LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Normal Period Exam - January 9, 2018

Duration: 1h15

Name:	**Number:**

1. Consider the two variable function defined by

$$f(x,y) = 4x^2 + y^2.$$

a. **[1,0 points]** Sketch the level curves of f corresponding to the values $k = 1$, $k = 4$ and $k = 0$.

b. **[1,0 points]** Consider the ellipse $\mathcal{E}$ of equation $4x^2 + y^2 = 5$.

Compute all vectors $\vec{u}$ perpendicular to $\mathcal{E}$ at point $(1, 1)$.

2. [2,0 points] Compute and classify the critical points of the function defined in $\mathbb{R}^2$ by

$$g(x, y) = x^3 + 3xy^2 - 15x - 12y.$$

3. [2,0 points] Find the maximum and the minimum of the function defined in $\mathbb{R}^2$ by

$$f(x, y) = x^2 y$$

over the set $M = \{(x, y) \in \mathbb{R}^2 \, : \, x^2 + y^2 = 1\}$.

4. [2,0 points] Using a double integral, compute the area of the region of the plane defined by

$$\mathcal{R} = \{(x,y) \in \mathbb{R}^2 \,:\, y \geq -2 - \frac{1}{2}x \wedge y \leq 8 - \frac{1}{2}x^2\}.$$

5. [2,0 points] Compute the general solution of the differential equation

$$y''(x) - 4y'(x) + 4y(x) = 4x^2$$

[Sheet for complementary resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Mid Term Exam - November 8th, 2018
Duration: 1h15m

1. Consider the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 4 & 0 & 0 \end{bmatrix}$.

a. [1,0 pts] Compute all eigenvalues of A and state their algebraic multiplicity.

b. [1,0 pts] Let λ_0 the largest eigeinvalue of A. Determine the eigenspace associated to λ_0. State also the geometric multiplicity of λ_0.

2. a. [0,5 pts] Consider the quadratic form $h(x, y, z, w, t) = -x^2 - 5xw + 4y^2 + w^2 + t^2 + ty$. Compute $h(1, 0, 0, -1, 0)$, $h(1, 0, 0, 0, 0)$ and deduce the classification of h.

b. [1,0 pts] Let A an invertible matrix. Explain why $\lambda = 0$ is not an eigenvalue of A.

c. [0,5 pts] Consider a set $A \subset \mathbb{R}^n$ and a a limit point of A. Explain why a is not an exterior point of A.

3. [1,5 pts] Classify the quadratic form $q(x, y, z) = x^2 + 2y^2 + 3z^2 + 8xy + 4xz$.

4. [1,5 pts] Consider the two variable function defined by the expression $f(x, y) = \dfrac{\ln(1-(x-1)^2-(y+2)^2)}{\sqrt{(x-1)(y+2)}}$.

Determine the domain D_f of f and represent it graphically. Is D_f open? Closed?

5. [1,5 pts] Consider the function F defined by $F(x, y) = f(u, v, w)$, where f is a function of class C^1 and $u = x^2 + y$, $v = x - y^2$, $w = \ln(xy)$.
Assuming that $\vec{\nabla} f(2, 0, 0) = (2, 2, 2)$, compute $\vec{\nabla} F(1, 1)$.

6. [1,5 pts] Consider the function f defined in $\mathbb{R}^2$ by

$$f(x,y) = \begin{cases} \dfrac{y^4 + 3yx^3}{\frac{1}{2}x^2 + y^2} & \text{if } (x,y) \neq (0,0); \\ \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Study the differentiability of f at point $(0,0)$.

[Sheet for complementary resolutions]

[Sheet for complementary resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management

Repeat Period Exam - January 6th, 2019
Duration: 2h15m

Name: ______________________ **Student Number:** ______________

1. Consider the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 4 & 0 \\ 2 & -2 & 1 \end{bmatrix}$.

a. (1,0 pts) Show that A admits one single real eigenvalue and compute it.

b. (1,0 pts) Determine the associated eigenspace.

