Inventory Management Exercises Solutions

IM_1

EOQ model

(a) $Q^* = \sqrt{\frac{2DS}{H}}$ = [(2 × D × S)/H]$^{1/2}$ = [(2 × 20000 × 90)/2,4]$^{1/2}$ = 1224,7 ≈ 1225 liters

(b) TC = (Q/2) × H + (D/Q) × S + **P** × D =

1225/2 × 2,4 + 20000/1225 × 90 + 12 × 20000 = 242939,30 €/year

(c) T = Q/D = 1225/20000 = 0,06125 years ≈15,31 days

(d) N = 20000/1225 ≈ 16,33 orders or 17 orders

(e) $Q_{t=25}$= $Q^{max.}$ – (t – T) × d = 1225 – (25-15,31) × 80 = 449,8 liters

(f) EOQ model

the EOQ was Q^* = 1225 liters. Now, we must analyze the costs for Q = 1200 liters and Q = 1300 liters, respectively, and choose the lowest of the two.

Cost $_{Q=1200}$ = 1200/2 × 2.4 + 20000/1200 × 90 = 2940,00 €/year

Cost $_{Q=1300}$ = 1300/2 × 2.4 + 20000/1300 × 90 = 2944,62 €/year

The cost variation is:

(2940,00-2939,39)/2939,39 × 100 = 0,0208% → 2,08%

Alternatively:

ΔC = C/C_0 = (D/Q × S + Q/2 × H)/(D/Q_0 × S + Q_0/2 × H)

= {[(2DS + Q^2 × H)/2Q]}/{[(2DS + Q_0^2 × H)/2 Q_0^2]}

= [Q_0 × (2DS/H + Q^2)]/[Q × (2DS/H + Q_0^2)]

= [Q_0 × (Q_0^2 + Q^2)]/ [Q × (Q_0^2 + Q_0^2)]

= (Q_0^2 + Q^2)/ 2Q Q_0

=1/2 × (Q_0/Q + Q/Q_0)

Solving for our values:

ΔC = C/C_0 = 1/2 × (1225/1200 + 1200/1225) = 1,000213, or a 0,213% cost increase.

IM_2

EOQ model

(a) $Q^* = \sqrt{\frac{2DS}{H}}$= [(2 × 72000 × 10)/1]$^{1/2}$ = 1200 units

(b) Annual cost = D/Q × S + Q/2 × H =

72000/1200 × 10 + 1200/2 × 1 = 1200,00 €/year

(c) ROP = LT × d = 4 × 288 = 1152 units

(d) $CV_{1000\ units}$= (72000/1000) × 10 + (1000/2) × 1 = 1220,00 €/year → Cheaper

$CV_{1500\ units}$= (72000/1500) × 10 + (1500/2) × 1 = 1230,00 €/year

IM_3

Quantity discounts

D = 86000 bottles/year; I = 0,3 × P euros/bottle/year; S = €10/order

P	10	9,5	9	7
Q	≤500	501-700	701-900	>900

First, we calculate the Q^* for the lowest possible price, €7.00

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 86\,000 \times 10}{0.3 \times 7}} = 905{,}25 \text{ bottles} = 906 \text{ bottles}$$

Because 906 > 900 units, this EOQ is ***feasible*** for the €7,00 price, therefore let's calculate the total cost for EOQ = 906 bottles.

P	*Q**	*Q*	*Annual holding cost (H*Q/2)*	*Annual ordering cost (D/Q*S)*	*Annual acquisition cost (P*D)*	*Annual Total Cost*
7	906	906	949,23	951,30	602 000,00	603 900,53

IM_4

Quantity discounts

D = 12000 units/year; I = 0,2 × P euros/unit/year; S = €30/order

p	€10	€9,5	€9,4
Q	<2000	2000≤Q<6000	Q≥6000

First, we calculate the *Q** for the lowest possible price, €9,40.

