1) Maria maximises expected utility, and her utility function with respect to wealth is $u(w) = w^{0,5}$. Her wealth consists of stocks which now have a market value of $w_0 = 121$. She believes the value of the stock will increase by 23 with 50% probability and fall by 21 also with 50% probability.
   a) What is to her the expected value of her final wealth?
   b) Without further calculations can you tell whether she would she be willing to sell her stock for this expected value?
   c) Compare the expected utility of the expected wealth with the expected utility of her final wealth, and check your previous conclusion.
   d) Would she be willing to sell her stock for its current market value? Explain.
   e) Now suppose she believes the value of the stock can go up or down by 23, each with 50% probability. What is the expected value of her final wealth now?
   f) What is the minimum price she would accept for her stock (if it can gain or lose 23)?

2) (Perloff 16.2.7, p. 616) Hugo has a concave utility function of $u(w) = w^{0.5}$. His only asset is shares in an Internet start-up company. Tomorrow he will learn the stock's value. He believes that it is worth \$144 with probability 2/3 and \$225 with probability 1/3. What is his expected utility? What is the minimum value he would sell the shares for today?

3) A casino offers its patron the game "Double of Nothing:" In this game you pay €100 and toss a coin; if you correctly guess which side will come up you receive €200; otherwise you receive nothing (lose the €100). Bob's initial wealth is €500, and his utility from wealth is given by the function $u(w)$
   a) Write Bob's expected utility function if he plays the game.
   b) Would Bob choose to play the game if (i) $u(w) = \ln w$? And if (ii) it is $u(w) = 2w$? And (iii) $u(w) = w^2$? What is Bob's attitude to risk in each case?

4) An aunt of yours has died and left behind two assets, which you and another heir will now inherit. The other heir is indifferent, so you can choose whichever asset you prefer. Asset *A* will be worth €1,000 with probability 30% or €11,000 with probability 70%. Asset *B* will be worth €5,000 with probability 70% or €15,000 with probability 30%.
   a) Which asset would you prefer?
   b) Calculate the expected value and variance of each asset.
   c) Would an expected-utility maximiser feel comfortable deciding between risky options knowing their expected values and variances only? Explain.
   d) Which of the assets above would Mr. Root, who has utility function $u(w) = w^{0.5}$ and maximises expected utility, prefer. For the sake of simplicity assume his initial wealth is zero (the result does not depend on initial wealth).
   e) Show that a risk-averse, expected-utility maximiser may prefer one asset over another with higher expected value and lower variance.

5) You have the opportunity to invest in a speculative stock: your investment will be totally lost with 50% probability and will treble in value with the remaining 50% probability.
   a) Would you invest if your only investment option were to invest €5 (so you would have equal chances of losing €5 or gaining €10)? And if the only investment option were €10? And €20? €50? What is the highest amount you would accept to invest if the only alternative were to invest nothing? What amount would you invest if you could freely choose the amount?
   b) Suppose someone would accept to invest any amount up to €300 (if the only alternative were to invest nothing) but not above €300. Is this attitude compatible with maximisation of expected utility? Explain.
   c) Mr. Log maximises expected utility and his utility of wealth is $u(w) = \ln w$, where ln is the natural logarithm. His initial wealth is €1000. Would Mr. Log accept any of the investments opportunities in part a)? What is the highest investment opportunity Mr. Log would accept if the only alternative were to invest nothing?
   d) How much would Mr. Log invest if he were free to choose the amount?
   e) Mr. Root also has an initial wealth of €1000, but his utility function is $v(w) = w^{0.5}$. Would he accept the investment opportunities of part a) What is the highest investment opportunity he would accept if the only alternative were to invest nothing? How much would he invest if he were free to choose the amount?
   f) Are these gentlemen risk averse or risk preferring? Which is more risk averse? Explain.

6) If an insurance is fair (the insurance premium as a percentage of capital insured is the same as the probability that the loss will occur) expected wealth does not depend on the amount of capital insured. Formally and intuitively prove this assertion. Also prove that if the insurance is more expensive that a fair one expected wealth will fall as the amount of capital insured increases.

