1) A monopolist sells in two markets with inverse demand curves $p_1 = 62 - y_1$ and $p_2 = 41 - 0{,}5y_2$. Costs are given by $c(y) = 10y$ where y is total output: $y = y_1 + y_2$.
   a) The firm is forced to charge the same price in both markets. Find the optimal quantity and price, and the profit.
   b) What are the marginal revenues and price-elasticities of demand in both markets? Explain the relationship between the prices and marginal revenues.
   c) Explain without calculations what the firm would do if it were allowed to charge different prices in each market.
   d) The law changed, and now the firm can charge different prices in each market. Find the optimal quantities and prices in each market, and total profit. Compare with your conclusions in part c).

2) A monopolist faces the same demand curves the in exercise 1, $p_1 = 62 - y_1$ and $p_2 = 41 - 0{,}5y_2$, but the cost curve is different: $c(y) = 0{,}05y^2 + 4{,}3y$, where y is again total output. The firm is free to charge different prices.
   a) Find the optimal quantities and prices in each market and total profit.
   b) Compare the results with those of the previous exercise and comment.
   c) Suppose demand increases in both markets, that is, demand curves shift upwards and to the right. Qualitatively compare the changes in output and prices we get with the two cost functions.

3) A monopolist's has the cost function $c(y) = y$. There are two markets, and the demand curves are $y_1(p_1) = 11 - p_1$ and $y_2(p_2) = 14 - 4p_2$.
   a) Presently the monopolist has access to market 1 only. Find the optimal quantity and price and profit.
   b) Now the monopolist is able to sell to both markets, but it has to charge the same price in both. Find the optimal quantity and price and profit. Compare with the results from part a) and explain.
   c) Now the firm can charge different prices. Explain without calculations whether the firm is going to sell in market 2 as well, and, if so, whether it changes the quantity and price in market 1.
   d) Find the optimal quantities and prices and o profit?
   e) Now the firm can perfectly price discriminate. How much is it going to sell in each market? Find the changes in profit, consumer and total surpluses.

4) Explain the long-run equilibrium in a monopolistically competitive industry. Why do firms produce less than the minimum efficient scale?

Multiple-Choice Questions

1. Which of the following is a tell-tale sign of price discrimination?
   a) Different consumers pay different prices for the same good.
   b) Consumer surplus is than in perfect competition.
   c) Price exceeds marginal cost.
   d) Profits tend to zero.

2. With perfect price discrimination marginal revenue:
   a) Exceeds the price.
   b) Equals the price.
   c) Is less than the price, not necessarily half of it.
   d) Is half the price.

3. If a hitherto single-price monopolist starts perfectly price discriminating. Then:
   a) Consumer surplus increases.
   b) Total surplus falls.
   c) The previous consumer surplus and deadweight loss turn into producer surplus.
   d) Deadweight loss increases.

4. To successfully practice 3rd degree price discrimination, a monopolist must be able to separate consumers into identifiable groups and:
   a) Increase prices to all consumers.
   b) Lower total output to increase prices.
   c) Prevent reselling form one group to another.
   d) All of the other options.

5. A monopolist has to charge the same price in all markets. Given the quantities sold at present, demand is more elastic in market *A* than in market *B*. If the monopolist is now allowed to charge different prices, what can it do to increase profit?
   a) Lower the price in market *A* an increase it in market *B*.
   b) Lower the price in market *B* an increase it in market *A*.
   c) Lower the price in both markets.
   d) Increase the price in both markets.

6. A monopolist is free to charge different prices in two markets. To maximise profit it will:
   a) Have a higher marginal revenue in the market with the more elastic demand.
   b) Have a higher marginal revenue in the market with the less elastic demand.
   c) Have equal marginal revenues in both markets.
   d) Have a higher marginal revenue in the market where it sells more.

7. In which of the following cases is total surplus the highest?
   a) It does depend on the cost and demand curves.
   b) Single-price monopolist.
   c) Third-degree price discrimination.
   d) Perfect price discrimination.

8. Which of the following is NOT typical of monopolistic competition?
   a) Barriers to entry.
   b) Product differentiation.
   c) In equilibrium firms operate in the downward section of their average cost curves.
   d) Each firm faces a downward-sloping demand curve.

9. A monopolistically competitive firm' marginal revenue is:
   a) Constant.
   b) Higher than the price.
   c) Equal to the price.
   d) Less than the price.

10. Do monopolistically competitive firms have market power?
    a) Yes, because there are barriers to entry.
    b) Yes, because of product differentiation.
    c) No, because firms compete against each other.
    d) No, because firms have zero profits in the long run.

