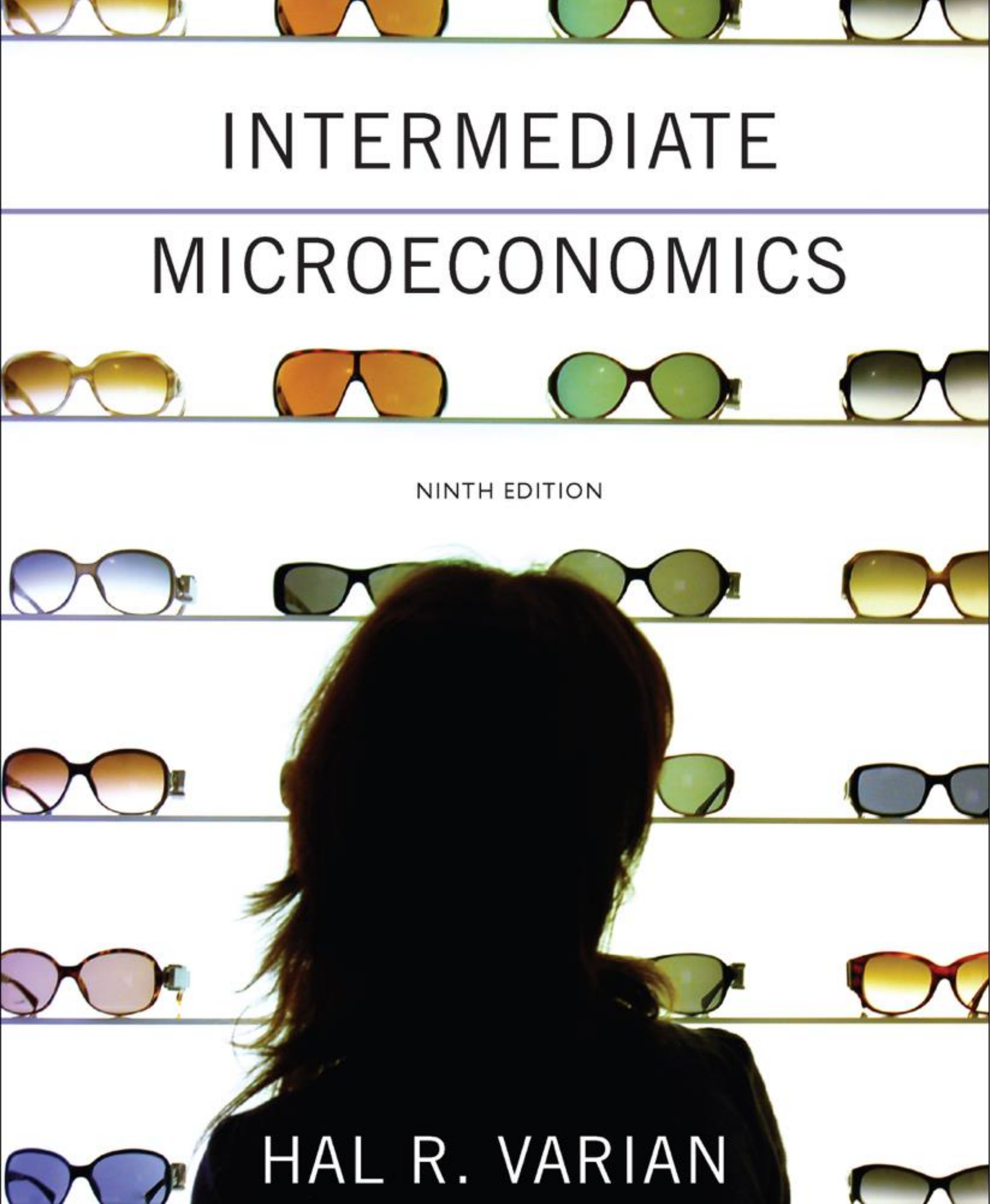


CHAPTER 28

Oligopoly

Oligopoly

A **perfect competitive** market is an industry consisting of many firms.