7) Ana has initial wealth of €50,000 and faces a 1% probability of losing €20,000. She maximises expected utility and her utility function is $u(w) = \ln w$, where ln is the natural logarithm.
   a) How much are her expected loss and her expected wealth?
   b) Without calculating any expected utilities, explain whether Ana would be willing to fully insure against the potential loss for a payment of €200.

c) Calculate the expected utilities with and without insurance, and check whether the results are compatible with your answer to part c).
d) Would a risk-averse person always choose to insure against a loss such as Ana faces? Explain.
e) What is the most Ana would be willing to pay to fully insure against her potential loss?
f) What amount of her wealth would Ana like to insure if she had to pay 1 cent for every euro insured? What if she had to pay 1.2 cents for every euro insured? And 2 cents for every euro insured?
g) How could you find the amount to insure in the case of 1 cent payment per euro without calculations (assuming you had correctly answered parts a) to g))?
h) What can the negative value you found in the case of the 2 cents per euro possibly mean?

8) (Perloff 16.2.9, p. 616) Would risk-neutral people ever buy insurance that is more expensive than fair one? Explain.

9) (Adapted from Perloff 16.3.3, p. 617) Jill possesses \$160,000 of valuables. She faces a 0.2 probability of burglary, where she would lose jewellery worth \$70,000. She can buy an insurance policy for \$15,000 that would fully reimburse the \$70,000 loss. Her utility function is $u(w) = 4w^{0.5}$.
a) What is the actuarially fair price for the insurance policy?
b) Should she buy this insurance policy?
c) What is the most she is willing to pay for an insurance policy that fully covers her against her loss?
d) Suppose now that Jill can buy a policy that insures only part of her loss. The insurance premium is reduced proportionally, that is, she pays \$15 for every \$70 insured. Write down her expected utility as a function of the capital insured. How much capital will she want to insure? Why is that less than the \$70,000?

10) (Adapted from Perloff 16.2.13, p. 616) Carolyn and Sanjay are neighbours. Each owns a car valued at \$10,000. Neither has comprehensive insurance (with covers losses due to theft) Carolyn's wealth, including the value of her car, is \$80,000. Sanjay's wealth, including the value of his car, is \$20,000. Carolyn and Sanjay have identical utility functions, $u(w) = w^{0.4}$. They can park their cars on the street or rent space in a garage. In their neighbourhood thee is a 50% probability that a street-parked car will be stolen during the year. A garage-parked car will not be stolen.
a) What is Carolyn´s or Sanjay´s expected loss if they park their cars on the street?
b) Are they willing to pay more than these expected losses to park their cars in a garage? Who is willing to pay more? Explain without calculations.
c) What is the largest amount Carolyn is willing to pay to park her car in a garage. What is the maximum amount Sanjay is willing to pay?
d) Compare Carolyn's willingness-to-pay to Sanjay´s. Why do they differ.

11) (Perloff 16.3.4, p. 617) An insurance agent (interviewed in Jonathan Clements, "Dare to Live Dangerously: Passing on Some Insurance Can Pay Off," *Wall Street Journal*, July 23, 2005, D1) states, "On paper, it never makes sense to have a policy with low deductibles or carry collision on an old car." But the agent notes that raising deductibles and dropping collision coverage can be a tough decision for people on low incomes or little savings. Collision insurance is the coverage on a policyholder's own car where another driver is not at fault.
a) Suppose that the loss is \$4,000 if an old car is in an accident. During the six-month coverage period, the probability that the insured person is found at fault in an accident is 1/36. Suppose that the price of the coverage for this loss is \$150. Should a wealthy person purchase the coverage? Should a poor person purchase the coverage? Do your answers depend on the policyholder's degree of risk aversion? Does the policyholder's degree of risk aversion depend on his or her wealth?
b) The agent advises wealthy people not to purchase insurance to protect against small losses. Why?

12) (Adapted from Perloff 16.3.1, p. 617) Lori, who is risk averse, has two pieces of jewellery, each worth \$1,000. She plans to send them to her sister's firm in Thailand to be sold there. She is concerned with the safety of shipping them. She believes that the probability that any box shipped will not reach its destination is θ.
c) Write down the probability distribution over the possible outcomes (in terms of the value arrived in Thailand) if the pieces are sent in a single box? What if they are sent separately?
d) What is the expected value if the pieces are sent together? And if they are sent separately. Are the variances se same or different? Explain. (You do not have to calculate the variances.)
e) Is Lori's expected utility higher if she sends the articles together or in two separate shipments? Explain.

