Homework 1
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Exercise 1: Let $V_M(t)$ and $V_C(t)$ be the the expected payoffs from holding money and the consumption good at the beginning of a generic period t, respectively. Money has no storage cost but the consumption good has a storage cost s. At the end of period t the trades occur and the storage costs are paid, and those agents that successfully sold the good produce another unit before the end of period t. Find the steady state symmetric Nash equilibria.

Solution: The indirect utility from holding money is:

$$V_M(t) = \frac{1}{1+r}\{(1-M)x\Pi U + (1-M)x\Pi V_C(t+1) + (1-(1-M)x\Pi)V_M(t+1)\}$$

and the indirect utility from holding a consumption good is:

$$V_C(t) = \frac{1}{1+r}\left\{-s + (1-M)x^2U + Mx\pi V_M(t+1) + (1-Mx\pi)V_C(t+1)\right\}$$

Substracting after imposing symmetry and assuming the steady-state,

$$V_C - V_M = \frac{(1-M)xU\,(x-\Pi) - s}{r + x\Pi}$$

The sign of the difference between expected utility from holding a good and money depends on the difference between the acceptance probability of goods, x, the acceptance probability of money, Π and the storage cost of the good, s. The higher the storage cost the lower the difference between expected utility from holding a good and money and the higher the probability of a monetary equilibrium.

Three Nash equilibria:

1. $x - \frac{s}{(1-M)xU} > \Pi \Rightarrow V_C > V_M \Rightarrow$ Agent never accepts money in exchange for a real commodity $\Rightarrow \pi = 0$

2. $x - \frac{s}{(1-M)xU} < \Pi \Rightarrow V_C < V_M \Rightarrow$ Agent always accepts money in exchange for a real commodity $\Rightarrow \pi = 1$

3. $x - \frac{s}{(1-M)xU} = \Pi \Rightarrow V_C = V_M \Rightarrow$ Agent is indifferent between money and a real commodity

$\Rightarrow \pi$ is anything between 0 and 1

Exercise 2: Consider that the representative agent of the Diamond-Dybvig model version considered in class has utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ where $\sigma > 1$. Assume that $\sigma = 2$, $t = 0.5$, and that $r = 0.44$. Compute the optimal contract. Show that a bank run can be an equilibrium outcome if the number of agents N is sufficiently large. What is the interval for the probability of a bank run that makes an agent prefer deposits to autarcy?

Solution: The optimal contract is $c_1 = 1.091$ and $c_2 = 1.309$.

In period 1 if each patient agent believes that all the other agents are going to withdraw their deposits in period 1 then he is better off by withdrawing his deposit also. If he waits he gets in period 2 what is left by the other agents which is: $\max \{0, (N - (N-1)1.091)\, 1.44\}$. For $N \geq 12$ this expression is 0, while if he decides to withdraw in period 1 he has a positive probability of getting 1.091.

The expected utility of autarcy is $0.5(-1) + 0.5(-0.694) = -0.847$

Autarcy is always better as long as the probability of a bank run is positive no matter how small it is. This result is due to the particular utility function considered. If an agent does not consume, because of the bank run, his utility is $-\frac{1}{0} = -\infty$.