Answers

1.a) We need the aggregate inverse demand curve. To find it easily let us first find the (direct) demand curves:
$p_1 = 62 – y_1 \Leftrightarrow y_1 = 62 – p_1$;
$p_2 = 41 – 0{,}5y_2 \Leftrightarrow y_2 = 82 – 2p_2$.
Now we can obtain the aggregate (direct) demand curve: for each price total quantity demanded is the sum of the individual quantities (there is a single price, $p_1 = p_2 = p$):
$y = y_1 + y_2 = 62 – p + 82 – 2p = 144 – 3p$.
The aggregate inverse demand curve is then:
$y = 144 – 3p \Leftrightarrow p = 48 – y/3$.
Revenue and marginal revenue are:
$r(y) = py = (48 – y/3)y = 48y – y^2/3$;
$MR(y) = r'(y) = 48 – 2y/3$.
Marginal cost is $MC(y) = c'(y) = 10$.
The firm maximises profit with:
$MC(y) = MR(y) \Leftrightarrow 10 = 48 – 2y/3 \Leftrightarrow y = 57$.
Optimal price and profit are:
$p(57) = 48 – 57/3 = 29$.
$\pi = py – c(y) = 29 \times 57 – 10 \times 57 = 1083$.

1.b) Marginal revenue functions are:
$MR_1(y_1) = 62 – 2y_1$;
$MR_2(y_2) = 41 – y_2$.
Quantities sold in each market are:
$y_1(29) = 62 – 29 = 33$.
$Y_2(29) = 82 – 2 \times 29 = 24$.
At these quantities marginal revenues are:
$MR_1(33) = 62 – 2 \times 33 = -4$;
$MR_2(24) = 41 – 24 = 17$.
Elasticity is (dy/dp) x (p/y). In our cases:
$\varepsilon_1(33) = -1 \times 29/33 = -0.879$.
$\varepsilon_2(24) = -2 \times 29/24 = -2.417$.
For price 29, demand in market 1 is inelastic, $|\varepsilon_1|<1$, so marginal revenue is negative; in market 2 demand is elastic, so marginal revenue is positive.

1.c) While marginal revenue in market 2 exceeds that of market 1, the firm increases profit by transferring units from market 1 to market 2. So it will do so, which means price increases in market 1 and falls in market 2.

1.d) A firm maximises profit equating marginal revenues and marginal cost. The marginal cost is constant and equal to 10, the firm maximises profit with both marginal revenues equal to 10:
$MR_1(y_1) = 62 – 2y_1 = 10 \Leftrightarrow y_1 = 26$.
$MR_2(y_2) = 41 – y_2 = 10 \Leftrightarrow y_2 = 31$.
$p_1(26) = 62 – 26 = 36$.
$p_1(31) = 41 – 0{,}5 \times 31 = 25{,}5$.
$\pi = p_1y_1+p_2y_2 – c(y_1+y_2) = 36x26 + 25{,}5x31 – 10x(26+31) =$
$= 1156.5$.
The results confirm our previous conclusions: quantity falls in market 1 and rises in market 2; prices move in the opposite direction. Profit increases

2.a) Now marginal cost is:
$MC(y) = c'(y) = 0{,}1y + 4.3 = 0{,}1(y_1 + y_2) + 4.3$.
That is, marginal constant is not constant, and none of the marginal revenues is constant either. So we cannot *a priori* tell what values these variables are going to take (in contrast to the previous exercise, where we knew all of them would be 10). We know though that both marginal revenues are the same at the optimal point.
$MR_1(y_1) = MR_2(y_2) \Leftrightarrow 62 – 2y_1 = 41 – y_2 \Leftrightarrow y_2 = 2y_1 – 21$..(1)
They will have to equal marginal cost too:
$MC(y_1 + y_2) = MR_1(y_1) \Leftrightarrow 0.1(y_1 + y_2) + 4.3 = 62 – y_1$.........(2)
Substitute (2) into (1):
$0{,}1(y_1 + 2y_1 – 21) + 4{,}3 = 62 – y_1 \Leftrightarrow y_1 = 26$.
From (1):
$y_2 = 2y_1 – 21 = 2 \times 26 – 21 = 31$.
The quantities are the same as in the previous exercise, so the prices are the same too: $p_1(26) = 36$; $p_1(31) = 25{,}5$.
Beware, profit is different though, because cost is different:
$\pi = p_1y_1+p_2y_2 – c(y_1+y_2) =$
$= 36 \times 26 + 25.5 \times 31 – [\,0.05 \times (26+31)^2 + 4.3 \times (26+31)\,]$
$= 1318.95$.

2.b) The demand curves are the same. The cost functions are different, mas the second function was chose so that marginal cost is 10 (constant marginal cost of the first function) for the optimal quantity of the previous exercise.

2.c) For each given quantity marginal revenue increases. So for the initial optimal quantities the marginal revenues will exceed marginal cost, so the firm will increase profit if it sells more. This will cause marginal revenues to fall until they equal marginal cost again. With the second cost function, marginal cost increases when output increases, so marginal cost and marginal revenues will meet again with lower quantity increases that in the case of the constant marginal cost function. This means prices will increases more with the increasing marginal cost case.

3.a) The inverse demand curve is

$y_1 = 11 - p_1 \Leftrightarrow p_1 = 11 - y_1$.