13) Quim is an expected-utility maximiser with utility function $u(w) = \ln w$, where ln is the natural logarithm, and w his wealth in million euros. His initial wealth is 1 (million euros). He may invest in asset A, which has a rate of return of -10% (it will lose 10% of its value) with 50% chance and 20% also with 50% chance.
a) Suppose he can invest any amount he likes. To invest more than his initial wealth he can borrow at no cost. Write down the probability distribution over his final wealth and his expected utility as functions of the amount invested (let x denote the amount invested).
b) Find his optimal amount to invest. What is the probability distribution over his final wealth for that level of investment? Compare this with the optimal

investment of Mr. Log in exercise 7.h). Can you explain the similarities?

c) How much would Quim invest if he were constrained to invest no more than his initial wealth?
d) Suppose now Quim can invest in assets *A* or *B* (or both). Asset *B* is identical to asset *A*, that is, its price falls by 10% or increases by 20%, each with 50% chance. What is Quim's expected rate of return if he invests in both assets? Does it depend of how much he invests in each asset?
e) Suppose the performances of the two assets are perfectly positively correlated. What does this mean exactly? How much will Quim invest in each asset if he can borrow at no cost? And in case he cannot invest more than his initial wealth?
f) Now suppose the performances of the two assets are independent. What does this mean exactly? Write down the probability distribution over Quim's final wealth and his expected utility as functions of his investment in asset *A* and his investment in asset *B*. (Denote the amounts invested in assets *A* and *B* *x* and *y* respectively.)
g) Suppose he can borrow at no cost. Without calculations explain whether he is going to invest in one asset only or in both. Calculate his expected utility with his investment of part b) in one asset only and then with that amount split between the two assets, 20% in one and 80% in the other, and then 50% in each. Do the numbers confirm you earlier conclusion?
h) Show that whatever amount Quim decides to invest he maximises his expected utility by splitting that amount equally between the two assets. (*Hint*: if his total investment is *k* and he invests *x* in asset *A* he will invest *k* – *x* in asset *B*. Substitute this in the probability distribution you found in part e), and it will become a function of one variable only. It will then be easy to maximise expected utility with respect to *x*.)
i) Without calculations explain whether his total investment in assets *A* and *B* will be less than, more than, or the same as he would invest if asset *A* alone was available (as in part b)). Check your conclusions by seeing what happens to the expected utilities you calculated in part f) after you increase the investments by 20%.
j) You already know that Quim will split his investment equally between the two assets. Write down the probability distribution over his final wealth and expected utility as a function of total investment *k*. How much will he invest in total? How does that compare with his investment when one asset only was available?
k) Suppose now the two assets are perfectly negatively correlated. What does this mean exactly? Write down the probability distribution over Quim's final wealth and his expected utility as functions of the amounts invested in each asset.
l) Suppose Quim cannot invest more than his initial wealth. It is possible to determine without calculations how much he will invest in each asset. Can you do that and explain it properly? Calculate the optimal amounts to check your conclusion. (*Hint*: use the method suggested in part h).) How much would he invest if he could borrow any amount at no cost? And if he could borrow any amount at a rate below 5%?

14) Quim of the previous exercise was pleased to learn that after all asset *A* has a rate of return of -10% with probability 50% and 30% (not 20% as previously thought) also with probability 50%. Asset *B* has a rate of return of -10% or 20% (as previously thought), each with 50% probability.
   a) How much will Quim invest in each asset if their performance is perfectly positively correlated?
   b) Suppose now that the performances of the assets are independent. If Quim is able to borrow at no cost can you tell, without calculations or reading the remaining parts of this exercise, whether he is going to invest in both assets or in asset *A* only?
   c) Quim is going to invest 5 (expected utility is maximised with an investment of 5.217, but Quim cannot borrow more than 4, and calculations are easier with 5 too). Write down Quim's expected utility as a function of his investment *x* in asset *A* (so investment in *B* is 5 – *x*), and find his optimal investment in each asset. Do the results confirm your expectation?
   d) Quim's optimal investment in asset *A* is 3.046 (61% of total investment) if he cannot invest more than 5, 1.908 (76% of total) if he cannot invest more than 2.5, and he will invest everything in asset *A* if he is limited to invest 1.4 or less. Can you make sense of this? (*Hint*: for each of those investment levels write down the probability distribution over Quim's final wealth, its expected value, and the expected rate of return on the total investment and to get an idea of the risk involved in those investments.)
   e) Is there a finite optimal investment if the performances of the two assets are perfectly negatively correlated? Will Quim invest in both assets or only in *A*? Explain.