A **monopoly** is an industry consisting of a single firm.

A **duopoly** (**oligopoly**) is an industry consisting of two (a few) firms.

In case of a duopoly or oligopoly, each firm's own price and output decisions affect its competitors' profits.

This is often referred to as **strategic interaction**: If firm 1 chooses y_1^t, then firm 2 optimally responds by y_2^{t+1}, then in turn firm 1 will optimally respond by $y_1^{t+2} \neq y_1^t$, then in turn firm 2 …

Oligopoly

How do we analyze markets that are oligopolistic?

In general, we want to find an **equilibrium**: a set of strategies where if firm 1 chooses y_1^*, then firm 2 optimally responds by y_2^*, then in turn firm 1 will optimally respond by $y_1^* = y_1^*$, then in turn firm 2 …

→ Firm 1 and 2 both do not have an incentive to deviate from (y_1^*, y_2^*)

We will mostly consider the **duopolistic case** of two firms supplying the same good (perfect substitutes).

Simultaneous Quantity Competition

Assume that firms **compete by choosing output levels** simultaneously. This often happens in markets with **capacity constraints**.

If firm 1 produces y_1 units and firm 2 produces y_2 units then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$

The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

Simultaneous Quantity Competition

Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\pi_1(y_1, y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

Given any output level y_2, what output level y_1 maximizes firm 1's profit?

Simultaneous Quantity Competition: An Example

Suppose that the market inverse demand function is $p(y) = 60 - y$ and that the firms' total cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.

Simultaneous Quantity Competition: An Example

Then, for given y_2, firm 1's profit function is $\pi(y_1, y_2) = (60 - y_1 - y_2)y_1 - y_1^2$.

So, given y_2, firm 1's profit-maximizing output level solves

$$\frac{\partial \pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$

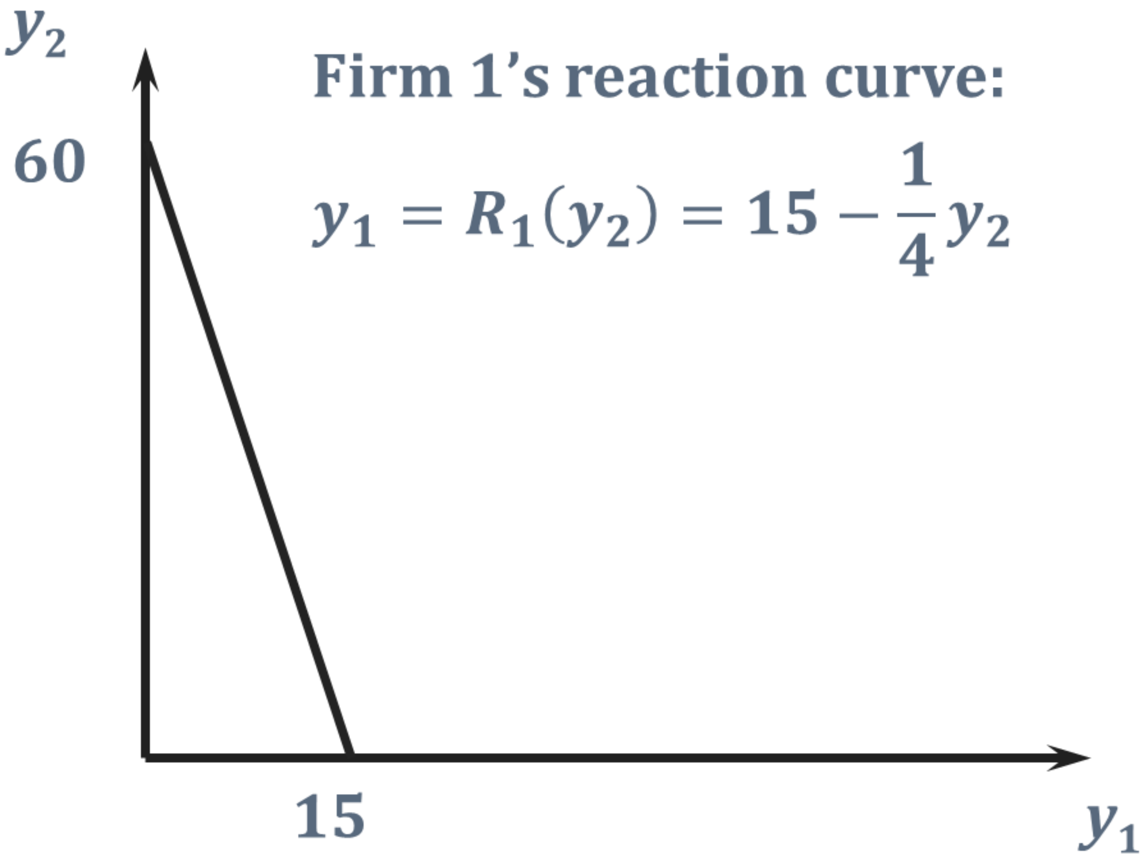
In other words, firm 1's **best reaction function** to y_2 is $y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2$.

