

- 1) (Adapted from Perloff 17.3.7, p. 650, and 17.4.2, p. 651) Suppose that the inverse demand curve for paper is $p=200 - Q$, the private marginal cost (unregulated market supply) is $MC^p = 80 + Q$, and the marginal harm from gunk (pollution caused by the production of paper) is $MC^g = Q$.
 - a) What is the unregulated competitive equilibrium? Show in a graph.
 - b) What is the social marginal cost curve, MC^s ?
 - c) What is the social optimum? What is the deadweight loss in the unregulated competitive equilibrium? Show in your graph.
 - d) What specific tax (per unit of output or gunk) results in the social optimum?
 - e) Now assume the market is monopolistic. What is the unregulated monopoly equilibrium?
 - f) How could you optimally regulate the monopoly? What is the resulting (socially optimal) equilibrium?
 - g) The socially optimal output and market price are the same regardless of whether the market is competitive or monopolistic. Is the distribution of the surplus between the various parties also the same when the socially optimal quantity is sold? Explain.
- 2) (Perloff 17.2.1, p. 649) Why isn't zero pollution the best solution for society? Can there be too little pollution? Why or why not?
- 3) In a market the inverse demand curve is $p = 100 - X$, and the private marginal cost (unregulated market supply) is $MC^p = 10 + X$; production causes pollution with external marginal cost $MC^E = X$.
 - a) Assume the market is perfectly competitive. How much will it produce in the absence of any corrective government policy? What is the socially optimal level of output? What Pigouvian tax would lead to the socially optimal level of output?
 - b) Now assume supply is controlled by a monopoly. In the absence of any government policy would welfare be higher under perfect competition or monopoly? (Hint: find the deadweight loss in both situations.)
- 4) In a perfectly competitive market the inverse demand curve is $p = 40 - X$, and the private marginal cost (unregulated market supply) is $MC^p = 10 + 0.5X$; production causes pollution with external marginal cost $MC^E = 10$.
 - a) How much will the unregulated market produce? How much should it produce to achieve the social optimum?
 - b) What Pigouvian tax that would lead to the social optimum? What is the reduction in the (total) external cost? What is the tax revenue.
 - c) How much is the gain in welfare. Show it in a graph.
- 5) There are several bakeries in Lapa. The inverse demand curve for their bread is $p = 200 - 2y$, and the private marginal cost (unregulated market supply) is $MC^p = 50$. Their pollution has external marginal cost $MC^E = 0.25y$. But the nice scent of freshly baked bread spreads through the neighbourhood; resident like this, and their marginal benefit from it is $MB^E = 12$.
 - a) Assume the market is competitive. How much will the unregulated market produce? How much should it produce to achieve the social optimum?
 - b) Find and explain the difference between the private and socially optimal equilibria.
- 6) In the city state of Urbania 1000 charcoal producers supply the city's 40,000 households in a perfectly competitive market. The supply and demand curves are respectively $Q^s = 10p - 200$ and $Q^d = 1000 - 5p$. The charcoal is used in cooking and heating, but it pollutes the air. The damage the production and use of each ton of charcoal causes the community is estimated at 60 urbos (the local currency).
 - a) The market is unregulated. Find the equilibrium, the social optimum, and deadweight loss in the unregulated equilibrium. Illustrate in a graph.
 - b) Discuss the feasibility of an agreement among the citizens to produce the socially optimal amount of charcoal. Consider the possibilities of the consumers using the same or different quantities of charcoal, and of their being equally or differently affected by pollution.
 - c) What could the city government do so that the social optimum were achieved? Give numbers for any policy you suggest.
 - d) Producers join in a cartel to maximise total profit. How does welfare change (from the unregulated competitive equilibrium)? How much does the cartel produce and at what price? What is the marginal social cost at the cartel's quantity? What is the new deadweight loss? Illustrate in your graph.
 - e) What could the government do to induce or force the cartel to produce and sell the socially optimal quantity?
- 7) A regional airport has just opened near the site of a planned small tourist resort. The annual profit of the airport (in thousands of euros) is $\pi_A = 40A - A^2$ where A is the number of annual flights in thousands. The estimated annual profit of the tourist resort (also in thousands of euros) is $\pi_C = 56C - C^2 - AC$ where C is the construction area of the tourist resort in thousands of square metres.
 - a) How can you explain the presence of the term AC in the resorts' profit function?
 - b) What will the two businesses' output (annual number of flights and construction area of the resort) be if they are run independently? What are the profits? Is this outcome socially optimal assuming profits correctly measure total welfare? If not are the socially optimal outputs larger or smaller?
 - c) Suppose the government orders the airport to shut down. Does social welfare rise or fall? By how much?
 - d) Suppose the government decides to order the airport to operate the socially optimal number of flights. What would the effect on social welfare be?