15) (Fairly advanced exercise) Filipe has an initial wealth of €2,000, which includes a spanking new tablet computer worth €600. There is a 5% probability that he will lose or damage his tablet, and he is going to pay €50 to insure against this loss. He is also willing to pay €5 for a 1 /10.000 chance of winning a €20.000 car in a raffle.
   a) Does his decision to insure against the possible tablet loss reveal risk-aversion or risk-preference? Why? How about is willingness to buy the raffle ticket?
   b) Assuming his behaviour can be properly described by expected utility theory, what would his utility curve look like?

c) Before buying the insurance for his tablet, Filipe received a cash inheritance that increased his wealth to €23,000. His is still willing to insure against the tablet loss. Can you find a utility curve that is compatible with his willingness to insure both before and after the inheritance? What do you conclude about the ability of expected utility theory to explain people's insurance and gambling decisions?

16) Imagine you are given one of these two gambles for free: *A* = (€100, 25%; €0, 75%) or *B* = (€140, 20%; €0, 80%). Which one would you choose? Now you are given a choice between €100 for sure or *X* = (€140, 80%; €0, 20%). Which one would you choose? Answer this questions about your own preferences before you proceed.
   a) Suppose David maximises expected utility and has utility function $u(w)$. For the sake of simplicity assume his initial wealth is zero (what follows does not depend on initial wealth). Suppose David would choose *B* over *A*. If given the other choice he would choose €100 over *X*. Write down his expected utility with *B* (in terms of the generic function $u(w)$) and the expected utility he would have if he chose *A* instead.
   b) Write down the inequality that expresses his preference, $EU(B) > EU(A)$ and solve if for $u(100)$ (that is, you should arrive at an inequality with the form $u(100) >$ something, or $u(100) <$ something).
   c) What does the result you arrived in part b) tell about David's preferences regarding €100 and *X*? Do his preferences conform to expected utility theory? Do yours? (Many, in some studies most, people would choose *B* over *A* and €100 over *X*. This is known as the common-ratio effect.)

17) Suppose that you can have one of these gambles for free: *G* = (€10m, 11%; €0, 89%) or *H* = (€50m, 10%; €0, 90%) where m stands for million. Which one would you choose? Now suppose you are given a choice between €10m for sure or *Y* = (€50m, 10%; €10m, 89%; €0, 1%). Which one would you choose?
   a) Suppose David from the previous exercise would choose *H* over *G* if given the first choice, and €10 over *Y* if given the second. Write down the inequalities that express these preferences.
   b) Do these preferences conform to expected utility theory? (*Hint*: add $0.89u(10)$ to and subtract $0.89u(0)$ from both sides of the inequality $EU(H) > EU(G)$. In many studies, most people answer as David; this is known as the common consequence effect.)

Multiple-Choice Questions

1) Mike is risk neutral. Then his utility curve from wealth is:
   a) Strictly convex.
   b) Strictly concave.
   c) Linear.
   d) Strictly concave for low levels of wealth and strictly convex for higher levels.

2) Mary's marginal utility from wealth fall as her wealth increases. Then she is:
   a) Risk averse.
   b) Risk loving.
   c) Risk neutral.
   d) None of the other options is correct.

3) According to expected utility theory an agent:
   a) Maximises the expected value of wealth.
   b) Maximises the expected utility.
   c) Minimises risk.
   d) All other options are correct.

4) A risk-averse expected utility maximiser's utility curve from wealth is:
   a) Decreasing.
   b) Increasing for low levels of wealth, and decreasing for high levels.
   c) Convex.
   d) None of the other options is correct.

5) An insurance is (actuarially) fair if and only if:
   a) The insurance premium is lower than the loss.
   b) The insurance company reimburses the policy holder if the loss occurs.
   c) If both a) and b) hold.
   d) None of the other options is correct.

6) Ana is risk neutral. Would she buy insurance if the insurance premium is higher than the expected loss?
   a) Yes.
   b) Never.
   c) She is indifferent between buying or not.
   d) The information is not enough to answer.

Answers

1.a) If she keeps the sock, she expects to have 121 + 23 = 144 or 121 – 21 = 100, each with 50% probability. So $EV(w) = 0.5(144) + 0,5(100) = 122$.

1.b) Her utility is a concave function of wealth. This means she is risk averse, so she will always be willing to exchange the risky option, her stock, for its expected value.

1.c) $u(122)=122^{0.5}=11.045$; $EU(w)=0.5 \times 144^{0.5}+0.5 \times 100^{0.5}=11$. As we concluded in part b) her utility from the expected wealth is higher than the expected utility of her wealth (11.045 > 11).

1.d) Her utility from the stock current market value is $121^{0.5} = 11$, the same as the expected utility of her final wealth (if she does not sell), so she would be indifferent between selling or not selling.

