1) (Adapted from Perloff, 13.1.5, p. 497) Lori employs Max. She wants him to work hard rather than loaf. She considers offering him a bonus. All else the same, Max prefers to loaf. The payoff matrix is

		Max	
		Work	*Loaf*
Lori	*Bonus*	1, 2	-1, 3
	No Bonus	3, -1	0, 0

a) Is this a game? Do Max´s and Lori's payoffs make sense? Explain.
b) Do either Lori or Max have dominant strategies? Explain.
c) Is (*Bonus, Work*) an equilibrium if Lori and Peter decide simultaneously with no previous conversation? Why or why not?
d) Are there pure-strategy equilibria if Lori and Peter decide simultaneously with no previous conversation? Explain.
e) Is there a prisoner's dilemma in this game? Explain.
f) Could Lori and Max achieve a better outcome the equilibria found in part d)? Would it be stable? Explain.

2) (Adapted from Perloff, 13.1.13 e 13.1.14, p. 499) Two guys (suffering from testosterone poisoning) engage in a game of chicken. They drive towards each other in the middle of the road. As they approach the impact point, each has the option of continuing driving down the middle of the road or to swerve. Both believe that if only one driver swerves, that driver loses face (payoff = 0) and the other gains self-esteem (payoff = 2). If neither swerves, they are maimed or killed (payoff = -10). If both swerve, no harm is done to either (payoff = 1).
a) Show the payoff matrix for the two drivers engaged in the game of chicken. Determine the Nash equilibria.
b) Modify the payoff matrix so that the payoff is -2 if neither driver swerves. How does the equilibrium change?

3) Two football teams are going to play each other in a championship game. Each can choose a defensive or attacking strategy. The strategy is chosen and rehearsed in secret, and cannot be changed during the game. The goals scored are going to be as follows depending on the strategies adopted

		SLB	
		Attack	*Defence*
FCP	*Attack*	2, 4	1, 2
	Defence	1, 1	0, 0

a) Is the game static or dynamic?
b) Study the existence of equilibrium.
c) Is there a prisoner's dilemma in this game? Explain

4) (Perloff, 13.1.1, p. 496) Show the payoff matrix and explain the reasoning in the prisoners' dilemma example where Larry and Duncan, possible criminals, will get one year in prison if neither talks; if one talks and the other does not, the one who does goes free and the other gets five years; and if both talk, both get two years.

5) (Perloff 13.1.9, p. 498) Suppose Toyota and General Motors are considering entering a new market for electric automobiles, and that their profits (in millions of dollars) from entering or staying out of the market are

		General Motors	
		Enter	*Stay Out*
Toyota	*Enter*	10, -40	250, 0
	Stay Out	0, 200	0, 0

a) What is the equilibrium in the static game?
b) How would your answer change if the U.S. government committed to paying GM a lump-sum subsidy of $50 million on the condition that GM would produce the electric car?

6) (Adapted from Perloff 13.1.10, p. 498) In the battle of sexes game, the husband likes to go to the mountains on vacation, and the wife prefers the ocean, but they both prefer to take their vacations together.

		Husband	
		Mountains	*Brach*
Wife	*Mountains*	2, 1	-1, -1
	Beach	0, 0	1, 2

a) Discuss whether these payoffs have a cardinal or merely ordinal interpretation?
b) What are the Nash equilibria? Discuss whether this game and equilibrium concept make sense for analysing a couple's decisions. How might you change the game's rules so that it makes more sense?

7) (Adapted from Perloff, 13.1.7, p. 497) Suppose that two firms face the following payoff matrix:

		Firm 1	
		Low Price	*High Price*
Firm 2	*Low Price*	2, 0	1, 2
	High Price	0, 7	6, 6

a) Do these payoffs imply that each firm wants to match the other's price?
b) What are the Nash equilibria?

8) (Adapted from Perloff, 13.1.3, p. 496) Two firms must simultaneously decide which quality to manufacture. The profit matrix (in tens of thousands of euros) is

		Firm 1		
		Low	*Medium*	*High*
Firm 2	*Low*	11, 1	19, 21	6, 12
	Medium	16, 25	3, 2	23, 4
	High	5, 24	14, 26	17, 11

a) Find and characterise all the equilibria in this game.
b) Change two payoffs so that there is an equilibrium in dominant strategies.
c) Find all possibilities of, with just one change in the payoffs (relative to the original matrix), having just one Nash equilibrium without any dominant strategies.