Total and marginal revenues are:

$r_1(y_1) = p_1y_1 = (11 - y_1)y_1 = 11y_1 - y_1^2$;

$MR_1(y_1) = r_1'(y_1) = 11 - 2y_1$.

Marginal cost is $MC(y) = c'(y) = 1$.

The firm maximises profit with:

$MC(y_1) = MR_1(y_1) \Leftrightarrow 1 = 11 - 2y_1. \Leftrightarrow y_1 = 5$.

The optimal price and profit are:

$p_1(5) = 11 - 5 = 6$.

$\pi = p_1y_1 - c(y_1) = 6 \text{ x } 5 - 1 \text{ x } 5 = 25$.

3.b) With $p_1 = p_2 = p$, the aggregate demand curve is:

$y = y_1 + y_2 = 11 - p + 14 - 4p = 25 - 5p$. (1)

The inverse demand curve is:

$y = 25 - 5p \Leftrightarrow p = 5 - 0.2y$. (2)

Total and marginal revenues are:

$r(y) = py = (5 - 0.2y)y = 5y - 0.2y^2$;

$MR(y) = r'(y) = 5 - 0{,}4y$. (3)

Equating marginal cost and marginal revenue:

$MC(y) = MR(y) \Leftrightarrow 1 = 5 - 0.4y \Leftrightarrow y = 10$.

For this quantity price and profit are:

$p(10) = 5 - 0.2 \text{ x } 10 = 3$.

$\pi = py - c(y) = 3 \text{ x } 10 - 1 \text{ x } 10 = 20$.

This profit is less than that of part a), so the firm prefers to keep charging a price of 6, selling nothing in market 2, and 5 units in market 1, with a profit of 25.

What is going on here is as follows. Demand curves are valid for certain price ranges only (because there can be no negative quantities): $p_1 \leq 11$ in market 1; $p_2 \leq 3.5$ in market 2. Then the aggregate demand, equation (1), is valid for $p \leq 3.5$ only (corresponding to $y \geq 7.5$). At higher prices quantity demanded in market 2 is zero, and aggregate demand coincides with that of market 1. This means the inverse demand curve, equation (2), and the aggregate marginal revenue curve, equation (3) are valid for $y \geq 7.5$ only (quantities corresponding to $p \leq 3.5$). For lower quantities ($p > 3.5$), the aggregate inverse demand curve and marginal revenue curve coincide with those of market 1, and it is there that the monopolist is going to operate, for it gets a higher profit there.

3.c) The firm is able to sell in market 2 at a price and marginal revenue (as the inverse demand curve and the da marginal revenue curve have the same vertical intercept) higher than marginal cost, so it will be profitable for them to sell in that market. As marginal cost is constant, the firm will keep selling the same quantity at the same price in market 1. The firm simply increases output to supply market 2 until marginal revenue in this market falls to the level of marginal cost. (If the firm had increasing marginal costs, as it increased output to supply market 2 marginal cost would increase, so the firm would sell less to market 1.)

3.d) the inverse demand curve in market 2 is:

$y_2 = 14 - 4p_1 \Leftrightarrow p_2 = 3.5 - 0.25y_2$.

Total and marginal revenues are:

$r_2(y_2) = p_2y_2 = (3.5 - 0.25y_2)y_2 = 3.5y_2 - y_2^2$;

$MR_2(y_2) = r_2'(y_2) = 3.5 - 0.5y_2$.

Marginal cost is $MC(y_1 + y_2) = c'(y_1 + y_2) = 1$. Note that total and marginal cost is always a function of total quantity, but in this case marginal cost is constant for any quantity.

The firm maximise profit with:

$MC(y_1 + y_2) = MR_2(y_2) \Leftrightarrow 1 = 3.5 - 0.5y_2 \Leftrightarrow y_2 = 5$.

O price optimal and o profit are:

$p_2(5) = 3.5 - 0.25 \text{ x } 5 = 2.25$.

$\pi = p_1y_1 + p_2y_2 - c(y_1 + y_2) = 6 \text{ x } 5 + 2.25 \text{ x } 5 - 1 \text{ x } 10 = 31.25$.

3.e) The firm sells in each market until the price it charges to the last customer equals marginal cost, that is 1. Therefore $y_1(1) = 10$, and $y_2(1) = 10$. Consumer surplus vanishes, but total surplus increases, because quantity increases. But all surplus is now profit. Surplus, or profit, is (draw a graph to see) (11 – 1) x 10/2 = 50 in market 1, and (3.5 – 1) x 10/2 = 12.5 in market 2, 62.5 in total.

4) See textbook. The reason firms produce less than the minimum efficient scale is the following. In the long-run equilibrium the demand firm facing the firm is tangent to the firm's average cost curve. As the demand curve is negatively sloped, this has to happen in the decreasing section of the average cost curve.

Answers to the multiple-choice questions

1a 2b 3c 4c 5a 6c 7d 8a 9d 10b.