- e) Could the government achieve the same goal by taxing or subsidising the airport or the resort? If so what should the tax or subsidy be? How would this policy impact the profits of the two businesses?
- f) After much litigation the courts decide that, as the airport was in operation before the resort was built, it is free to operate as many flights as it pleases without being liable to pay any compensation or extra taxes. Could negotiation between the two parties lead to the socially optimal outcome? Explain. What agreements could be reached?
- g) What difference would it make if the courts decided instead that any flights the airport decided to operate required express consent from the resort? Quantify your answer as much as possible.
- h) Discuss the relevance of the Coase theorem in general and in this particular case.
- 8) Firms X and Y operating in a perfectly competitive market have the profit functions $\pi_Y = 90y - 0.5y^2 + 40x - 0.5x^2$ and $\pi_X = 120x - 0.5x^2$ where x and y are firm X 's output and firm Y 's output respectively. Nobody else is affected by the activities of these firms.
- Explain the nature of the externality between these firms.
 - How much will each produce and how much profit will they earn if they act independently? What is the marginal effect of firm X 's output on firm Y 's profit at these output levels? Without further calculations, explain whether this outcome is Pareto efficient.
 - What are the socially optimal outputs? What is the marginal effect of firm X 's output on its own profit and on firm Y 's profit? Explain the relationship between them.
 - What tax or subsidy would induce both firms to produce the socially optimal outputs?
- e) Suppose there is no legal restriction, taxes or subsidies on the activities of the firms. Could they reach an agreement that led to the socially optimal outputs? Would one firm need to pay a compensation to the other? Within what limits would that compensation have to be?
- f) How would your answer to part e) change if firm X required a permission from firm Y to produce $x > 40$? What would be the requirement for such requirement?
- 9) Critically discuss the following statements.
- If there are positive production externalities the competitive market will produce less than the socially optimal. This is so because, at the private equilibrium the social marginal benefit is less than the private one.
 - The Coase theorem shows that if there are externalities but property rights are clearly defined and there are no costs to private negotiations who has the property rights has no distributive effects.
- 10) (Perloff 17.6.3, p.651) Are broadcast TV and cable TV public goods? If either is a public good, why is it privately provided?
- 11) (Expanded from Perloff 17.6.13, p.652) Anna and Bess are assigned to write a joint paper within a 24-hour period about the Pareto optimal provision of public goods. Let A denote the number of hours that Anna contributes to the project and B the number of hours that Bess contributes. The numeric grade (0 to 100) that Anna and Bess earn is a function, $23\ln(A + B)$, of the total number of hours that they contribute to the project. If Anna contributes A hours, then she has $24 - A$ hours in the day for leisure. Anna's utility function is $u_A = 23\ln(A + B) + \ln(24 - A)$; and Bess's utility function is $u_B = 23\ln(A + B) + \ln(24 - B)$.
- If they choose the hours to contribute simultaneously and independently, what is the Nash equilibrium number of hours that each will provide? What grade will they get?
 - What is the number of hours each should contribute to the project that maximises the sum of their utilities? What grade will they get? Is this outcome Pareto efficient?
 - You know that when the opportunity cost of the public good is one unit of the private good (as in this case) the optimal provision of the public good requires that the sum of marginal rates of substitution of the agents be equal to 1. Can you tell without calculations whether the correct result of part b) meets this condition? Calculate the marginal rates of substitution to check your answer.
 - Are there any Pareto-efficient numbers of hours that are different from the result of part b)? What is the general condition for the Pareto optimal hours contributed to the assignment?
- 12) The aim of this exercise is to show that if individuals have identical Cobb-Douglas preferences regarding a public good and a private good, the socially optimal quantity of the public good is the same regardless of how they share its cost. For simplicity consider two individuals. Each has utility function $u_i(G, x_i) = a\ln G + b\ln x_i$ where G is quantity of the public good, and x_i the quantity of the private good consumed by individual i . Each has income m_i and contributes g_i towards the public good, so $G = g_1 + g_2$. Suppose for the sake of simplicity that the prices of the public and private goods are both 1.
- What is each individual's budget constraint?
 - What is each individual's marginal rate of substitution?
 - Write down the condition for the socially optimal quantity of the public good.
 - Solve the condition above to find the socially optimal quantity of the public good as a function of m_1, m_2, a and b . (Remember $x_i = m_i - g_i$ and $g_1 + g_2 = G$.)