9) (Adapted from Perloff, 13.2.1, p. 499) Two firms are planning to sell 10 or 20 units of their goods and face the following payoff matrix:

		Firm 2	
		10	*20*
Firm 1	*10*	30, 30	50, 35
	20	60, 40	20, 20

a) Identify and characterise the equilibria if both firms make their decision simultaneously.
b) How does your analysis change if the government imposes a lump-sum tax of 40 on each firm (that is, the payoffs in the matrix are reduced by 40). Explain how your analysis would change if the firms have an additional option of shutting down to avoid paying the tax.
c) Draw the game tree if firm 1 can decide first (and there is no tax). What is the outcome? Why?
d) Identify all Nash equilibria and the subgame-perfect Nash equilibrium in the case above. What is the difference between subgame-perfection and absence of perfection?
e) Draw the game tree if firm 2 can decide first. What is the outcome? Why?

10) Two firms can choose to broadcast or not to broadcast adverts, and face the following profit matrix:

		Firm B	
		No Adverts	*Adverts*
Firm A	*No Adverts*	70, 50	40, 65
	Adverts	60, 35	30, 30

a) How could you explain the effects of adverts on the firms' profits?
b) Are there dominant strategies in the static game?
c) Analyse the equilibria in the static game.
d) Suppose firm *A* can choose first. Which strategies would the firms adopt?
e) If firm *B* is able to play first, which strategies would the firms adopt?
f) Do the firms gain from playing first?
g) Do firms choose in the dynamic game the same moves they do in the static game? Why or why not?

11) (Perloff, 13.2.8, p. 500) Levi Strauss and Wrangler are planning new generation jeans and must decide on the colours. The possible colours are white, black, and violet. The payoff to each firm depends on the colours both firms choose, as the profit matrix below shows:
a) Given that firms move simultaneously, identify any dominant strategies and find all Nash equilibria.
b) How many strategies can Levi Strauss choose from?
c) Now suppose firms move sequentially, with Wrangler moving first. Draw the game tree and identify the subgame-perfect Nash equilibrium.

		Levi Strauss		
		White	*Black*	*Violet*
Wrangler	*White*	10, 10	20, 30	30, 40
	Black	30, 20	0, 0	15, 35
	Violet	40, 15	35, 20	0, 0

12) There is something funny about some payoff matrices. If the game is static one player has a dominant strategy; let us call it strategy *X*. But if the game is sequential and that player moves first, that strategy *X* is no longer optimal. This is the case with firm *A*'s strategy *No Adverts* in exercise 11. Check whether this is also possible in a championship football game where teams can choose one of two tactics as in exercise 3 (you can try any scores you like). For the sake of simplicity assume that each team only care whether they win, draw or lose, and secondarily about the goal difference.

13) (Adapted from Perloff, 13.2.4, p. 500) A small tourist town has two Italian restaurants, Romano's and Giardino's. Normally both restaurants prosper with no advertising. Romano's could take some of Giardino's customers by running raidio ads, and Giardino's could do the same thing. The one-month payoff matrix (in thousands of dollars) is
a) What is the Nash equilibrium in the static (one month) game?
b) Is interaction between the two restaurants plausibly described by a static game?

		Romano's	
		No Ads	*Ads*
Giardino's	*No Ads*	3, 3	0, 4
	Ads	4, 0	1, 1

c) Describe one or more possible Nash equilibrium if the game is repeated indefinitely?
d) Are there multiple equilibria if the game is repeated indefinitely?

14) (Adapted from Perloff, 13.2.3, p. 500) Two airline companies can choose between two capacities, yielding the profit matrix (Perloff's example, p. 471):

		Americam Airlines	
		64	*48*
United Airlines	*64*	4.1, 4.1	5.1, 3.8
	48	3.8, 5.1	4.6, 4.6

a) What is the equilibrium in the static game?
b) What will happen if the players know the game will last five periods? What happens if the game is repeated indefinitely but one or both firms care only about current profit?