The best reaction function captures how firm 1's output choice depends upon the output choice of firm 2.

Simultaneous Quantity Competition: An Example

Firm 1's reaction curve:

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2$$



Simultaneous Quantity Competition: An Example

Similarly, given y_1, firm 2's profit function is

$$\pi(y_1, y_2) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.$$

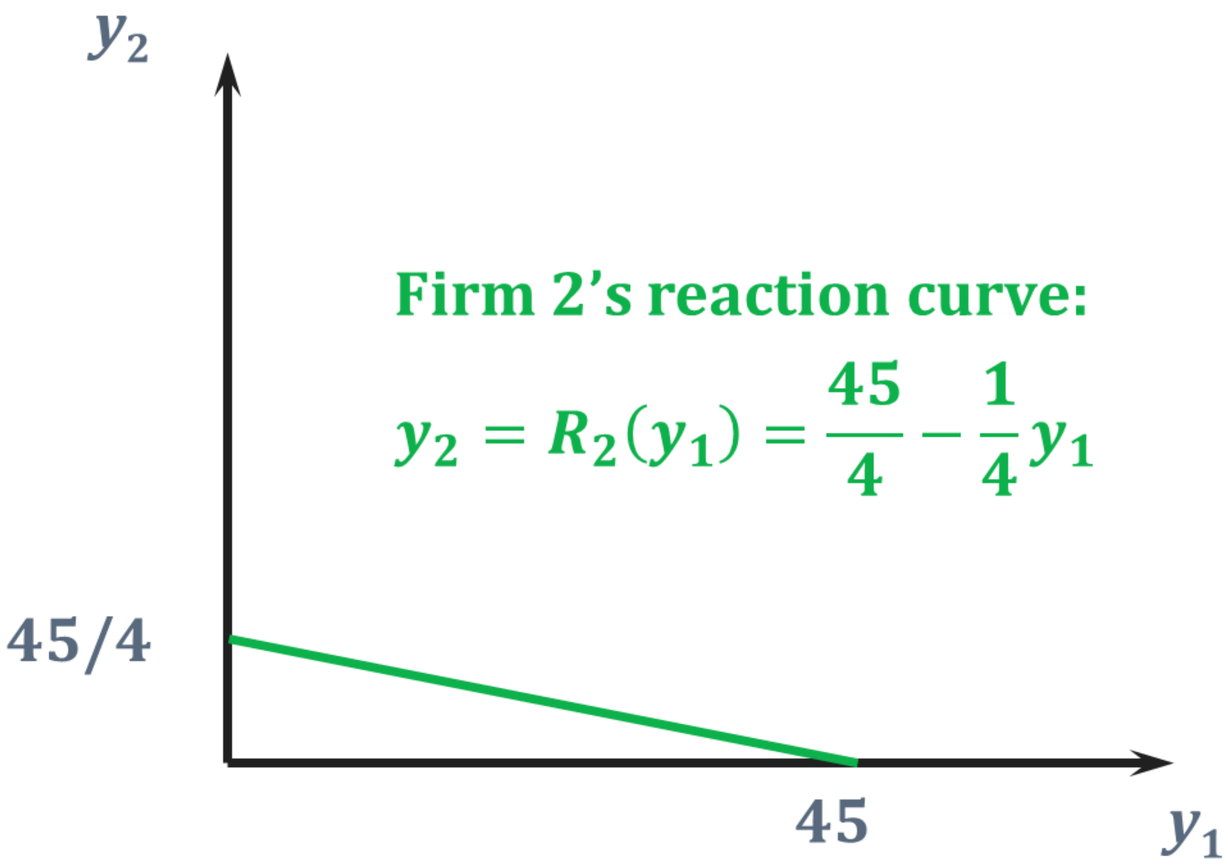
So, given y_1, firm 2's profit-maximizing output level solves