13) Ana and Bela live at the end of an unpaved road. The utility of each of them depends on total spending on the maintenance of their road and on the money each has left for everything else. Nobody else spends anything on that road. Their utility functions are $u_A = 0.4 \ln G + \ln x_A$ and $u_B = 0.1 \ln G + \ln x_B$ where G is total spending on the road, and x_i the money each has for everything else after contributing towards the maintenance of the road. Ana's income is 150 and Bela's 300.

- a) Assume first they decide simultaneously and independently how much to contribute towards the maintenance of the road. Find the Nash equilibrium.
- b) In the Nash equilibrium above how much private good is Ana willing to forgo to obtain another unit of public good? And Bela? Is this outcome Pareto efficient? Explain.
- c) Now the neighbours decide to cooperate. They first think of each contributing equal amounts towards the maintenance of the road. How much would Bela like that contribution to be? (*Hint*: She now assumes that in her utility function g_A is going to be the same as g_B .) Would both neighbours be better off if this solution were adopted than they were in the Nash equilibrium of part a)? Is this solution Pareto efficient? If not, would Pareto efficiency require more or less public good? (*Hint*: Check the marginal rates of substitution.)
- d) How much would Ana like the contribution to be if Bela were to contribute the same? Would they be better off relative to the solution of part a)? Is the outcome Pareto efficient? If not should there be more or less public good?
- e) Given the shortcomings of the previous solutions Ana and Bela ask you to determine the Pareto-efficient quantity of public good subject to both sharing the cost equally. Would both be better off than in part a)? Given what each is paying for a unit of public good would each like to have the amount you determined, more or less?
- f) Now Ana and Bela decide to explore the possibility of contributing towards road maintenance proportionally to their incomes. What proportion t of their income would ensure Pareto efficiency? Given what each is paying towards one unit of the public good would each like the amount of public good you found, more or less?
- g) Given the neighbours grievances regarding all previous solutions you devise the following scheme. You set a personalised price for each neighbour, that is, the amount that each will pay for every euro spent on the road. As you know their utility functions you can predict how much public good each would like to have given her price. So you set the prices so that both neighbours want the same amount of public good and this amount is Pareto efficient. What are the prices and the optimal amount of public good?
- h) Why do different funding schemes lead to different Pareto-efficient quantities of the public good?

- i) Is there an incentive to free ride in any of the previous solutions? That is, would any of the neighbours like to contribute less if she believed that the other would not change her contribution?