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 12\,000 \times 30}{0,2 \times 9,4}} = 618,85 \text{ units}$$

Because 618,85 < 6000, this EOQ is ***infeasible*** for the €9,4 price, therefore let's calculate the total cost for the smallest quantity that allows us to take advantage of the €9,4 price ➔ 6000.

Now we calculate the EOQ for the next price, €9,50.

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 12\,000 \times 30}{0,2 \times 9,5}} = 615,59 \text{ units}$$

Because 615,59 < 2000, this EOQ is ***infeasible*** for the €9,5 price, therefore let's calculate the total cost for the smallest quantity that allows us to take advantage of the €9,5 price ➔ 2000.

Now we calculate the EOQ for the next price, €10,00.

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 12\,000 \times 30}{0,2 \times 10,0}} = 600 \text{ units}$$

Because 600 < 2000, this EOQ is ***feasible*** for the €10,00 price, therefore let's calculate the total cost for EOQ = 600 bottles.

P	*Q**	*Q*	*Annual holding cost (H*Q/2)*	*Annual ordering cost (D/Q*S)*	*Annual acquisition cost (P*D)*	*Annual Total Cost*
9,4	618,85	6000	60	5640	112800,00	118500,00
9,5	615,59	2000	180	1900	114000,00	116080,00
10,00	600	600	600	600	120000,00	121200,00

(a) Q^* = 2000 units for a total cost of €116080,00/year

(b) If Q = 6000

(6000/2 × 0,2 × **P)** + (12000/6000 × 30) + (**P** × 12000) ≤ 116080,00

P ≤ 9,208 €

IM_5

D_{ARC} = 750 units/year; ➔ D_{CZ} = 3 x 750 = 2250 units/year;

I = 0,1 × P €/unit/year; S = €200/order.

P	€25	€20
Q	Q < 1000	Q ≥ 1000

First, we calculate the Q^* for the lowest possible price, €20.00

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 2250 \times 200}{0,1 \times 20}} = 670,5 \text{ units} = 671 \text{ units}$$

Because 671 < 1000 units, this EOQ is ***infeasible*** for the €20,00 price, therefore let's calculate the total cost for the smallest quantity that allows us to take advantage of the €20,00 price ➔ 1000.

$TC_{Q=1000}$ = (1000/2)×2 + 2250/1000×200 + 20×2250 = €46450,00/year

Now we calculate the EOQ for the next price, €25.00.

$$EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 2250 \times 200}{0,1 \times 25}} = 600 \text{ units} = 600 \text{ units}$$

Because 600 < 1000, this EOQ is ***feasible*** for the €25,00 price, therefore let's calculate the total cost for EOQ = 600 units.

$TC_{Q=600}$ = (600/2)×2 + 2250/600×200 + 25×2250 = €57750,00/year

The EOQ is 1000 units. The total annual cost associated with this is €46450,00/year.

IM_6

POQ model

(a) $Q^* = \sqrt{(2 \times D \times S)/[H \times (1 - d/p)]}$

$= \{(2 \times 60000 \times 200)/[3{,}84 \times (1 - 240/320)]\}^{1/2} = 5000$ lenses

$$QEF = \sqrt{\frac{2 \times 60\,000 \times 200}{3{,}84 \times (1 - \frac{240}{320})}}$$

(b) $TC = [Q/2 \times (1\text{-}d/p)] \times H + (D/Q \times S) + (\mathbf{P} \times D)$

(In this case, we have no production cost **P**, so we compute the variables costs)

$= [5000 \times (1 - 240/320)]/2 \times 3{,}84 + 60000/5000 \times 200 = 4800{,}00$ €/year

