is $(30 - 28)\times(32 - 28)/2 = 4$. Total surplus increase by 100 + 4 = 104.

8.c) The firm is again able to charge the price it likes in the domestic market; and it can export at a price of 28. So domestic marginal revenue is again $MR(y) = y = 60 - 2y$. In the foreign market marginal revenue is 28 (when it sells one additional unit, it gets 28 for that unit, and does not have to reduce the price it charges for the previous units). So in the domestic market, marginal revenue will have to be 28 as well: $28: MR(y_i) = 28 \Leftrightarrow y_i = 16$. Marginal cost will have to be 28 too, so $y_t = 28$, and exports will be $y_e = 12$ ($y_t = y_i + y_e$, output = domestic demand + exports). Domestic price is $p(16) = 44$. Profit is $\pi = 28y_e + 44y_i - c(28) = 28 \times 12 + 44 \times 16 - 0{,}5 \times 28^2 = 648$. Even though it can sell additional units in the domestic market at a price exceeding the export price, the firm prefers not to do it. To sell more domestically it would have to lower the price, which would cause domestic marginal revenue to fall below the export price.

Answers to the multiple-choice questions

1a 2d 3b 4d 5c 6d 7a 8c 9a 10c.