Multiple-Choice Questions

- 1) When a firm emits pollution, the unregulated equilibrium emits too much pollution because:
 - a) Pollution does not affect people's welfare.
 - b) Firms ignore the external cost of pollution.
 - c) Firms overestimate the external cost of pollution.
 - d) Firms underestimate the external cost of pollution.
- 2) If there are no externalities, social marginal cost:
 - a) Cannot be compared to the private marginal cost.
 - b) Exceeds the private marginal cost.
 - c) Is less than the private marginal cost.
 - d) Equal the private marginal cost.
- 3) If producing a good generates a negative externality:
 - a) Too much good will be produced.
 - b) Too little good will be produced.
 - c) The cost of producing the good will be higher.
 - d) There will be excess demand in the market for that good.
- 4) When a common resource has no price:
 - a) People do not use that resource.
 - b) People use that resource up to the point where the social marginal benefit equals the private marginal cost.
 - c) People use that resource up to the point where the social average benefit equals the private marginal cost.
 - d) None of the other options is correct.
- 5) Common resources such as fish stocks in the oceans tend to be overexploited because:
 - a) Social marginal benefit exceeds the private one.
 - b) Social marginal benefit equals the private one.
 - c) Social marginal benefit is less than the private one.
 - d) External marginal cost is less than the private one.
- 6) In a large condominium some the residents leave litter on the condominium lawn. According to the Coase theorem, negotiation among residents:
 - a) Would lead to the optimal amount of litter.
 - b) Would reduce the amount of litter to zero.
 - c) Might be too costly (owing to the large number of residents) and fail to reach an efficient outcome.
 - d) Would lead to an unfair outcome.
- 7) Aggregate demand for a public good is:
 - a) The vertical sum of individual demand curves.
 - b) The horizontal sum of individual demand curves.
 - c) The average willingness to pay curve.
 - d) For each quantity the highest marginal willingness to pay among all consumers.

- 8) Public goods:
- May be produced by private firms.
 - May be produced by state-owned firms.
 - May be produced by a non-firm state organisation.
 - All other options are correct.
- 9) At the efficient provision level of a public good its marginal cost equals:
- Each consumer's marginal benefit.
 - The sum of all consumers' marginal benefit.
 - The average marginal benefit among all consumers.
 - Note of the other options is correct.
- 10) The private market fails to provide a public good at the efficient level because:
- There is no private demand for the good.
 - Firms cannot prevent people from consuming the good for free.
 - The government can produce the public good at a lower cost than private firms.
 - Both b) and c) are correct.

Answers

- $Q = 60, p = 140$.
 - $MC^S = 80 + 2Q$.
 - $Q = 40$. Deadweight loss = 600 (triangle above the demand curve and below the MC^S curve between quantities 40 and 60).
 - Specific tax is 40, the marginal harm at the socially optimum output.
 - $Q = 40, p = 160$.
 - Do nothing: the monopoly is by sheer coincidence producing the socially optimal output.
 - Consumer surplus is the same in both cases (it depends only on the market price). In the regulated competitive market the producer price is 120, and there is a tax revenue of 1,600; with the monopoly there is no tax and the producer price is 160, so the tax revenue is absorbed into the producer surplus.
- There is too little pollution if the marginal benefit of polluting a little more (benefit from increased consumption or reduction in production costs) exceeds the marginal cost caused by pollution. That would be the case in the previous exercise if paper output were reduced below 40.
- Unregulated equilibrium: $MC^P = p \Leftrightarrow X = 45$. Socially optimal level of output is achieved when the social marginal cost equal the price (marginal benefit): $MC^S = MC^P + MC^E = p \Leftrightarrow X = 30$. The Pigouvian tax must equal the marginal external cost at the socially optimal output, $t = MC^E(30) = 30$ (per unit).
 - It will help if you show the equilibria in a graph. Under perfect competition the deadweight loss is the area of the triangle to the right of the socially optimal equilibrium, between the demand curve and the social

marginal cost curve; the width of the triangle is $45 - 30$ (difference between output levels in the unregulated and socially optimal equilibria), and the height is the external marginal cost at $X = 45$, which is 45. So $DWL = 15 \times 45/2 = 337.5$.