$$\frac{\partial \pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0.$$

In other words, firm 2's **best reaction function** to y_1 is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Simultaneous Quantity Competition: An Example



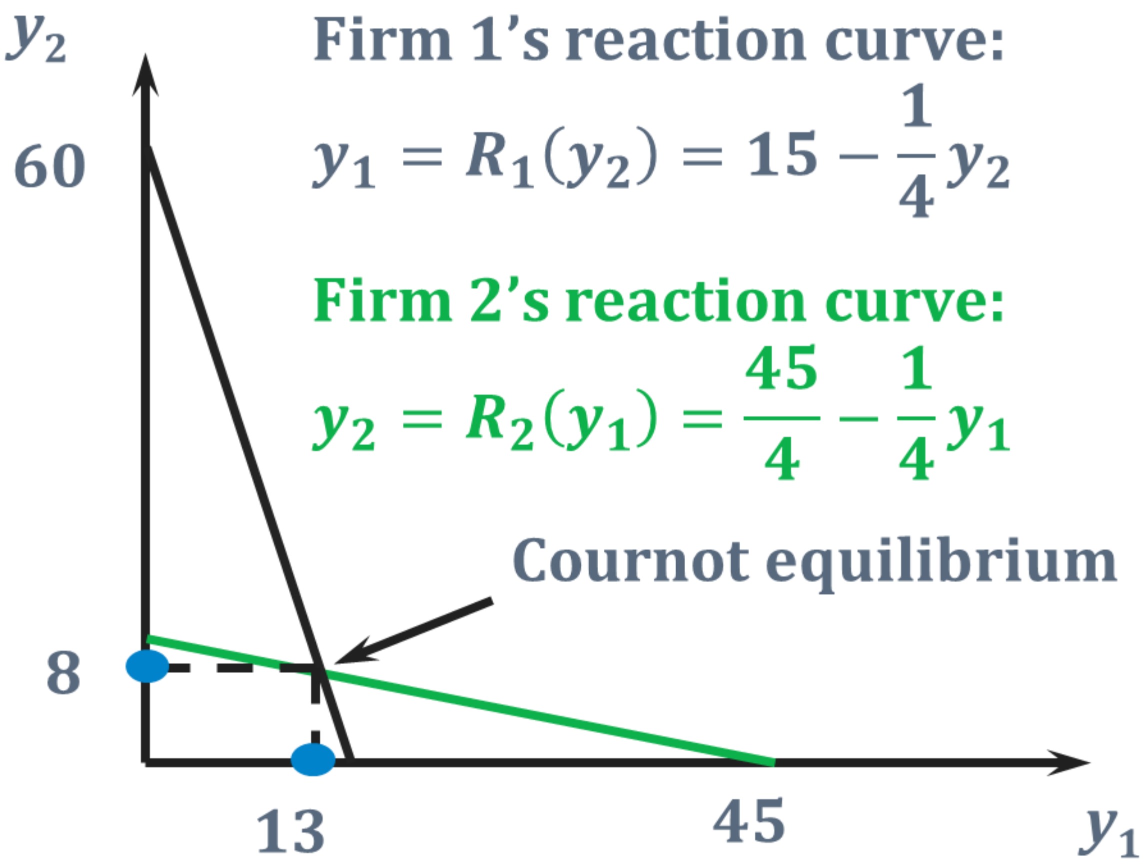
Simultaneous Quantity Competition: An Example

An **equilibrium** is when each firm's output level is a best response to the other firm's output level, for then neither wants to deviate from its output level.