(c) $T = 5000/60000 \approx 0{,}0833$ years or 20,83 days

(d) $t_1 = Q/p = 5000/320 = 15{,}625$ days

(e) $t_2 = T - t_1 = 5{,}205$ days

(f) $I^{Max.} = Q \times (1 - d/p) = 1250$ lenses

(g) $I^{Avg.} = 0{,}5 \times [Q \times (1 - d/p)] = 625$ lenses

(h) $I_{t=10} = 10 \text{ days} \times (320 - 240) = 800$ lenses

(i) $I_{máx} - d \times (T - t_1 - t) = 1250 - 240 \times (5{,}205 - 3) = 720{,}8$ lenses ≈ 720 lenses

or $t \times d = 3 \times 240 = 720$ lenses

IM_7

Quantity discounts (economies of scale)

If **P** = 1500 €/unit

$Q^* = \sqrt{(2 \times D \times S)/[H \times (1 - d/p)]}$

$Q_{P=1500} = \{(2 \times 60000 \times 300)/[150 \times (1 - 60000/100000)]\}^{1/2} \approx 774{,}6$ units

$$POQ = \sqrt{\frac{2 \times 60\,000 \times 300}{1500 \times \left(1 - \frac{60000}{100000}\right)}} = 774{,}6\ unidades$$

If Q = 600 (lower 800 units)

TC = [600 × (1-0,6)]/2 × 150 + 60000/600 × 300 + 1500 × 60000 = 90048000,00€/year

Notice that Q = 800 is not defined for the **P** = 1500 €/unit interval, so the total cost calculation is not applicable in such case.

If **P** = 1400 €/unit

$Q_{P=1400} = \{(2 \times 60000 \times 300)/[140 \times (1 - 60000/100000)]\}^{1/2} \approx 801{,}7$ units

$$POQ = \sqrt{\frac{2 \times 60\,000 \times 300}{1400 \times \left(1 - \frac{60000}{100000}\right)}} = 801{,}7\ unidades$$

If Q = 800

TC = [800 × (1-0,6)]/2 × 140 + 60000/800 × 300 + 1400 × 60000 = 84044900,00€/year

If Q = 1000

TC = [1000 × (1-0,6)]/2 × 140 + 60000/1000 × 300 + 1400 × 60000

= 84046000,00€/year

The production order quantity is 800 units.

IM_8

(a1) I_{max} = Q×(1-d/p) = 600 ×(1-150/200) = 150 units

(a2) Annual holding cost = Q/2 × (1-2/p) × H = 150/2 × 25€ = €1875,00/year

(a3) t_1 = Q/p = 600/200 = 3 weeks

(a4) We must first determine the cycle time: T = Q/D = 600/7500 = 0,08 years = 20 days (250 work days/year)

$I_{t=18}$ = d × (T – t) = 150/5 × (20-18) = 60 units

(b)

$$\text{POQ} = \sqrt{\frac{2 \times 7500 \times 250}{0{,}25 \times 100 \times \left(1 - \frac{150}{200}\right)}} = 774{,}6\,units \approx 775 \text{ units}$$

The lot size should be increased from 600 units to 775 units.

IM_9

(a) $Q^* = \sqrt{(2 \times \text{D x S})/(\text{I} \times \text{P})}$

$Q^* = \sqrt{(2 \times 1250 \times 150)/(1)}$ = 612,37 ➔ 613 units

(b) SS = Z x $(d \times \sigma_{LT})$ ➔ 1,96 x (25 x 2) = 98 units

(c) Annual holding cost: (Q^*/2 + SS) x H

(613/2 + 98)× 1 = €404,50

(d) ROP = (d x LT) + $Z \times \sigma_{dLT}$ ➔ 183 = 25 x 4 + Z x 50

Z = 1,66

Service level is approximately 95,15%

IM_10

μ_{d} = 140 bottles/day

σ_d= 10 bottles/day

μ_{LT} = 6 days

σ_{LT}= 1 day

SS = 329 bottles, Q = 4375 bottles, H = 1,5/bottles/year, S = 50 euros/order

a) ROP = $\mu_d \times \mu_{LT}$ + SS = 140 × 6 + 329 = 1169 bottles

b) SS = $Z_\alpha \times \sigma_{dLT}$

$\sigma_{dLT} = (\mu_d^2 \times \sigma_{LT}^2 + \mu_{LT} \times \sigma_d^2)^{1/2} = (140^2 \times 1^2 + 6 \times 10^2)^{1/2}$ = 142,13 bottles