2. Let B a invertible square matrix such that $B^2 + 3B$ is not invertible.

a. (0,5 pts) Show that $\lambda_1 = 0$ is not an eigenvalue of B.

b. (1,0 pts) Show that $\lambda_2 = -3$ is an eigenvalue of B.

3. (2,0 pts) For $\alpha \in \mathbb{R}$, $|\alpha| \neq \sqrt{2}$, consider the quadratic form

$$q(x, y, z) = z^2 + y^2 + \frac{1}{4}x^2 - azy + \frac{a}{2}zx.$$

Classify q in terms of the parameter a.

4. (1,5 pts) Consider the two variable function defined by the expression

$$f(x,y) = \ln((x^2 + x - 2)(y^2 - 3y + 2)).$$

Determine the domain D_f of f and represent it graphically. Is D_f an open set?

5. Consider the function f defined in $\mathbb{R}^2$ by

$$f(x,y) = \begin{cases} \dfrac{5yx^3}{y^2+6x^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

a. (1,5 pts) Study the continuity of f at point $(0,0)$.

b. (1,5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$, $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist and determine for which vectors u the directional derivative $\partial_u f(0,0)$ exists.

6. (1,5 pts) Compute the coordinates of a vector orthogonal to the line of equation

$$x^3 + 3y^3 + e^{xy} = 4$$

at point $(0, 1)$. Hint: Interpret the line as a level curve.

7. (2,5 pts) Solve the initial value problem

$$\begin{cases} x^2y' = x - y' \\ y(1) = 0 \end{cases}$$

8. (3,5 pts) Compute the absolute maximum and minimum of the function f defined by

$$f(x, y) = 2x^2 + y^2$$

over the set $A = \{(x, y) \in \mathbb{R}^2 \, : \, x^2 + 4y^2 \leq 1\}$.

9. (2,5 pts) Compute

$$\iint_A y\,dx\,dy$$

where $A = \{(x, y) \in \mathbb{R}^2 \,:\, x + y^2 \leq 0 \wedge x + y + 2 \geq 0\}$.

[Extra sheet for additional resolutions]

[Extra sheet for additional resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Mid Term Exam - April 11th, 2019
Duration: 1h15m

Name:
Number:

1. [2,0 pts] Classify the quadratic form

$$q(x, y, z) = 5ax^2 + 2ay^2 - z^2 - 6axy$$

for all possible values of the parameter $a \in \mathbb{R}$.

2. a. [1,0 pts] Consider a square matrix $A \in \mathcal{M}_{2\times2}(\mathbb{R})$ such that

$$A\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \text{ and } A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}.$$

Write down the characteristic polynomial of A and deduce the value of $\det(A)$.

b. [0,5 pts] Give an example of a 2 by 2 square matrix with no eigenvalues.

3. [1,5 pts] Compute the equation of the tangent straight line to the curve

$$e^{xy} + y\cos(x) + x^2y = 2$$

at point $(0, 1)$.

4. [2,0 pts] Consider the two variable function defined by the expression $f(x,y) = \dfrac{\ln\left(1 - x^2 - \frac{1}{4}y^2\right)}{\sqrt{xy}}$.

Determine the domain D_f of f and represent it graphically. Sketch also the boundary of D_f.

5. Consider the function f defined in $\mathbb{R}^2$ by

$$f(x,y) = \begin{cases} \dfrac{x^5 + x^3 + 3yx^2 + xy^2 + 3y^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0); \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

a. [2,0 pts] Show that f is differentiable at point $(0,0)$.

b. [1,0 pts] Using the fact that f is differentiable at $(0,0)$, give an approximation of

$$f(0.05; 0.01).$$

[Sheet for complementary resolutions]

[Sheet for complementary resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Regular Period Exam - June 6, 2019

Duration: 1h15

Name:	**Number:**

1. [2,0 points] Let $u(x,y)$ and $v(x,y)$ two functions such that

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Setting $U(r,\theta) = u(r\cos(\theta), r\sin\theta)$ and $V(r,\theta) = v(r\cos(\theta), r\sin\theta)$, show that

$$\frac{\partial U}{\partial r} = \frac{1}{r}\frac{\partial V}{\partial \theta}.$$

Hint: Start by computing $\frac{\partial U}{\partial r}$ and $\frac{\partial V}{\partial \theta}$ in terms of the partial derivatives of u and v.