A pair of output levels $(y_1{}^*, y_2{}^*)$ is a **Cournot equilibrium** if $y_1^* = R_1(y_2^*)$
And $y_2^* = R_2(y_1^*)$.

That is, a **Cournot equilibrium** is that firm 1 chooses y_1^*, then firm 2 optimally responds by y_2^*, then firm 1 will optimally respond by $y_1^* = y_1^*$, then in turn firm 2…

Simultaneous Quantity Competition: An Example



Firm 1's reaction curve:

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2$$

Firm 2's reaction curve:

$$y_2 = R_2(y_1) = \frac{45}{4} - \frac{1}{4}y_1$$

Simultaneous Quantity Competition: An Example

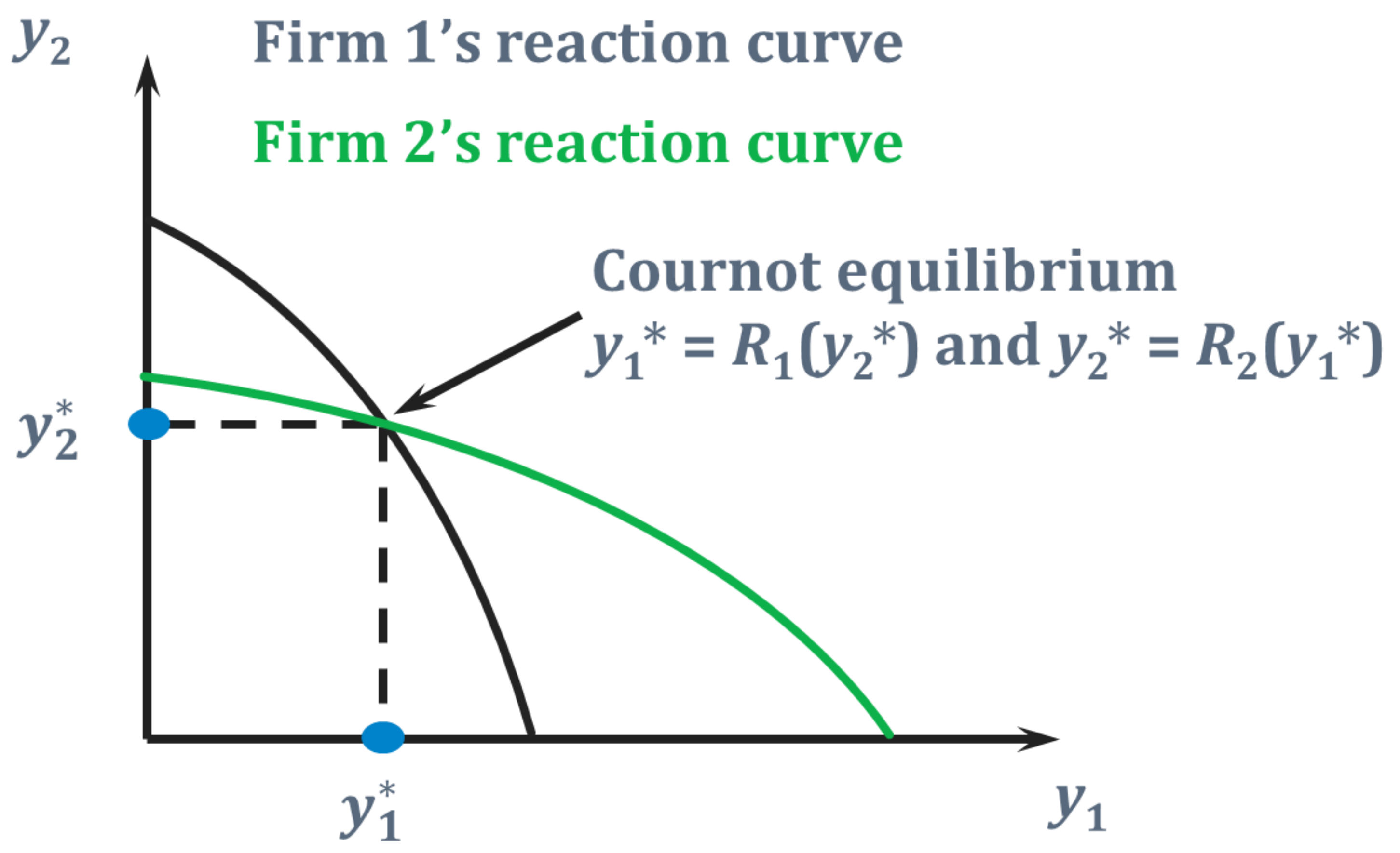
$$y_1^* = R_1(y_2^*) = 15 - \frac{1}{4}y_2^* \text{ and } y_2^* = R_2(y_1^*) = \frac{45}{4} - \frac{y_1^*}{4}$$