Z_α= (329/142,13) = 2,315 ≈ 2,32 ➔ service level ≈ 98,99%, percentile 99.

c) Annual holding cost = (Q/2 + SS) × H = = (4375/2 + 329) × 1,5 = 3774,75 euros/year

IM_11

(a) $t_1 = Q/p \rightarrow Q = t_p \times p$ = 20 × (100000/250 dias) = 8000 units

(b) N = D/Q = (1200 × 50)/8000 = 7,5 ≈ 8 times

(c)

$$T = \frac{Q}{D} \Rightarrow \quad T = \frac{8000}{60000} = 0{,}133 \text{ years ou } 33{,}33 \text{ days}$$

$$I_{máx} = Q \times \left(1 - \frac{d}{p}\right) = 8000 \times \left(1 - \frac{240}{400}\right)$$
$$= 3200 \text{ units}$$
$$I_{máx} - (T - t) = 3200 - (30 - 20) \times 240$$
$$= 800 \text{ units}$$

(d) $\text{QEE} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2\times 60000\times 180}{3{,}75}} = 2400\ units$

(e) ROP = d × μ_{LT} + SS = 240 × 10 + ($z_{0,025} \times \sigma_{dlt}$) = 2400 + [1,96 × $(240^2 \times 2^2)^{1/2}$] = 3340,8 ≈ 3341 units

(f) INTERNAL annual holding cost = Q/2 × (1-d/p) × H = 8000/2 × 0,4 × (0,25 × 18) = 7200€/year

TOPÁS annual holding cost = (Q/2 + SS) × H = (2400/2 + 941) × 3,75 = 8028,75€

The best option is to produce internally.

IM_12

D = 90000 units/year ⇒ d = 1800 units/week; P = €40/unit; S = €1000/setup cost

p = 2500 units/week; H = (20 + 5) euros/unit/year = €25/unit/year

a)

$$\text{POQ} = \sqrt{\frac{2 \times 90\,000 \times 1000}{25 \times \left(1 - \frac{1800}{2500}\right)}} = 5070{,}9 \text{ ou } 5071 \text{ units}$$

b)

Imáx =Q× (1-d/p) = 5071× (1-1800/2500) = 1420 units ⇒ Phase 1

t × p - t × d = 700 units ⇔ t ×(p-d) = 700 ⇔ t = 700/(2500-1800)= 1 week

c) Cost of a production run = 1000 + 5071 × 40 = €203840/fabric

SH4 data:

D = 90 000 ×2 =180000 units/year ⇒ d = 3600 units/week; S = €500/order;

P = €6/unit if Q< 50 000 or P = €5/unit if Q ≥ 50000; H = 0,25 × P euros/unit/year

First, we calculate the Q^* for the lowest possible price, €5,00

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 180000 \times 500}{0{,}25 \times 5}} = 12000 \text{ units}$$

Because 12000 < 50000, this EOQ is ***infeasible*** for the €5,00 price, therefore let's calculate the total cost for the smallest quantity that allows us to take advantage of the €5,00 price ➔ 50000.

Now we calculate the EOQ for the next price, €6,00.

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 180000 \times 500}{0{,}25 \times 6}} = 10954{,}45 \text{ units} ==> 10955 \text{ units}$$

Because 10955 < 50000, this EOQ is ***feasible*** for the €6,00 price, therefore let's calculate the total cost for EOQ = 10955 units.