1.e) $EV(w) = 0.5(144) + 0,5(98) = 121$: with equal probabilities of winning or losing 23, the expected wealth is the same as the initial wealth.

1.f) She believes that if she keeps the stock she will have 144 or 98, each with 50% probability. Therefore $EU(w)=0.5 \times 144^{0.5}+0.5 \times 98^{0.5}=10.95$. If she sells the stock by k her utility will be $u(k) = k^{0.5}$. The minimum price k she will accept is such that her utility $u(k)$ is the same as if she does not sell: $u(k) = k^{0.5} = 10.95 \Leftrightarrow k = 119.897$. She is risk averse and would be willing to lose 121 – 119.897 = 1.103 on the expected value of the sock to get rid of the risk.

2) EU(w)=(2/3) x $144^{0.5}$+ (1/3) x $225^{0.5}$=14. He will sell for k as long as $u(k) = k^{0.5} \geq 14 \Leftrightarrow k \geq 196$. The shares expected value is 198, so he is willing to lose 2 on the expected value to get rid of the risk.

3.a) Bob will end up with €600 if he guesses correctly and €400 otherwise. So $EU(w) = 0.5u(600) + 0.5u(400)$.

3.b) (i) Bob's expected utility is $u(500)$ = ln 500 = 6.2146 if he does not play; EU = 0.5ln 600 + 0.5ln 400 = 6.1942 if he plays. Expected utility is higher if he does not play, so he won't. We could tell that in advance: $u(w) = \ln w$ is a concave function, so it denotes risk aversion. The expected value of his wealth is the same whether he plays or not (equal chances of winning or losing the same amount), and a risk averse person always prefers the certain amount over a lottery with the same expected value. (ii) The expected utility is the same whether he plays or not; Bob is risk neutral, and indifferent between playing or not. (iii) Expected utility is higher if he plays (260,000) than if he does not play (250,000); he is risk loving, and will play the game.

4.a) You tell us. There are no right or wrong answers.

4.b) EV = 8,000 and σ^2 = 21 million for both assets.

4.c) The present example suggests not: it is not difficult to imagine that someone might prefer one these assets above the other even though they have the same expected value and variance.

4.d) $EU(B) = 86.24 > EU(A) = 82.903$, so Mr. Root would prefer asset B.

4.e) Any increase (below €20,000) in the lower outcome of asset A will increase its expected value and lower its variance. However if the increase is small enough Mr. Root, for instance, will still prefer asset B over asset A.

5.a) You alone can answer these questions. There are no right or wrong answers.

5.b) It is. A risk-averse person might want to invest small amounts, because the expected value of the investment is positive and could outweigh the risk, and refuse to invest large amounts. See the examples of Messrs. Log and Root.

5.c) Mr. Log would accept to invest any amount up to €500.

5.d) €250.

5.e) He would accept any investment opportunity up to €888.88 if the only alternative were to invest nothing, and would choose to invest €500.

5.f) They are both risk averse: they would refuse investments that would increase the expected value of their wealth. Mr. Log is more risk averse than Mr. Root: he would reject investments (above €500 and below €888.88) that Mr. Root would accept.

6) Let w_0, L, and p denote initial wealth, potential loss, and probability of loss respectively. Let A de note the capital insured (if the loss occurs the insurance company pays the policy holder A; in the real world $A \leq L$, but the following result holds for $A > L$ as well). The insurance premium is pA. Final wealth is $w_0 - pA$ if no loss occurs, and $w_0 - pA - L + A$ if the loss occurs. The expected value is $w_0 - pL$, so it does not depend on A. This is so because the insurance premium is pA, the same as the expected compensation the insurance company will pay the policy holder (the company pays A with probability p). With an insurance more expensive than a fair one the insurance premium is larger than the expected compensation received from the insurance company, so reducing expected wealth, and the more so the larger the capital insured.

7.a) Expected loss is €200, expected wealth is €49,800.

7.b) If she fully insures for a payment of €200 she will have a certain wealth of €49,800, which is the same as the expected wealth without insurance but with no risk. As she is risk averse, given two options with the same expected value, she prefers the one without risk. So she would take the insurance.

7.c) EU(49,800) = 10.8158 > EU without insurance = 10,8147.

7.d) No, if the insurance is sufficiently expensive the person may prefer to partially insure or not to insure at all.

7.e) €254.76.

7.f) Respectively €20,000, €11,605.94, and –€5,306.12.