2. [2,0 points] Show that the function defined in $\mathbb{R}^2$ by $f(x,y) = 3xe^y - x^3 - e^{3y}$ has one single critical point and classify it. Does f has an absolute minimum?

3. [2,0 points] The temperature at a point (x, y) of a metal plate is given by

$$T(x, y) = xy.$$

An ant, walking on the plate, moves in the ellipse of equation $2x^2 + 8y^2 = 25$, making a complete loop. What are the highest and lowest temperatures encountered by the ant? Answer this question using Lagrange multipliers.

4. [2,0 points] Compute the double integral

$$\iint_{\mathcal{R}} y dx dy$$

where $\mathcal{R}$ is the region bounded by the lines of equation $y = \ln(x)$, $y = 0$ and $x = e$.
Note: choose the order of integration carefully.

5. [2,0 points] Find a function $y : x \in \mathbb{R} \to y(x) \in \mathbb{R}$ such that

$$\begin{cases} y' = -4xy^2 \\ y(0) = 1. \end{cases}$$

[Sheet for complementary resolutions]

[Sheet for complementary resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Regular Period Exam - January 10, 2020

Duration: 1h15

Name:	**Number:**

1. [2,0 points] Consider a two-variable function F such that $\dfrac{\partial F}{\partial x}(1,1) = \dfrac{\partial^2 F}{\partial y^2}(1,1) = 1$ and $\dfrac{\partial F}{\partial y}(1,1) = \dfrac{\partial^2 F}{\partial x^2}(1,1) = 0$.

Setting $g(r,\theta) = F(r\cos(\theta), r\sin(\theta))$, compute $\dfrac{\partial^2 g}{\partial \theta \partial r}\left(\sqrt{2}, \dfrac{\pi}{4}\right)$.

2. [2,0 points] Show that the function defined in $\mathbb{R}^2$ by $f(x, y) = e^{-(x^2+y^2+2x)}$ has one single critical point and classify it.

3. [2,0 points] Consider the function $f(x, y, z) = \sin(x)\sin(y)\sin(z)$, $x, y, z \geq 0$. Show that if $x + y + z = \frac{\pi}{2}$, then

$$f(x, y, z) \leq \frac{1}{8}.$$

Hint: Apply the Lagrange multipliers method to f in a convenient set.

4. [2,0 points] Compute the double integral

$$\iint_{\mathcal{R}} xe^y dxdy$$

where $\mathcal{R}$ is the region bounded by the lines of equation $y = x^2$ and $y = 8 - x^2$.

5. [2,0 points] Find a function $y : x \in \mathbb{R}^+ \to y(x) \in \mathbb{R}$ such that

$$\begin{cases} \dfrac{e^y}{\sqrt{x^2+1}}y' - 2x = 0 \\ y(0) = 0. \end{cases}$$

[Sheet for complementary resolutions]

[Sheet for complementary resolutions]

LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management

Special Exam - March 2, 2020
Duration: 2h

Name: ______________________ **Student Number:** __________

1. Consider the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ -4 & 1 & 4 \\ -5 & 1 & 7 \end{bmatrix}$.

a. (1,0 pts) Knowing that $\lambda = 1$ is an eigenvalue of A, compute all its eigenvalues.

b. (1,0 pts) Determine the eigenspace associated to the largest eigenvalue of A.