P	*Q**	*Q*	*Annual holding cost (H*Q/2)*	*Annual ordering cost (D/Q*S)*	*Annual acquisition cost (P*D)*	*Annual Total Cost*
5,0	12000	50000	31250,00	1800,00	900000,00	933050,00
6,0	10955	10955	8216,25	8215,43	1080000,00	1096431,68

d1) The economic order quantity is 50000 units, because it has the lowest total annual cost, €933050,00.

D = 90000 ×2 =180000 units/year ⇒ d = 3600 units/week ⇒ d = 720 units/day

P = €7,00/unit; S = €100,00/order; H = 0,25 × P euros/unit/year;

μ_{LT} = 3 days and σ_{LT}= 2 days; Z_α= $Z_{0,02}$=2,054; Q = 6000 units.

d2) **ROP** = $d \times \mu_{LT}$ + SS = 720 × 3 + 2958 = **5118 units**

SS = $Z_\alpha \times \sigma_{dLT}$ = 2,054 × 1440 = 2957,76 ≈ 2958 units

σ_{dLT} = d x σ_{LT} = 720 x 2 = 1440 units

d3) $Annual\ holding\ cost = \left(\frac{Q}{2} + SS\right) \times H = \left(\frac{6000}{2} + 2958\right) \times (0{,}25 \times 7) = €10\ 426{,}50/year$

IM_13

D = 40000 units/year ⇒ d = 160 units/day; p = 60000 units/year ⇒ p = 240 units/day

P = 15€/unit; H = €10/unit/year; S = €300/setup cost; T = 15 days

a) T= Q/D ↔15 days = Q/160 units/day ⇒ Q = 2400 units

b) Units stored per day = (240 × 1) – (160 × 1) = 80 units/day

$$POQ = \sqrt{\frac{2 \times 40\,000 \times 300}{10 \times \left(1 - \frac{160}{240}\right)}} = 2683,28 \text{ unidades ou } 2683 \text{ unidades}$$

The lot size used is not the one that allows minimizing the total costs

d)

d1) $D_{Bot.}$ = 2 × 40000 = 80000 units/year ⇒ d = 320 units/day; $\mathbf{P}_{Bot}$ = 4 €/unit; H_{Bot} = 0,5 × 4 = €2/unit/year; S_{Bot} = €50/order; LT ~ Norm.(μ_{LT} = 3 days; σ_{LT} = 1 day); ROP = 1617 units

ROP = $\mu_{LT} \times d + Z_\alpha \times \sigma_{DLT}$ ⇔ 1617 = 3 × 3200 + Z_α × (320 × 1)

Z_α = 2.053 → α ≈ 0.02 ⇒ Stockout probability 2%

SS = 1617-3 × 320 = 657 units

d2) N = D/Q = 80000/2000 = 40 orders

$$Q^* = \sqrt{(2 \times D \times S)/H} = \sqrt{(2 \times 80000 \times 50)/2} = 2000 \text{ units}$$

d3)

SS = 1617-3 × 320 = 657 units

Annual total cost = (Q/2 + SS) × H + D/Q × S + **P** × D =

= (2000/2 + 657) × 2 + 40 × 50 + 4 × 80000 = €325 314/year

IM_14

a) $N=(D/Q)=12500/600=20{,}8 \approx 21$ production runs

b) Annual holding cost $=(Q/2) \times (1-d/P) \times H = (600/2) \times (1-50/100) \times 9 =$ €1350,00/year

c) $T = 600/50 = 12$ days $t1 = 600/100 = 6$ days

Stock $= Imáx - 2 \times d = 300 - 100 = 200$ units or

$(12-8) \times d = 200$ units

d)

d1) $SS = Z_{\alpha} \times \sigma_{dLT}$

$\alpha = 0{,}025 \Rightarrow Z_{0,025} = 500 \times 1{,}96 = 980$ units

$\sigma_{dLT} = d \times \sigma_{LT} = 250 \times 2 = 500$

d2)
Sometal

$Q = 764 \Rightarrow Ct = (764/2) \times 0{,}2 \times 15 + (12500/764) \times 70 + 12500 \times 15 = 189791{,}3$ euros/year

METALIC
$Q = 791 \Rightarrow Ct = ((791/2)+980) \times 0{,}2 \times 10 + (12500/791) \times 50 + 12500 \times 10 = 1\ 128541,\ 1$ euros/year

The supplier is METALIC.