7.g) The insurance premium is equal to the expected compensation Ana will receive from the insurance company (say, if Ana insure €500, she pays €5 insurance premium; if the loss occurs the insurance company pays Ana €500; this happens with 1% probability; so the expected compensation is €5). So Ana´s expected wealth is the same regardless of how much she insures. Then she will prefer to eliminate the risk altogether, which means insuring against the full €20,000 potential loss.

7.h) This means Ana would like to "sell insurance:" the insurance company would pay Ana an "insurance premium" of 0.02 x €5,306.12 = €106.12, and Ana would pay the company €5,306.12 if the loss occurred. This means Ana would have a wealth of €50,106.12 if no loss occurred, and €51,106.12 – €20,000 – €5,306.12 = €24,800 if the loss occurred. This goes to show that even a risk averse person is willing to incur risks if the reward is high enough.

8) No. An insurance that is more expensive than a fair one reduces expected wealth, and a risk-neutral person maximises expected wealth.

9.a) $14,000.

9.b) Her expected utility is 1,468.7 without insurance, and 1,520 with the insurance, so she should buy the insurance policy.

9.c) $25,000.

9.d) Let k denote the capital to insure. Insurance premium is $3k/14$, $EU = 0.8(90{,}000 + 11k/14) + 3.2(160{,}000 - 3k/14)$. Differentiating with respect to k and equating to zero yields k = \$57,697. The price insurance price exceeds the fair price (which would keep her expected wealth constant no matte how much she insured) so she prefer not to insure fully.

10.a) \$5,000.

10.b) Their utility function is concave, so they are risk averse. Therefore they will be willing to pay more than \$5,000 to prevent a loss. See part c) for who is willing to pay more.

10.c) Carolyn would be willing to pay up to \$5,100.11, Sanjay up to \$5,515.13.

10.d) With this utility function people become less risk averse as their wealth increase. Carolyn is a lot less risk averse than Sanjay because she is much wealthier, so she is not willing to pay as much as him to prevent a possible loss.

11.a) The expected loss is \$111.11, much less than the price of insuring against it. So buying this insurance eliminates the risk of the loss but reduces expected wealth. Someone who can afford to lose \$4,000 may prefer no to buy the insurance. But if you are very poor, and, say, could not afford to buy another car if you wrecked your old one completely in an accident, you might prefer to buy the insurance. The degree of risk aversion matters, off course. Even a wealthy person may dislike so much losing \$4,000 as to prefer to buy the insurance. Or a poor person might be risk-neutral or even risk preferring. The lever of wealth matters. Two people may have the same preferences (same utility function), but different degrees of risk aversion because one is wealthier than the other. See previous exercise for an example.

11.b) Precisely because insurance is not fair (the insurance premium exceeds the expected loss, or else insurance companies would go bust), so you lose expected wealth when buying insurance. Wealthy people can easily afford a small loss, so there may have no compelling reason to reduce their expected wealth to insure against them. Besides, whereas in any given short period, say a couple of months, you may face a high degree of uncertainty over small losses (there is some probability that you will suffer a small loss, such as losing you mobile phone or causing a damage to your car, and a large probability that your will suffer no such loss), over the course of your lifetime the uncertainty over small losses is small (you will with almost certainty lose a few mobile phones or drop them in your bath, but with almost equal near certainty, unless you are a pathological case, you will not lose many). So it will be cheaper to bear the small loss now and then when they occur than to insure against them. But if you have no savings and earn very little even a small loss might be disrupting enough, so that insurance may be preferable.

12.a) Assume shipment costs c_1 for a single shipment and c_2 for two separate shipments (possibly $c_2 = 2c_1$). Assume also that the outcomes of any two shipments are independent. With a single shipment, the Thai store will get no piece, so the outcome is $-c_1$, with probability θ and both pieces, outcome is \$2,000 $- c_1$, with probability $1 - \theta$. With two separate shipments the Thai store will receive no piece, $-c_2$, with probability θ^2, exactly one piece, \$1,000 $- c_2$, with probability $2\theta(1 - \theta)$, and both pieces, \$2000 $- c_2$, with probability $(1 - \theta)^2$.

12.b) Apart from the shipment costs the expected values are the same: $2000(1 - \theta) - c_1$ with a single shipment and $2000(1 - \theta) - c_2$ with two shipments. Subtracting a constant from a random variable does not affect its variance, so we may ignore the shipment costs when comparing the variances. The variance is lower with two shipments, because the probabilities of the extreme outcomes, \$0 and \$2,000, falls ($\theta^2 < \theta$ because $\theta < 1$, and $(1 - \theta)^2 < (1 - \theta)$ because $(1 - \theta) < 1$) and the probability of the middle outcome increases (from zero to $2\theta(1 - \theta)$).