Multiple-Choice Questions

1) A dominant strategy:
a) Is the best response to any of the other player's strategies.
b) Is the best response to at least one of the other player's strategies, not necessarily to all of them.
c) It is the best response to the strategy the other player is likely to choose, not necessarily to the other.
d) None of the other options is correct.

2) A pure strategy:
a) Is a strategy a player will play regardless of what he expects the other will play.
b) Is a best response to some but not all of the other player's strategies.
c) Gives the player an advantage over the other player.
d) None of the other options is correct.

3) A mixed strategy:
a) Only exists in repeated games, and involves taking one action in some rounds, and another in other rounds.
b) Is one that avoids the worst possible payoff even if that means forgoing the best possible payoff.
c) Consists in choosing at random with some probability the action to be followed.
d) None of the other options is correct.

4) In a Nash equilibrium each strategy is:
a) A dominant strategy.
b) A pure strategy.
c) A best response to the other equilibrium strategies.
d) None of the other options is correct.

5) Which of the following is true of a Nash equilibrium?
a) One will always exist in mixed strategies, but not necessarily in pure strategies.
b) There is always at least one in pure strategies.
c) There is never more than one.
d) If one exist in mixed strategies, one will also exist in pure strategies.

6) In a prisoner's dilemma:
a) There is only one Nash equilibrium.
b) There is one equilibrium in dominant strategies.
c) The Nash equilibrium is not Pareto efficient.
d) All other options are correct.

7) Suppose in static game (non-repeated game) there is a prisoner's dilemma. If the game is repeated a finite number of rounds, and players care sufficiently about the future:
a) In each round the equilibrium is the same as in the non-repeated game.
b) Pareto efficiency will be achieved in all rounds.
c) Pareto efficiency will be achieve in some rounds.
d) None of the other options is correct.

8) Suppose in static game (non-repeated game) there is a prisoner's dilemma. If the game is repeated an infinite number of rounds, and players care sufficiently about the future:
a) In each round the equilibrium is the same as in the non-repeated game.
b) Pareto efficiency will be achieved in all rounds.
c) Pareto efficiency will be achieve in some rounds.
d) None of the other options is correct.

Answers

1.a) They do. Lori's most preferred outcome is Max working without bonus, followed by Max working with bonus, followed by Max loafing without bonus, and the worst outcome would be Max loafing and getting a bonus. Max's payoffs are equally intelligible.
1.b) (*No Bonus, Loaf*) is an equilibrium in dominant strategies.
1.c) Yes: both players would get larger payoffs with (*Bonus, work*) than with the equilibrium in dominant strategies.

2.a) There are two Nash equilibria: (*Swerve, Go Ahead*) and (*Go Ahead, Swerve*)

		Driver 1	
		Swerve	*Go Ahead*
Driver 2	*Swerve*	1, 1	0, 2
	Go Ahead	2, 0	-10, -10

2.b) The equilibria do no change (the preference ordering is still the same for both players).

3.a) Static: each team make their decision without knowing the other's strategy.

3.b) The first thing to note is that the goal matrix would be the payoff matrix only if each team wanted to maximise the number of goals scored, but presumably each team prefers winning one-to-nil than scoring three goals and losing or drawing. The table below is a possible payoff matrix, reflecting the following preferences. Winning is better than drawing, which is better than losing. Winning by two goals is better than by one goal only (if two teams have the same number of points, the one with the largest goal difference will be ahead). Drawing one-all is better zero-all (more goals scored can be an advantage in the classification table when the number of points and goal difference are the same). (Football fans will possibly be able to think of circumstances in which some of the criteria above will not be valid.) For each team the payoffs range from zero, for the worst outcome, to three, for the best outcome.

		SLB	
		Attack	*Defence*
FCP	*Attack*	0, 3	1, 2
	Defence	3, 1	2, 0

(*Defence, Attack*) is an equilibrium in dominant strategies.

3.c) No: there is no other outcome that both teams would prefer.

