Homework 2
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**Exercise 1: The Money Multiplier**

Please derive the expression for the money multiplier you have seen in class. Let $R$ denote total reserves, $RR$ required reserves, $ER$ excess reserves, $C$ currency, $D$ deposits, $M$ money supply, $MB$ the monetary base, $r$ the required reserve ratio, $c$ the currency ratio, and $e$ the excess reserve ratio.

Discuss why the money multiplier is normally greater than 1.

1. Now evaluate the money multiplier empirically. To do this, access the website of the Federal Reserve Economic Data (FRED) http://research.stlouisfed.org/fred2. Find data for the "M1 Money Multiplier (MULT)" and plot a graph for the sample period 1984-02-15 to 2015-02-18 (Frequency: Bi-Weekly; Unit: Ratio).

2. What do you observe up to 2008 and what crucial change occurs in 2008 and stays after? Relate your observation to what you derived in the beginning of the exercise. Interpret the size of the multiplier that is reported for 2015-02-18 (e.g. If the central bank increases xxx by one unit,...).

3. How is the money multiplier calculated? Plot the two data series that underly the money multiplier in a graph for the same sample period as above (Frequency: Bi-Weekly; Unit: Billions of Dollars). Interpret your observation from point 2 by using the graph, i.e. which series increased more and what does this indicate?

(Hint: The multiplier is a ratio of two other data series. When you look at a data series - before you click on "Edit Graph" -, you will find the option "View Data" on the left and can check "Notes".)

4. Try to find an explanation for the observation and relate it to the derivation of the money multiplier. In doing so, plot a graph for the same sample period as before in which you plot the monetary base with its three components that you derived above (Frequency: Monthly; Unit: Billions of Dollars). Further, describe what your explanation implies about the size of $R$ relative to $D$. Do you see this in the data when you plot Total Checkable Deposits (TCDNS), the Reserves (ADJRESSL), and the Multiplier (MULT)?

(Hint: Think about what you have seen and heard about the behavior of banks during the financial crisis. Also feel free to check google or other sources.)

**Exercise 2: Open Market Operations and the Money Supply**

Consider the following initial situation: $MB = 1000$, $r = 0.1$, $e = 0.1$, $c = 0$. Calculate the money multiplier $m$, the money supply $M$, the deposits $D$, the excess reserves $ER$ and the required reserves $RR$.

(Hint: You can make use of the formula for the money multiplier that you derived in the previous exercise.)

Now consider the three following changes in monetary policy, in each case starting from the initial situation.

1. The Fed increases the monetary base through an open market operation of size 500. First, represent with the help of a graph the effect on the federal funds rate in the market for reserves, assuming that both before and after the open market operation the supply and the demand curves of reserves intersect in the vertical portion of the first and the downward sloping portion of the second. Second, calculate the changes in $MB$, $M$, $RR$, $ER$, $D$, and $m$. Discuss.

2. The Fed increases the monetary base through an open market operation of size 500. First, represent with the help of a graph the effect on the federal funds rate in the market for reserves, assuming that both before and after the open market operation the supply and demand curves intersect in the vertical portion of the first and the flat portion of the second. Second, calculate the changes in $MB$, $M$, $RR$, $ER$, $D$, and $m$. Discuss.

3. Initially the supply and demand curves of reserves intersect in the vertical portion of the first and the flat portion of the second. The Fed raises the interest rate on reserves. First, represent with the help of a graph the effect on the federal funds rate in the market for reserves. Second, explain why this policy will lead to an increase in the excess reserve ratio $e$. Third, assume $e$ raises to 0.3 and calculate the changes in the $MB$, $M$, $RR$, $ER$, $D$, and $m$. Discuss.

**Exercise 3: Investment curve**

Consider a two period economy with a representative firm. The profits of the firm in period 1 are

$$\pi = y - wN - I,$$

where $y = F(K, N)$ is the output, $K$ is the stock of capital, $N$ are hours of work, $w$ is the real wage and $I$ is investment. The profits of the firm in period 2 are

$$\pi' = y' - w'N' + (1 - d)K',$$

where the variables with the superscript "$\prime$" are period 2 variables and $y' = F(K', N')$. The present value of profits is

$$V = \pi + \frac{\pi'}{1 + r},$$

where $r$ is the real interest rate. The firm's problem is to maximize the present value of its profits:

$$\begin{aligned} \max_{\{N,I,K',N'\}} V &= y - wN - I + \frac{y' - w'N' + (1 - d)K'}{1 + r} \\ &\quad \text{s.t.} \\ &\quad K \text{ is given} \\ K' &= (1 - d)K + I \end{aligned}$$

Show that the firm's optimal investment rule, obtained by equating the marginal benefit and marginal cost of investment is

$$F_{K'} = d + r.$$

Use this equation to show that the relationship between $I$ and $r$ is negative.

**Exercise 4: IS curve**

Consider an economy which lasts for 2 periods. Suppose the representative agent's lifetime utility is given by

$$U(C_1, C_2) = \log C_1 + \beta \log C_2,$$

where $0 < \beta < 1$ is the intertemporal discount factor and $C_1$ and $C_2$ are consumption in real terms in the first and the second period, respectively. Denote income in real terms in the first and the second period with $Y_1$ and $Y_2$, respectively. Furthermore, assume that initially agents have a positive wealth $A_1$. In the first period agents either consume or invest in nominal bonds which yield a return $i_1$. In the second period they spend all of their income.

1. Write the budget constraint for periods 1 and 2 in nominal terms and derive the intertemporal budget constraint.

2. Recall that the exact relationship between the real and the nominal interest rate is $1+r_1 = \frac{1+i_1}{1+\pi_2}$ where $\pi_2$ denotes net inflation. Use this relationship to express the intertemporal budget constraint in real terms.

3. State the agent's optimization problem and derive the optimal consumption path.

4. What is the condition for the consumption path to be constant? Provide an economic explanation.

5. Derive the log linear dynamic IS curve and explain what are the major differences with the standard IS curve.