MULTIPLE CHOICE QUESTIONS

1. Daily demand for the product PERFUMAIS from D. Maria's shop is equal to 60. The cost of each order is 62.5 euros and D. Maria estimates the yearly holding cost to be of 3 euros per unit. Which of the following is the economic order quantity?

1		1560 units
2	X	791 units
3		50 units
4		300 units

2. According to the EOQ model, if the ordered quantity is 1400 units and the time between orders is 7 weeks, the stored quantity 4 weeks after the reception of the previous order is:

1	X	600 units
2		200 units
3		400 units
4		800 units

3. The company MO.CA produces cardboard furniture and appliances, such as the cardboard tree NATAL.CA, with annual demand of 1500 units. Currently the production capacity is of 10 units per day. Start-up cost of production is 120 euros and the yearly holding cost is of 30 euros. At present, MO.CA is producing batches of 150 units of NATAL.CA.

What is the annual holding cost associated with the batch size defined by the company?		
1		1038 euros/year
2		2250 euros/year
3		2595 euros/year
4	X	900 euros/year

What is the production time in each production run?		
1	X	3 weeks
2		15 weeks
3		5 weeks
4		22 weeks

4. The reputed Pharaoh's cigar factory consumes 1300 tobacco crates per year. The yearly holding cost per crate is of 3 euros. The lead time for the supplier of this type of tobacco is normally distributed with mean equal to 10 weeks and standard deviation of 5 weeks. Currently Mr Partágas, the factory director, is ordering batches of 500 crates. The factory operates 52 weeks per year.

Assuming the factory director wishes to maintain a stock out probability less than or equal to 2.5%, what level of safety stock do you recommend?		
1		1175 crates
2		125 crates
3	X	245 crates
4		12740 crates

Assuming the firm works with a safety stock of 750 boxes, what should the reorder point be?		
1		800 crates
2		1425 crates
3		13750 crates
4	X	1000 crates

If the company works with a safety stock of 750 crates, what should the annual holding cost associated with this policy be?		
1	X	3000 euros
2		1875 euros
3		1125 euros
4		2250 euros

5. Weekly demand for wholegrain flour at GOODBUY supermarket follows a Normal distribution with mean of 60 packages and standard deviation of 10 packages. The yearly holding cost of each package is 2 euros. The lead time is 8 weeks. Currently the supermarket owner orders batches of 500 packages.

Assuming the GOODBUY supermarket owner follows a safety stock of 70 packages, what is the service level provided to the customers?		
1	x	99.32%
2		95.0%
3		81.06%
4		85%

Assuming the GOODBUY supermarket owner follows a safety stock of 70 packages, what is the yearly holding cost associated with this inventory policy?		
1	x	640 euros/year
2		500 euros/ year
3		1140 euros/ year
4		140 euros/ year

6. Annual demand for TVPLUS television sets at ELECTRICA store is of 10 000 units. The order cost is of 30 euros and the weekly holding cost per unit is 0.50 euros. How many orders should ELECTRICA make in a year?		
1	x	65 orders
2		9 orders
3		10 orders
4		24 orders

7. Weekly demand for tea bags at Mrs Amélia's tea store equals 125 bags. Ordering costs are 10 euros and Mrs Amélia estimates the yearly holding cost per tea bag to be of 0.50 euros.

Calculate the periodicity between tea bag orders.		
1	x	20 days
2		12.5 days
3		10 days
4		25 days

If the lead-time is 4 days, which of the following is the re-order point?		
1	x	100 units
2		20 days
3		500 units
4		2500 unidades