2. Let B a square matrix and $\lambda \in \mathbb{R}$ an eigenvalue of B.

a. (1,0 pts) Show that λ^3 is an eigenvalue of B^3.

b. (0,5 pts) Show that if $\lambda = 0$ B is not invertible.

3. (2,0 pts) For $\alpha \in \mathbb{R}$, $|\alpha| \neq \sqrt{2}$, consider the quadratic form

$$q(x, y, z) = z^2 + y^2 + \frac{1}{4}x^2 - azy + \frac{a}{2}zx.$$

Classify q in terms of the parameter a.

4. (1,5 pts) Consider the two variable function defined by the expression

$$f(x, y) = \arctan(x) \arcsin(4x^2 + y^2) \ln(xy).$$

Determine the domain D_f of f and represent it graphically. Is D_f an open set?

5. Consider the function f defined in $\mathbb{R}^2$ by

$$f(x,y) = \begin{cases} \dfrac{2yx^3}{7y^2+6x^8} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

a. (1,5 pts) Study the continuity of f at point $(0,0)$.

b. (1,5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$, $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist and determine for which vectors u the directional derivative $\partial_u f(0,0)$ exists.

6. (1,5 pts) Compute the coordinates of a vector orthogonal to the line of equation

$$\ln(1 + xy) + e^x + e^y = 2$$

at point $(0, 0)$. Hint: Interpret the line as a level curve.

7. (2,5 pts) Solve the initial value problem

$$\begin{cases} \sqrt{1 - x^2}y' + 2 + 2y^2 = 0 \\ y(0) = 1 \end{cases}$$

8. (3,5 pts) Compute the absolute maximum and minimum of the function f defined by

$$f(x, y) = x^2 + 2y^2$$

over the set $A = \{(x, y) \in \mathbb{R}^2 \,:\, 4x^2 + y^2 \leq 1\}$.

9. (2,5 pts) Compute

$$\iint_A xy dx dy$$

where A is the region buonded by the curves $y = -x^2 + 4$, $y = 3\sqrt{x}$ and $y = 0$.

[Extra sheet for additional resolutions]

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 6, 2022

Duration: 2h

Exam Notebook I

Name: ______________________________ **Student Number:** __________

(1) Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$.

(a) (1.0 pts) Compute the eigenvalues of A.

(b) (1.5 pts) Determine if A is diagonalizable.

(2) Let B be a square matrix and -1 an eigenvalue of B.

(a) (1.5 pts) Show that $B^2 + B$ is not invertible.

(b) (1.0 pts) Show that -1 is an eigenvalue of B^T.

(3) (2.0 pts) For $a \in \mathbb{R}$ with $|a| \neq 2$, consider the quadratic form

$$q(x, y, z) = x^2 + y^2 + 4z^2 - 2axz.$$

Classify q in terms of the parameter a.

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 6, 2022

Duration: 2h

Exam Notebook II

Name: ______________________________ **Student Number:** __________

(4) (2,0 pts) Consider the following function

$$f(x,y) = \frac{\sqrt{1-x^2-y^2}\,\log(x+y)}{1-x^2-y^2} + \arctan(x^2+y^2).$$

Determine the domain D_f of f and represent it graphically. Is D_f an open set? Justify your answer.

(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{yx^2}{x^4 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (1.5 pts) Study the continuity of f at $(0,0)$.

(b) (1.5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist. Is f differentiable at $(0,0)$?

[Extra sheet for additional resolutions]

Lisbon School of Economics & Management
Universidade de Lisboa

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 6, 2022

Duration: 2h

Exam Notebook III

Name: ______________________________ **Student Number:** __________

(6) (2,0 pts) Consider the function $f\colon \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f(x, y, z) = e^x \sin y + \frac{z^2}{2}.$$

Compute the 2nd order Taylor approximation of f around the point $(0, 0, 0)$.

(7) (2.0 pts) Compute the optimal points of

$$f(x,y) = \frac{x^2}{2} + xy$$

over the set $D = \{(x,y) \in \mathbb{R}^2 \colon 2x^2 + y^2 \leq 1\}$.

