Midterm II

## General Guidelines

- You may use a calculator;
- You may not use a programmable calculator;
- You may not use notes or books;
- You may have some beverages on your desk;
- All other belongings, including phones, must be on the floor;
- Write all your answers on the blank answer sheets;
- Write your name and student number on every answer sheet;
- If a question does not ask for an explanation, there is no need to give one;
- Any form of fraud will, at least, imply an invalid grade for this course.


## 1. Game theory (8 points)

Consider a game with two players: A and B. Both players have two strategies: player A can go up or down, and player B can go left or right. The game is played simultaneously. The corresponding payoff matrix is shown below, where for each combination of strategies the first number indicates the pay-off of player $A$ and the second number the pay-off of player $B$.

|  |  | Player B |  |
| :---: | :---: | :---: | :---: |
|  |  | left | right |
| Player A | up | $(12,2)$ | $(3,9)$ |
|  | down | $(5,8)$ | $(4,2)$ |

a. Is there a Nash Equilibrium in pure strategies? If yes, what is it? (2 points)
b. What is the Nash Equilibrium in mixed strategies? (3 points)
c. Imagine the game is now played sequentially: player A plays first, and player B plays second. Is there a Nash Equilibrium in pure strategies? If yes, what is it? (3 points)

## 2. Uncertainty and information asymmetry (12 points)

Pedro's utility of wealth $w$ is $u=\ln (w)$. His initial wealth is given by $w=\$ 100,000$. However, with probability $p=0.2$ Pedro gets sick and must pay for treatment so that his wealth drops to $\$ 40,000$, and with probability $(1-p)=0.8$ he stays healthy, and his wealth remains $\$ 100,000$. Note that Pedro maximizes expected utility.
a. What would be the fair insurance premium for Pedro? (Note that in class we denoted the insurance premium by $\gamma$ ). (3 points)
b. Consider that Pedro can either fully insure against the fair premium or cannot insure at all. What would Pedro choose? (3 points)
c. Consider now that (i) the insurance premium $\gamma$ is equal to 0.25 and (ii) Pedro can choose any level of insurance (fully insure, not insure at all, and anything in between). What will be the optimal amount of insurance that Pedro chooses? (3 points)

Afonso's situation is like Pedro's: initial wealth of Afonso is $\$ 100,000$, with probability $p$ he gets sick, and his wealth drops to $\$ 40,000$, and with probability $(1-p)$ he stays healthy, and his wealth remains $\$ 100,000$. However, in case of Afonso, $p>0.2$ and $(1-p)<0.8$. Next to Pedro, also Afonso considers an insurance.
d. Imagine that the insurer cannot tell who Pedro and Afonso is, and therefore charges the same premium to both, based upon the average risk profile. Explain in words what may likely happen in this market. There is no need to do any calculations here. (3 points)

## Answers

## 1. Game theory

a. There is no NE in pure strategies.
b. Player $A: P($ up $)=6 / 13$ and $P($ down $)=7 / 13$. Player $B$ : $P($ left $)=1 / 8$ and $P($ right $)=7 / 8$.
c. Yes, there is a NE in pure strategies. $\mathrm{NE}=\{$ down, left $\}$.

## 2. Uncertainty and Information Asymmetry

a. Insurer makes zero profit if $\gamma=p=0.2$. That is, per dollar insured, Pedro pays 20 cents.
b. Pedro will fully insure. Why? With a fair insurance premium, Pedro can get expected wealth $\$ 88,000$ for sure (Pedro pays $0.2 * 60,000=12,000$ to the insurer, so that if he gets sick, he receives $\$ 60,000$. Hence, if Pedro stays healthy, he has $\$ 88,000=100,000-$ $12,000$, and if he gets sick he has $\$ 88,000=40,000-12,000+60,000)$. And since Pedro is risk-averse, he prefers utility of expected wealth over expected utility. You can verify the latter with $\ln (88,000)>0.2 * \ln (100,000)+0.8 * \ln (40,000)$.
c. Optimal amount of insurance $k^{*}$ is $37,333.333$. Why? To find the optimal amount of insurance, we need to know how much wealth Pedro wants when healthy versus when sick. That is, we need to find the optimal bundle: wealth when healthy (we will denote this by $w_{h}$ ) and wealth when sick (we will denote this by $w_{s}$ ). To find this bundle, we need to set the slope of the budget line equal to the MRS. We start by doing this,

$$
\frac{0.25}{1-0.25}=\frac{0.20 * \frac{1}{w_{s}}}{0.80 * \frac{1}{w_{h}}} .
$$

We can simplify this towards,

$$
1.333 w_{s}=w_{h} .
$$

Next, we realize that $w_{s}$ and $w_{h}$ depend upon the amount of insurance $k$,

$$
\begin{gathered}
w_{h}=100,000-0.25 k \\
w_{s}=40,000-0.25 k+k,
\end{gathered}
$$

Finally, we substitute this into the equation above and solve for $k$,

$$
\begin{gathered}
1.333(40,000-0.25 k+k)=100,000-0.25 k, \\
1.333 * 40.000+k=100,000-0.25 k \\
1.25 k=46,666.6666 \\
k^{*}=37,333.333
\end{gathered}
$$

d. Adverse selection will likely take place. For Pedro the average premium may be too expensive, and so he may choose not to insure at all. Then, only Afonso will be left in the market. The insurer knows this and since the average insurance premium is too low to cover the loses of someone with Afonso's risk profile, the insurer will raise the insurance premium.