So, the Cournot equilibrium is $y_1^* = 13$.

Substitute for y_2^* to get $y_2^* = 8$.

Hence, $(y_1^*, y_2^*) = (13, 8)$.

Simultaneous Quantity Competition: More Generally



Collusion

Are the Cournot equilibrium profits the largest that the firms can earn in total? Or can they do better?

Firms could get together and attempt to set output low and prices high as to maximize **total industry profits** rather than **individual firm profits.** Subsequently the firms could divide the industry profits amongst them.

This is called a **cartel**, and competition authorities have the task to dissolve them since they tend to decrease total (and consumer) surplus.

Cartels always tend to increase profits compared to Cournot competition, since the cartel could always decide to produce at Cournot levels.

Collusion

The problem facing two firms that form a cartel is to choose their outputs y_1 and y_2 as to maximize **industry profits**:

$$\max_{y_1, y_2} p(y_1 + y_2)[y_1 + y_2] - c_1(y_1) - c_2(y_2)$$

If we define $y = y_1 + y_2$, then we can also write:

$$\max_{y_1, y_2} p(y)y - c_1(y_1) - c_2(y_2)$$

Note that this profit function is different from the one of a single monopolist that can third-degree price discriminate between different types of consumers:

$$\max_{y_1, y_2} p_1(y_1)y_1 + p_2(y_2)y_2 - c(y)$$

Collusion and Stability

Is such a cartel stable? Does one firm have an incentive to cheat on the other?

In other words, if firm 2 continues to produce y_2^m units, is it profit-maximizing for firm 1 to continue to produce y_1^m units?

The answer is no. Firm 1 has an incentive to **cheat** and start producing more. The intuition is that the cartel keeps output low and prices high to maximize joint profits. In this case, a single firm can exploit these higher prices and start producing more to maximize individual profits.

Collusion and Stability

The two first order conditions to the profit maximization problem are:

$$\frac{\partial \pi}{\partial y_1} = \frac{\partial p(y)}{\partial y}\frac{\partial y}{\partial y_1}y + p(y)\frac{\partial y}{\partial y_1} - mc_1(y_1) = 0$$

$$\frac{\partial \pi}{\partial y_2} = \frac{\partial p(y)}{\partial y}\frac{\partial y}{\partial y_2}y + p(y)\frac{\partial y}{\partial y_2} - mc_2(y_2) = 0$$

Note that $\frac{\partial y}{\partial y_1} = \frac{\partial y}{\partial y_2} = 1$, and hence it can be written as:

$$p(y) + \frac{\partial p(y)}{\partial y}y - mc_1(y_1) = 0$$

$$p(y) + \frac{\partial p(y)}{\partial y}y - mc_2(y_2) = 0$$

Collusion and Stability

And since $y = y_1 + y_2$, we can write:

$$\underbrace{p(y) + \frac{\partial p(y)}{\partial y}[y_1 + y_2]}_{mr\ of\ cartel} = mc_1(y_1)$$

The mr has the usual two effects: if firm 1 increases output y_1 then (i) the revenue increases by the price, but (ii) the revenue decreases because it decreases the prices for all products sold.