(8) (2.0 pts) Compute

$$\iint_A 4xy - y^3 \, dxdy$$

where A is the region bounded by the curves $y = x$ and $y = \sqrt{x}$.

(9) (2.0 pts) Solve the initial value problem

$$\begin{cases} (x+1)y' - xy = 0 \\ y(0) = 1 \end{cases}$$

[Extra sheet for additional resolutions]

Lisbon School of Economics & Management
Universidade de Lisboa

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Resit Exam - July 1, 2022
Duration: 2h
Exam Notebook I

Name: ______________________________ **Student Number:** __________

(1) Consider the matrix $A = \begin{bmatrix} 1 & 1 & -3 \\ 0 & -1 & 0 \\ 0 & -5 & 4 \end{bmatrix}$.

(a) (1.0 pts) Compute the eigenvalues of A.

(b) (1.5 pts) Determine the eigenspace associated to the largest eigenvalue of A.

(2) Let B be an invertible square matrix and (λ, v) an eigenpair of B.

(a) (1.5 pts) Show that (λ^{-1}, v) is an eigenpair of B^{-1}.

(b) (1.0 pts) Show that $B^3 - \lambda B^2 + B - \lambda I$ is not invertible.

(3) (2.0 pts) For $a \in \mathbb{R}$, consider the quadratic form

$$q(x, y) = ax^2 + ay^2 + xy.$$

Determine the values of the parameter a such that $q(x, y) < 0$ for every $(x, y) \neq (0, 0)$.

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Resit Exam - July 1, 2022
Duration: 2h
Exam Notebook II

Name: ______________________________ **Student Number:** __________

(4) (2,0 pts) Consider the following function

$$f(x,y) = \frac{\sqrt{1-xy}}{\log(xy)}.$$

Determine the domain D_f of f and represent it graphically. Is D_f a compact set? Justify your answer.

(5) (2.0 pts) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} x + y + \dfrac{y^2 x}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that f is differentiable at $(0,0)$ and compute its differential.

(6) Consider a differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ and let $g : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$g(u, v) = f(u^2 - v^2, 2uv).$$

(a) (1,5 pts) Show that $(0, 0)$ is a critical point of g.

(b) (1,5 pts) Suppose that $f(x, y) = x + y$. Classify the critical point of g.

[Extra sheet for additional resolutions]

Resit Exam - July 1, 2022
Duration: 2h
Exam Notebook III

Name: ______________________________ **Student Number:** __________

(7) (2.0 pts) Compute the optimal points of

$$f(x,y) = x^2 + y^2$$

over the set $D = \{(x,y) \in \mathbb{R}^2 \colon (x-3)^2 + y^2 \leq 4\}$.

(8) (2.0 pts) Compute

$$\iint_A x + y \, dxdy$$

where A is the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

(9) (2.0 pts) Solve the initial value problem

$$\begin{cases} (1+x)e^y y' = 1 + e^y \\ y(0) = 0 \end{cases}$$

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 5, 2023

Duration: 2h

Exam Notebook I

Name: ________________________________ **Student Number:** ________

(1) Consider the matrix $A = \begin{bmatrix} a & 0 & 1 \\ 1 & 2 & 1 \\ a & 0 & 1 \end{bmatrix}, \quad a \in \mathbb{R}.$

(a) (1.0 pts) Compute the eigenvalues of A depending on the parameter a.

(b) (1.5 pts) Assuming $a = 1$, determine if A is diagonalizable.

(2) Let B be a square matrix such that $B^3 = 0$.

(a) (1.5 pts) Show that 0 is an eigenvalue of B.

(b) (1.0 pts) Can we conclude that $B = 0$? Justify your answer.

(3) (2.0 pts) For $a \in \mathbb{R}$, consider the quadratic form

$$q(x, y, z) = y^2 + 2ayz + 2xy + x^2.$$

Classify q in terms of the parameter a.