12.c) Lori is risk averse, so if the cost of separate shipments is not much higher than of a single shipment, shipping them separately yields a higher expected utility.

13.a) $(1 - 0.1x, 50\%; 1 + 0.2x, 50\%)$.
$EU = 0.5\ln(1 - 0.1x) + 0.5\ln(1 + 0.2x)$.

13.b) 2.5. (0.75, 50%; 1.5, 50%). The proportions of the good and bad outcomes relative to the initial wealth are the same in both cases. This is because in both cases the utility function is the same, the probabilities of the bad and good outcomes are the same, the ratio of the possible increase in wealth to its possible reduction (2 to 1) is the same, and with this utility function the ideal proportion of the wealth to risk is the same for all levels of wealth, as we had seen in 7.h) (so it does not matter that the initial wealth in exercise 7 was not the same as in this case).

13.c) All his initial wealth.

13.d) Both asset have the same rate of return, 5%. So any combination of both will also have a rate of return of 5%.

13.e) It means their performances will be the same. If one yields -10% so will the other. The same for the 20%. So it is irrelevant whether all capital is invested in one or split in any way between the two. (In the real world there might be higher transaction costs when investing in both.)

13.f) Independence in this case means the probability of one asset yielding -10% or 20% is 50% regardless of what actually happens with the other asset. The probability distribution is $(1 - 0.1x - 0.1y, 25\%; 1 - 0.1x + 0.2y, 25\%; 1 + 0.2x - 0.1y, 25\%; 1 + 0.2x + 0.2y, 25\%)$. EU =

0.25[ln(1 – 0.1x – 0.1y) + ln(1 -0.1x + 0.2y) + ln(1 + 0.2x – 0.1y) + ln(1 + 0.2x + 0.2y)].

13.g) He is going to invest in both, because the expected rate of return is the same but the risk is lower: the probability of the worst outcome, that both assets yield -10%, is now only 25%, whereas the probability that only one yields -10% is 50%. His expected utility is 0.0589 if he invests 2.5 in one asset only, 0, 0781 if he invests 0,5 in one and 2 in the other, and 0,0883 in he invests 1,25 in each, so confirming our earlier conclusion.

13.h) EU = 0.25[ln(1 – 0.1k) + ln(1 + 0.2k – 0.3x) + ln(1 – 0.1k + 0.3x) + ln(1 + 0.2k)]. Differentiating with respect to x and equating to zero yields $x = k/2$. So $y = k – x = k/2$ as well.

13.i) He is going to invest more. There is a trade-off in investing: the more you invest the larger the expected gain (5% of the capital invested) but the higher the risk. As splitting the investment between the two assets is less risky than investing in one alone it will pay off to increase investment beyond the optimal level when one asset only was available. Increasing total investment by 20% yields expected utility 0.0566 when investing in one asset only (less than before), 0.0840 if investing 20% in one asset and 80% in the other (more than before), and 0.0982 (again more than before). So it is beneficial to invest more when two independent assets are available.

13.j) Substituting $x = y = k/2$ into the results obtained in 13.f) yields (1 – 0.1k, 25%; 1 + 0.05k, 50%, 1 + 0.2k, 25%) and EU = 0.25ln(1 – 0.1k) 0.5ln(1 + 0.05k) + 0.25ln(1 + 0.2k). Differentiating with respect to k and equating to zero yields, after solving a quadratic equation, k = 4.606, a lot more that when one asset only was available.

13.k) It means that if one asset yields –10% the other yields 20%. (1 – 0.1x + 0.2y, 50%; 1 + 0.2x – 0.1y, 50%). EU = 0.5ln(1 – 0.1x + 0.2y) + 0,5ln(1 + 0.2x – 0.1y).

13.l) The expected rate of return is 5% regardless of how the investment is split between the two assets, as seen in part d) (this does not depend on the correlation between the assets). But if he invests the same amount in each asset the rate of return is guaranteed to be 5%, so there will be no risk. Substituting $y = k – x$ into the EU function in part k) yields EU = 0.5ln(1 + 0.2k – 0.3x) + 0,5ln(1 – 0.1k + 0.3x). Differentiating with respect to x and equating to zero yields $x = k/2$. If he could borrow any amount at less than 5% interest the more he invested the more he would profit (with no risk).