With (ii), the cartel considers the effect of the lower price on **total industry output** y (and not just y_1). We knew that for a monopolist $mr < p$ due to the second effect. In case of a cartel $mr < p$ as well, and effect (ii) is even stronger so mr is even lower.

Collusion and Stability

Imagine the cartel maximizes joint profits with y_1^m and y_2^m. Now what would happen if firm 1 considers her own profit function (and not the cartel's)?

$$\max_{y_1} p(y)y_1 - c_1(y_1)$$

She takes the first order condition for profit maximization:

$$\frac{\partial \pi_1}{\partial y_1} = \frac{\partial p(y)}{\partial y}\frac{\partial y}{\partial y_1}y_1 + p(y)\frac{\partial y_1}{\partial y_1} - mc_1(y_1) = 0$$

And since $\frac{\partial y}{\partial y_1} = \frac{\partial y_1}{\partial y_1} = 1$, we can write this as:

$$p(y) + \frac{\partial p(y)}{\partial y}y_1 - mc_1(y_1) = 0$$

Collusion and Stability

The first order condition for firm 1 is:

$$\underbrace{p(y) + \frac{\partial p(y)}{\partial y} y_1 - mc_1(y_1)}_{\frac{\partial \pi}{\partial y_1}} = 0$$

Recall that the first order condition for the cartel was:

$$p(y) + \frac{\partial p(y)}{\partial y} [y_1 + y_2] - mc_1(y_1) = 0$$

Rewriting this cartel solution, we know that:

$$\underbrace{p(y) + \frac{\partial p(y)}{\partial y} y_1 - mc_1(y_1)}_{\frac{\partial \pi}{\partial y_1}} = \underbrace{-\frac{\partial p(y)}{\partial y} y_2}_{>0}$$

Hence it is the case that $\frac{\partial \pi_1}{\partial y_1} > 0$.

Collusion and Stability

$\frac{\partial \pi_1}{\partial y_1} > 0$: Firm 1 could increase its individual profits by **cheating** and start producing more. This is also true for firm 2.

Again, the intuition is that the cartel keeps output low and prices high to **maximize joint profits**. Both firm 1 and 2 have an incentive to exploit the high prices and start producing more to **maximize individual profits**.

The mathematics makes clear where this incentive comes from: For a single firm the marginal revenue is higher than for the cartel.

Collusion and Punishment

So, a profit-maximizing cartel in which firms jointly set their output levels is fundamentally **unstable**.

But is the cartel unstable if the game is repeated many times, instead of being played only once? Then there is an opportunity to **punish** a cheater and make a cartel stable.

Collusion and Punishment

Consider that there are two firms that try to make a cartel stable. One firm says to the other: *"We should both produce the output level that maximizes cartel profits. But if you ever deviate and produce more, I will punish you by producing the Cournot output for ever."*

Then define:

π_m: profit under cartel

π_d: profit if firm deviates from cartel

π_c: profit under Cournot competition, with

$$\pi_d > \pi_m > \pi_c$$

Collusion and Punishment

This threat may stabilize the cartel if the PV of the cartel is larger than the PV of cheating. Let r be the discount rate, then:

$$present\ value\ of\ cartel\ = \pi_m + \frac{\pi_m}{r}$$

$$present\ value\ of\ cheating = \pi_d + \frac{\pi_c}{r}$$

Hence, firms keep cooperating if:

$$r < \frac{\pi_m - \pi_c}{\pi_d - \pi_m}$$

Cooperating is likely if the discount rate is sufficiently small, so that the future is important.