4) One possibility is for the payoffs to be simply the negative of years in jail. It has to be negative because the fewer years in jail the better.

		Duncan	
		Talk	*Keep Silent*
Larry	*Talk*	-2, -2	0, -5
	Keep Silent	-5, 0	1, 1

There are infinite other possibilities: choose a payoff for five years in jail, a higher one for two years (which is better than five), an even higher payoff for one year, and the highest payoff for zero years.

5.a) For Toyota *Enter* is a dominant strategy, so it enters. GM expects this and stays out.

5.b) Then *Enter* will also be a dominant strategy for GM, and both firms enter the market.

6.a) There is no obvious cardinal interpretation, so any payoff could be changed as long as each player's preference ordering remains the same (but only the couple could provide a definitive answer to this question).

6.b) The static game has two Nash equilibria: (*Mountain, Mountain*) and (*Beach, Beach*). This does not make much sense. In a static game each player chooses his strategy without knowing the other player's strategy. In this case it would be in the couple's best interest to coordinate their decisions.

7.a) Firm 1 prefers to match firm 2's price, but firm 2 prefers not to match the other firm's price.

7.b) There is no Nash equilibrium.

8.a) There are two Nash equilibria: (*Medium, Low*) e (*Low, Medium*).

8.b) There is one possibility only: in (*Medium, Medium*) make the payoff of firm 2 higher than 19, and that of firm 1 higher than 25. (*Medium, Medium*) will then be an equilibrium in dominant strategies.

8.c) There are two possibilities: make the payoff of firm 1 in (*Low, Medium*) less than 12, (*Medium, Low*) will then be the only Nash equilibrium; make the payoff of firm 2 in (*Medium, Low*) less than 11, and (*Low, Medium*) will then be the only Nash equilibrium.

9.a) Two Nash equilibria: (10, 20) and (20, 10). With (20, 10) both firms get higher payoffs than with (10, 20). Therefore (20, 10) is the likely outcome of the game.

9.b) If the options are 10 and 20 only the likely outcome is still the same (adding or subtracting the same constant to all payoffs does not change the preference ordering), but firms now get payoffs (20, 0). With the extra option of shutting down firm 1 keeps choosing 20 (under the somewhat doubtful assumption that their payoff is the same whether firm 2 shuts down or produces 10) and firm 3 is indifferent between producing 10 or shutting down.

9.c) Firm 2 responds to 10 by firm 1 by selling 20, and to 20 by selling 10. Firm 1 foresees this responses and chooses to sell 20. The outcome is the same as in the static game.

9.d) The subgame-perfect Nash equilibrium is the equilibrium identified in 10.c). There are two more Nash equilibria, which are not subgame-perfect. In one firm 2 always responds with 10, and firm 1 chooses 20. The outcome is the same as before, but this equilibrium is not subgame-perfect because 10 is not firm 2's best response if firm 1 chooses 10. This is not relevant to the outcome because firm 1 chooses 20, not 10. But if firm 1 made a mistake and chose 10, it would be irrational of firm 2 to respond with 10. In the other non-subgame-perfect Nash equilibrium firm 2 always responds with 20, and firm 1 chooses 10. If firm 1 believed that firm 2 would always choose 20, 10 would be firm 1's best choice, and if firm 1 chooses 10, choosing 20 anyway is a best strategy for firm 2. However it would not be rational for firm 2 to choose 20 if firm 1 chose 20, and that is why this equilibrium is no subgame-perfect.

9.e) Firm 1 responds to 10 by firm 2 with 20, and to 20 with 10. Firm 2 foresees this and chooses 10, giving rise again to the same outcome as in the other cases.

10.a) Adverts lower the rival's profits and do little to increase own profits. This suggests that in this case adverts have little efficacy (it takes a lot of money to attract more customers) and their main effect is mainly to steal customers from the rival.

10.b) Firm *A*'s dominant strategy is *No Adverts*; firm *B* has no dominant strategy.

10.c) There is one Nash equilibrium: (*No Adverts, Adverts*); firm *A* adopts their dominant strategy, and firm *B* can foresee that and choose *Adverts*.