(4) (2,0 pts) Consider the following function

$$f(x, y) = \frac{\ln(\sqrt{1 - x^2 - y^2})}{x^2 - y^2}.$$

Determine the domain D_f of f and represent it graphically. Is D_f a compact set? Justify your answer.

(5) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x \sin y}{\sqrt{x^2+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (1.5 pts) Study the continuity of f at $(0,0)$.

(b) (1.5 pts) Compute $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$ if such quantities exist. Is f differentiable at $(0,0)$? Justify your answer.

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Regular Period Exam - June 5, 2023
Duration: 2h
Exam Notebook II

Name: ____________________ **Student Number:** __________

(6) (2,0 pts) Let $w(x, y, z) = e^x y + \frac{z^2}{2}$ and

$$x(s,t) = \sin(s - t), \quad y(s,t) = s - t, \quad z(s,t) = \sqrt{s + t}.$$

Use the chain rule to show that

$$\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 1.$$

(7) (2.0 pts) Compute the optimal points of

$$f(x, y, z) = x^2 - 10y$$

over the set $D = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 25\}$.

(8) (2.0 pts) Compute

$$\iint_A xe^{-2y}\, dxdy$$

where A is the region bounded by the axes of the plane and the curves $y = 1$ and $y = \ln x$.

(9) (2.0 pts) Solve the boundary value problem

$$\begin{cases} y'' + 4y = 0 \\ y(0) = 0,\ y(\frac{\pi}{4}) = 0 \end{cases}$$

[Extra sheet for additional resolutions]

Lisbon School of Economics & Management
Universidade de Lisboa

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Resit Period Exam - June 29, 2023

Duration: 2h

Exam Notebook I

Name: ______________________ **Student Number:** __________

(1) Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) (1.0 pts) Compute the eigenvalues of A.

(b) (1.5 pts) Determine if A is diagonalizable and compute the eigenspace associated to the smallest eigenvalue of A.

(2) Let B be a 3×3 matrix with eigenvalues equal to 0, 1 and 2.

(a) (1.5 pts) Determine the eigenvalues of $I + B$.

(b) (1.0 pts) Is $I + B$ an invertible matrix? Justify your answer.

(3) (2,0 pts) Consider the following function

$$f(x, y) = \frac{\sqrt{y - x^2}}{\ln(1 + x^2 + y^2 + \sqrt{1 - y})}$$

Determine the domain D_f of f and represent it graphically. Is D_f a compact set? Justify your answer.

(4) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^3 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(a) (1.5 pts) Study the continuity of f at $(0, 0)$.

(b) (1.5 pts) Compute $\dfrac{\partial f}{\partial y}(0, 0)$. Is f differentiable at $(0, 0)$? Justify your answer.

[Extra sheet for additional resolutions]

Department of Mathematics
Undergraduate degrees in Economics, Finance and Management
Mathematics II

Resit Period Exam - June 29, 2023

Duration: 2h

Exam Notebook II

Name: ______________________________ **Student Number:** __________

(5) (2.0 pts) For $|a| \neq 1$, consider the quadratic form

$$q(x, y, z) = x^2 + y^2 + z^2 + 2ayz.$$

Classify q in terms of the parameter a.

(6) (2,0 pts) Consider the function

$$f(x,y) = xe^{-x}(y^2 - 4y)$$

Determine and classify the critical points of f.

(7) (2.0 pts) Compute the optimal points of

$$f(x, y, z) = x + y + z$$

over the set $D = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 2\}$.

(8) (2.0 pts) Use the double integral to calculate the area of the planar region

$$A = \left\{(x, y) \in \mathbb{R}^2 \colon y^2 \le x \le 4,\ 0 \le xy \le 1\right\}.$$

(9) (2.0 pts) Solve the initial value problem

$$\begin{cases} xy' + y + x = 0 \\ y(1) = 0 \end{cases}$$

[Extra sheet for additional resolutions]