The Order of Play

In the quantity competition discussed so far, it has been assumed that firms choose their output levels **simultaneously**.

The competition between the firms is then a **simultaneous game** in which the output levels are the strategic variables.

The Order of Play

What if firm 1 chooses its output level first and then firm 2 responds to this choice?

Firm 1 is a **leader**. Firm 2 is a **follower**.

The competition is a sequential game in which quantity is the strategic variable. Such games are **Stackelberg** games (rather than **Cournot** games)

Sequential Quantity Competition

What is the best response that follower firm 2 can make to the choice y_1 already made by the leader, firm 1?

For firm 2 this is simply the **best reaction function** we have seen in simultaneous quantity competition. Choose $y_2 = R_2(y_1)$.

Firm 1 knows this and so anticipates firm 2's reaction to any y_1 .

Sequential Quantity Competition

This makes the **leader's** profit function

$$\pi_1^s(y_1) = p\big(y_1 + R_2(y_1)\big)y_1 - c_1(y_1).$$

The leader chooses y_1 to maximize its profit taking into account $R_2(y_1)$.

Sequential Quantity Competition

The leader could choose its Cournot output level, knowing that the follower would then also choose its Cournot output level. The leader's profit would then be its Cournot profit.

But the leader does not have to do this, so **the leader's profit must be at least as large as with Cournot**.

In fact, the leader can make a slightly higher profit than under Cournot. The intuition is that it will set its output a bit higher than Cournot, knowing that firm 2 will follow with $R_2(y_1)$ and setting its output somewhat lower. Firm 1 can put firm 2 "*with its back against the wall*" by choosing a high y_1.

Sequential Quantity Competition: An Example

The example uses identical demand and cost functions as with simultaneous quantity competition.

The market inverse demand function is $p(y) = 60 - y$

The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.

Firm 2 is the follower. Its **best reaction function** is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Sequential Quantity Competition: An Example

The **leader's profit function** is therefore:

$$\pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= \left(60 - y_1 - \frac{45 - y_1}{4}\right)y_1 - y_1^2 = \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

To maximize profit for firm 1, $\frac{195}{4} = \frac{7}{2}y_1$ or $y_1^s = \frac{195}{14} \sim 13.9.$

Sequential Quantity Competition: An Example

Firm 2's **best response** to the leader's choice is therefore:

$$y_2^s = R_2(y_1^s) = \frac{45 - 13.9}{4} = 7.8.$$

The Cournot levels were (13; 8).

So, the leader produces more than its Cournot output (13.9>13) and the follower produces less than its Cournot output (7.8<8).

Price Competition

What if firms compete using price-setting strategies instead of using quantity-setting strategies?

This often happens in markets **without capacity constraints**. That is, prices are chosen, and the firm can subsequently produce as many products as there are consumers.

Games in which firms use only price strategies and play simultaneously are called **Bertrand** games.

Simultaneous Price Competition

Each firm's marginal cost is constant at c.

All firms set their prices p simultaneously.

Firms supply the same product (perfect substitutes), as was the case for quantity competition as well.

Is there an equilibrium?

Simultaneous Price Competition

Is there an equilibrium?

Yes. Exactly one. All firms set their prices equal to the marginal cost. Why?

Suppose a setting with two firms, and firm 1 believes that firm 2 sets its price higher than marginal cost.

Then firm 1 could set the price just below the price of firm 2. All consumers would buy from firm 1 (since perfect substitutes). Firm 1 makes a profit (since $p > mc$) and firm 2 is out of business.

But firm 2 can reason the same way. The only price where this undercutting stops, and both firms do not have an incentive to deviate, is when $p = mc$.

Overview of the Various Settings

Simultaneous **quantity**: Cournot competition

Collusion

→ Highest price and lowest output

Quantity leadership: Stackelberg competition

Simultaneous **price**: Bertrand competition

→Lowest price and highest output

Price leadership: not discussed in lecture