Exercise SET 1 (EST.II)

Erida Gjini
Solution

ISEG, Universidade de Lisboa

erida.gjini@iseg.ulisboa.pt

04.03.2024

$X \sim Binomial(n, \theta)$

$\underline{n = 18, \theta = 0.3}$

- $Prob(X = 8) = 0.0811$
- $Prob(X = 16) \approx 0$

- $Prob(X \leq 10) = 0.0386 + 0.0811 + .. = 0.9939$

$\underline{n = 10, \theta = 0.6}$

$n - X \sim Binomial(n, 1 - \theta)$ – na tabela ver $1 - \theta = 0.4$

- $Prob(X = 8) = Prob(n - X = 2) = 0.1209$ (Tabela 1, linha 2, coluna 0.4)
- $Prob(X = 0) = Prob(n - X = 10) = 0.0001$
- $Prob(X > 3) = 1 - Prob(X \leq 3) = Prob(n - X \leq 7) = 0.9877$ (Tabela 1, linha 7, coluna 0.4)

$X \sim Poisson(\lambda)$

$\underline{\lambda = 4.7}$

- $Prob(X = 4) = 0.1849$
- $Prob(X > 4) = 1 - Prob(X \leq 4) = 1 - 0.4946$
- $Prob(X \leq 10) = 0.9910$

$Z \sim Normal(0, 1)$

- $Prob(Z = 2.1) = 0.0440$ (Tabela 3)
- $Prob(Z > 1.92) = 1 - Prob(Z \leq 1.92) = 1 - 0.9726$ (Tabela 4)
- $Prob(Z > b) = 0.05,\ b = 2.576$ (Tabela 5)
- $Prob(Z < a) = 0.75, a =?\ Prob(Z < a) \approx Prob(Z \leq a) = 1 - Prob(Z > a) = 0.75 \implies Prob(Z > a) = 0.25, a \in [0.524, 0.842]$ (Tabela 5)

$X_i \sim Normal(\mu, \sigma^2)$

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$\mu = 2, \sigma^2 = 2$

Usar:

$$Z := \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

- $(n = 10)$ $Prob(\bar{X} = 2) = Prob(Z = \frac{2-2}{\sqrt{2}/\sqrt{10}}) = Prob(Z = 0) = 0.3989$
- $(n = 100)$ $Prob(\bar{X} > 3) = 1 - Prob(\bar{X} \leq 3) = 1 - Prob(Z < \frac{3-2}{\sqrt{2}/10}) = 1 - Prob(Z < 10/\sqrt{2}) = 1 - Prob(Z < 7.07) \approx 1 - 1 = 0$
- $(n = 50)$ $Prob(\bar{X} > b) = 0.1, b = ?$ $Prob(Z > \frac{b-2}{\sqrt{2}/\sqrt{50}}) = 0.1 \implies \frac{b-2}{\sqrt{2}/\sqrt{50}} = \Phi^{-1}(0.1) = 1.282 \implies b = 1.282 * 0.2 + 2 = 2.2564$

$Q \sim \chi^2_n$

$\underline{n = 11}$

- $Prob(Q > q_1) = 0.05, q_1 = 19.675$
- $Prob(Q < q_2) = 0.95$ $q_2 = 19.675$
- $Prob(Q > q_3) = 0.25$ $q_3 = 13.701$

$T \sim t(n)(t - student)$

$\underline{n = 12}$

- $Prob(T > t_1) = 0.025$ $t_1 = 2.179$
- $Prob(T < t_2) = 0.95$
  $1 - Prob(T > t_2) = 0.95 \rightarrow Prob(T > t_2) = 0.05 \rightarrow t_2 = 1.782$

$\underline{n = 5}$

- $Prob(T < t_3) = 0.75$, $t_3 = 0.727$