For the monopoly, revenue is $pX = 100X - X^2$; marginal revenue is $MR = 100 - 2X$; from $MC^P = MR \Leftrightarrow X = 30$. So the monopolist maximises profit by producing the socially optimal output level, so no deadweight loss. This is of course sheer coincidence.

- Unregulated equilibrium: $MC^P = p \Leftrightarrow X = 20$. Socially optimal level of output is achieved when the social marginal cost equal the price (marginal benefit): $MC^S = MC^P + MC^E = p \Leftrightarrow X = 13.333$.
- The Pigouvian tax must equal the marginal external cost at the socially optimal output, $t = MC^E = 10$ (it is constant). To find the reduction in the external cost note that the external marginal cost is constant and equal to 10, so the average external cost is also constant and 10. Output fell 6.666, so the reduction in external cost is 66.666. You could also integrate the external marginal cost to obtain the external cost function, which gives you $C^E = 10X + K$ (K is a constant, fixed external cost if you will, so it does not affect the reduction. You could also see this reduction in the graph if you do 4.c) first; it is the area between the private and social marginal cost curves corresponding to the reduction in output. The tax revenue is 133.333.
- See 3.b). The gain in welfare is 33.333.
- Unregulated equilibrium: $MC^P = p \Leftrightarrow y = 75$. Socially optimal level of output is achieved when the social marginal cost equal the social marginal benefit: $MC^S = MB^S \Leftrightarrow MC^P + MC^E = p + MB^E \Leftrightarrow 50 + 0.25y = 200 - 2y + 12 \Leftrightarrow y = 72$.
 - There are both negative and positive externalities. The former leads to too much output, and the latter to too little, from the social point of view. In this case private equilibrium output exceeds the socially optimal output. This means that at $y = 75$, $MC^E > MB^E$ ($0.25 \times 75 = 18.75 > 12$), so output should be reduced to improve efficiency.
 - Unregulated equilibrium: $Q = 600, p = 80$. Social optimum: $Q = 400$. Deadweight loss = 6,000 (graphically it is the triangle above the demand curve and below the marginal social cost, $MC^S = 80 + 0.1Q$, between quantities 400 and 600).
 - Reaching an agreement would be difficult in any case given the large number of people affected. But it would be especially hard if people used different amounts of coal or were differently affected by pollution.
 - The simplest policy would be a tax of 60 euros per ton of charcoal. The private marginal cost of charcoal would then equal its social marginal cost.