10.d) Firm *B* responds to *No Adverts* with *Adverts* and to *Adverts* with *No Adverts*. Firm *A* foresees this and chooses *Adverts*.

10.e) Firm *A* always responds with *No Adverts* (their dominant strategy in the static game). Firm *B* foresees this and chooses *Adverts*.

10.f) Both firms are better off playing first rather than playing second.

10.g) Not always. Firm *A* always choose *No Adverts* when they the second to play, as they would do in the static game, for *No Adverts* is their best response to any of the rival's actions. But firm *A* choose *Adverts* if playing first (*best response* is irrelevant here because the first mover does not 'respond'), because by so doing it influences the rival's behaviour to their own advantage.

11.a) There are no dominant strategies. There are two Nash equilibria: (*Violet, Black*) and (*White, Violet*).

11.b)Levi Strauss can choose among nine different strategies.

11.c) Levi Strauss responds to *White* and *Black* with *Violet*, and to *Violet* with *Black*. Foreseeing this, Wrangler chooses *Violet*.

12) It is not possible. The trick to get a strategy that is dominant in the static game but not optimal if the player plays first in a sequential game as illustrated in the following payoff matrix. Note that the numbers there are not goals, they are the payoffs already: the higher the better.

		SLB	
		A	*D*
FCP	*A*	*Y*, *a*	*Z*, *b*
	D	4, *c*	2, *d*

Let *D* be *FCP*'s dominant strategy in the static game. Then $Y < 4$ and $Z < 2$. Now *A* is *FCP*'s optimal strategy when they choose first only if $Y > 2$, $d > c$ e $a > b$. Then if *FCP* choose *A*, *SLB* choose *A*, because $a > b$, and *FCP* gets *Y*; if *FCP* chose *D*, *SLB* would choose *D*, because $d > c$, and *FCP* would get 2; as $Y > 2$, *FCP* choose *A*. But there is a problem. $Y > 2$ and $2 > Z$, then $Y > Z$. But in this game if $Y > Z$, it must be $a < b$ (the better the result to *FCP* the worse to *SLB*). And if $a < b$ *SLB* would choose *D* if *FCP* chose *A*, and *FCP* would get *Z*; because $Z < 2$, *A* would no longer be *FCP's* optimal strategy. The relevant difference between this game and that of exercise 11 is that this is a zero-sum game (the better a result to one player, the worse to the other), whereas the game is exercise 11 is not.

13.a) (*Ads, Ads*) is an equilibrium in dominant strategies.

13.b) It is not. The restaurants interact over the course of many months, so the interaction is best described by a repeated game, where each restaurant can observe what the other has done and modify its decision.

13.c) A possible Nash equilibrium consists in both firms adopting the following strategy (explained in the textbook): in the first month do not advertise (*No Ads*), and the keep refraining from advertising while the other restaurant does the same; if the rival ever advertises, then from the next month on always advertise. Thus no firm will ever advertise: if one did, it would earn larger profits in that month (4 rather than 3), but would then provoke the rival's retaliation and earn at best 1 for ever.

13.d) There are multiple Nash equilibria. In addition to the strategies described in 13.c) there is the so called tit-for-tat: each firm refrains from advertising in the first month; from the second month on each does what the rival has done in the previous month. If a firm ever advertises they know that in the following month the other will advertise, and the two firms will be locked in a war alternating forever between (*Ads, No Ads*) and (*No Ads, Ads*), which yields lower average profits than if the two firms never advertise. It is possible to think of other retaliating strategies that, like the two previous examples, dissuade the firms from ever advertising.

14.a) (*64*, *64*) is an equilibrium in dominant strategies.

14.b) When the game is repeated indefinitely there's a possibility both players would choose 48, to ensure future cooperation and higher payoffs. But if the game is repeated is repeated five times or any other finite number of times, (*64*, *64*) will be the equilibrium in each round; the same will happen if it is repeated an indefinitely large number of times but one player only cares about the current payoff.

Answers to multiple choice questions

1a 2d 3c 4c 5a 6d 7a 8b.

Source for exercises adapted from Perloff:

Perloff, Jeffrey M., *Microeconomics with Calculus*, 3rd edition, Pearson 2014.