14.a) The assets are perfectly positively correlated, so either A yields 30% and B yields 20% or both yield -10%. So Quim will invest in A only.

14.b) It is a tough question. Asset B has a lower expected rate of return than asset A. so investing in both rather than in A only reduces the expected rate of return on total investment. But it reduces risk. So, for a risk-averse investor, such as Quim, it may worth it to invest in both assets.

14.c) EU = 0.25[ln0.5 + ln(2–0.3x) + ln(0.5+0.4x) + ln(2+0.2x)]. Differentiating with respect to x and equating to zero, yields, after solving a quadratic equation, x = 3.046, which means investment in asset B is 5 – 3.046 = 1.954.

14.d) (0.86, 50%; 1.42, 50%), EV = 1.14, expected rate of return 10% if he invests 1.4.
(0.75, 25%; 0.93, 25%; 1.51, 25%, 1.69, 25%), EV = 1.22, expected rate of return 8.82% if he invests 2.5.
(0.5, 25%; 1.09, 25%; 1.72, 25%, 2.31, 25%), EV = 1.402. expected rate of return 8.05% if he invests 5.
The reason to invest in B, despite its lower expected rate of return, is to reduce risk: half of the time when A yields -10%, B yields 20%, so the probability of getting the worst outcome (losing 10% of the investment) falls from 50% to 25%. Of course, if Quim invests very little in B relative to A the second worst outcome (A yields -10% and B yields 20%) is nearly as bad as the worst. So, for any given total investment, the closer to 50% is the share of B in total investment the more you reduce the risk. Increasing the share of B in total investment has a cost though: it reduces the expected rate of return on total investment. For any given share of B in total investment, risk is the higher the more Quim invest in total. When Quim invests only 1.4, the risk is fairly small even if all is invested in A (the most he can lose is 14% of his initial wealth), so he does not find it worthwhile to reduce risk further (by investing in B) at the cost of reducing his expected rate of return and expected wealth. As he invest more and more, the larger total investment increases the risk, so he decides to forgo some expected wealth in order to reduce that risk (by increasing the share of B in total investment).

14.e) Just as in exercise 14 there is no finite optimal investment. If Quim invests, say, equal amounts in each asset he is guaranteed a positive rate of return no matter what happens (in this case 5% or 10%, each with probability 50%). So the more he invests in total the richer he will be with certainty. He would have to invest in both assets to guarantee a positive rate of return. Just as in the case of independent performances he would not invest in B if he is constrained to invest little (about 1.4 or less, the threshold is the same in both cases). For any given level of total investment the share of asset B is a little lower in the case of perfectly negative correlation than in the case of independence (perfectly negative correlation reduces the risk further than independence, so less investment in B is needed to achieve the same reduction in risk). If you care to calculate it (it's easy), the optimal investment x in asset A as a function of total investment k is $x = (5 + 11k)/18$, so its share is very high for low k and declines towards 11/18 = 0.611 as total investment increases towards infinity.

15.a) The expected loss is €30, less than the insurance premium, so Felipe is willing to reduce expected wealth to eliminate the risk. So this decision reveals risk-aversion. The expected gain in the car raffle is €2, less than the cost of participating. So he is willing to bear a certain loss of €5 for an uncertain gain with expected value of €2. This reveals preference for risk.

15.b) The utility curve could have to be concave for wealth between €1,400 and €2,000 and convex for wealth between €2,000 and €23,000.

15.c) The curve would look like a snake: it would have to be concave for wealth between €22,400 and €23,000. People with very different wealth levels buy insurance and gamble. It is unlikely that their utility curves just happen to switch from being concave to convex around the right wealth levels.

16.a) $EU(B)$=0.2u(140)+0.8u(0), $EU(A)$ = 0.25u(100) + 0.75u(0).

16.b) $u(100) < 0.8u(100) + 0.2u(0)$.

16.c) The result above implies David prefers €100 over X, which contrary to his choice.

17.a) $EU(H)>EU(G) \Leftrightarrow 0.1u(50)+0.9u(0) > 0.11u(10)+0.89u(0)$.
$EU(10)>EU(X) \Leftrightarrow u(10) > 0.1u(50)+ 0.89u(50)+0.01u(0)$.

17.b) If you follow the hint you will find that $EU(H)>EU(G)$ implies that David ought to prefer X over €10m, which is the opposite of his choice.

Answers to Multiple-Choice Questions

1c 2a 3b 4d 5d 6b