- 6.d) $Q = 360$, $p = 128$. $MC^S = 116$. Deadweight loss = 240 (graphically it is the triangle above MC^S and below the demand curve between quantities 360 and 400).
- 6.e) The monopoly is producing less than the social optimum. Perloff suggests a subsidy per unit of output. You decide what you think of subsidising a monopoly that is already making large profits at the expense of the consumer (although you could couple the production subsidy with a tax on profits; this tax does not change the optimal output as long as it is levied on economic profit). The government could impose a price ceiling equal to 120, and the monopoly would then maximise profit producing the socially optimal output.
- 7.a) The flights cause a negative externality on the resort. For instance the airplane noise reduces the rates the resort can charge.
- 7.b) $A = 20$, $C = 18$, $\pi_A = 400$, $\pi_C = 324$. The outcome is not socially optimal. The flights cause an external cost, so their socially optimal number is lower. If their number is lower, the resorts' marginal profit increases, so its optimal output increases.
- 7.c) If $A = 0$, $C = 28$, $\pi_A = 0$, $\pi_C = 784$, so welfare increases by 60.
- 7.d) Maximisation of total profit yields $A=8$, $C=24$, $\pi_A = 256$, $\pi_C = 576$, so total profit is 832, larger than in the previous situations.
- 7.e) The socially optimum can be achieved with a tax per flight equal to the flights' marginal external cost at the optimal outputs, which is 24 (euros per flight, or a few cents per passenger!). This would reduce the airports' profit by the total tax bill, $24 \times 8 = 192$, to 64 (thousand euros).
- 7.f) The socially optimal outcome maximises total profit (in this case) so the two parties could benefit from it. Without any agreement we would have the results of part b). Both parties would benefit if the airport reduced the number of flights to 8 (thousand) and the resort paid it more than 144 and less than 254. Alternatively one firm could buy the other, or they could merge.
- 7.g) The only difference lies in the distribution of profits. Without agreement now we would have the results of part d). So the airport could pay more than 208 and less than 400 to induce the resort to let it operate 8 thousand flights a year, and both would benefit. In case of merger or one firm buying the other, the value of the resort would be higher than in the previous case, and that of the airport would be lower.
- 7.h) In many, if not most, situations of interest direct negotiation is impractical because the affected parties are too many. In this case this would not be a problem.
- 8.a) Firm X causes an externality on firm Y. The marginal external effect is $\partial\pi_Y/\partial x = 40 - x$, positive for $x < 40$, and negative if $x > 40$.
- 8.b) $x = 120$, $\pi_X = 7.200$, $y = 90$, $\pi_Y = 1.650$. $\partial\pi_Y/\partial x = 40 - x$, which is -80 for $x = 120$. Firm X increases output until one additional unit does not change its profit. By this time each additional unit is decreasing firm Y's profit, so it is decreasing the sum of the two firms' profits.
- 8.c) $x = 80$, $y = 90$. At these output levels $\partial\pi_Y/\partial x = -40$ and $\partial\pi_X/\partial x = 40$. So to produce the socially optimal output (maximise joint profit) firm X should expand output only while one additional unit increases its own profit by more than it decrease firm Y's.
- 8.d) A tax of 40 per unit of output of firm X, equal to its marginal cost on firm Y.
- 8.e) At the social optimum $\pi_X = 6.400$, $y = 90$, $\pi_Y = 4.050$. So relative to the previous situation π_X falls by 800 but π_Y rises by 2400. So both firms will benefit if firm Y pays firm X more than 600 and less than 2400.
- 8.f) The rationale for the requirement is that for $x > 40$, firm X causes a negative (marginal) effect on firm Y. So, in the language of Coase theorem, this requirement gives firm Y the "property rights". If firm X produced only 40, $\pi_X = 4.000$, $y = 90$, $\pi_Y = 4.850$. By producing at the social optimum instead firm X's profit would increase by 2.400 and firm Y's would fall by 800. So if firm X paid firm Y more than 800 and less than 2.400 both would benefit.
- 9.a) False. In the private, unregulated, equilibrium firms produce too little, from a social point of view, that is true; but the reason is the opposite of the one stated: they produce too little because social marginal benefit exceeds the private one; they produce to the point where their marginal cost equals their private marginal benefit (the price); but additional output would still benefit society, because of the positive external benefit.
- 9.b) False as well. In the stated conditions, negotiations will always lead to same efficient output levels; but a party will be able to get larger share of benefits if it has the property rights than if it has not. See exercises 7 and 8.
- 10) Cable television is not a public good, because provider can a do deny provision to those who don't pay (technically they are called a *club good*). Broadcast TV is a public good. Private firms can profitably provide it by selling adverts.
- 11.a) In the Nash equilibrium each girl maximises her utility given the number of hours the other contributes. Ana's best response function is $A = 23 - B/24$, and Bess's is $B = 23 - A/24$. These intercept at $A = B = 22.08$. Their grade will be 87.1.
- 11.b) $A = B = 23$. Their grade will be 88.1 (one less hour of sleep for an extra percentage point). This outcome maximises the sum of utilities. So it is not possible to increase one's utility without reducing the other's: the outcome is Pareto efficient.
- 11.c) The condition that the sum of marginal rates of substitution be 1 (or whatever the opportunity cost of the public good in term of the private good is) is the

necessary condition for Pareto optimal provision of the public good. As the previous outcome is Pareto efficient it has to meet that condition.

$MRS_A = (\partial u_A / \partial A) / (\partial u_A / \partial (24 - A)) = (23 / (A + B)) / (1 / (24 - A)) = 23(24 - A) / (A + B)$, and similarly for Bess; this is 0.5 for $A = B = 23$, and so is Bess's, so they add up to 1.

11.d) $MRS_A + MRS_B = 1 \Leftrightarrow A + B = 46$. So there is Pareto efficiency as long as $A + B = 46$. Of course $A, B \leq 24$.

12.a) $x_i + g_i = m_i$.

12.b) $MRS_i = MU_G / MU_{x_i} = ax_i / (bG)$.

12.c) $ax_1 / (bG) + ax_2 / (bG) = 1$.

12.d) $G = a(m_1 + m_2) / (a + b)$.

13.a) Ana maximises utility function $0.4 \ln(g_A + g_B) + \ln x_A$. Her best-response functions is $g_A = (60 - g_B) / 1.4$. Similarly for Bela, $g_B = (30 - g_A) / 1.1$. The best-response functions would intercept at $g_A = 66.67$ and $g_B = 33.33$. So Bela contributes nothing, and Ana's best response to $g_B = 0$ is 42.857.

13.b) $MRS_A = 1$, $MRS_B = 0.7$. It is not Pareto efficient. Together they would be willing to forgo a lot more of private good, 1.7, to obtain one more unit of public good than they would have to, 1.

13.c) Bela would prefer that each contribute 27.273. She would be worse off ($u_B = 5.939$, less than the $u_B = 6.08$ she would get if Ana alone contributed 42.857); Ana would be better off. $MRS_A = 0.9$, $MRS_B = 0.5$, so the quantity of public good is still lower than Pareto efficiency requires.

13.d) Ana would prefer that each contribute 42.857 (the same as if Bela contributed zero) Ana would be better off than if she alone contributed 42.857, but Bela would be worse off. $MRS_A = 0.5$, $MRS_B = 0.3$, so now there is too much public good for Pareto efficiency.

13.e) $MRS_A + MRS_B = 1$, after substituting $x_i = m_i - G/2$, yields $G = 72$. Ana is better off, but again Bela would prefer that she contributed nothing and Ana contributed 42.857. Each is paying €0.50 for each €1 spent on the road. This is less than what Ana is willing to pay, $MRS_A = 0.633$, and more than Bela is willing to pay, $MRS_B = 0.3667$. So Ana would like more public good (we knew already that under equal cost sharing she would prefer 2×42.857), and Bela would prefer less.

13.f) $MRS_A + MRS_B = 1$, after substituting $G = t(m_A + m_B)$ and $x_i = (1 - t)m_i$, yields $t = 1/6$, and $G = 75$. Now Ana is paying 1/3 for each unit of public good, which is less than her MRS , 0.3667, so she would prefer more public good. Bela is paying 2/3 for each unit, twice as much as her MRS , so she would like less public good.

13.g) Let p_i be the personalised price neighbour i pays per unit (euro) of public good. The quantity of public good neighbour i wants at price p_i , and because the price of the private good is 1, is such that $MRS_i = p_i$. Substituting $x_i = m_i - p_i G$ into the previous equation and solving for p_i yields $p_A = 60 / (1.4G)$ for Ana and $p_B = 30 / (1.1G)$ for Bela. Pareto efficiency requires $MRS_A + MRS_B = 1$.

Because $MRS_i = p_i$, $60 / (1.4G) + 30 / (1.1G) = 1$, which yields $G = 70.13$. Substituting back into the inverse demand curves for G yields $p_A = 0.6111$ and $p_B = 0.3889$.

13.h) Given the same quantity of the public good, different funding schemes change each individual's amount of private good, and hence their marginal rates of substitution. However in some cases the sum of the marginal rates of substitution does not change, and so the optimal quantity of the public good does not change either. See previous exercise for an example.

13.i) Except for Ana in part a) both of them have always an incentive to free ride, as each has marginal rate of substitution less than 1, that is, each is willing to forgo one unit of public good for less than one unit of private good.

Multiple-Choice Questions

1b 2d 3a 4c 5c 6c 7a 8d 9b 10b.