Microeconomics

Chapter 8
Choice

Fall 2024

Comparative statistics

We will examine some important **comparative statistics** of consumer demand behavior.

For instance, with the Marshallian demand function we can ask: How does demand changes when the price changes while income is kept fixed?

This question reflects a comparative statics exercise, since the Marshallian demand can be interpreted as an equilibrium situation, which we compare at two different price levels.

Most importantly, we will learn how changes in the Marshallian demand due to changes in prices can be decomposed into two effects.

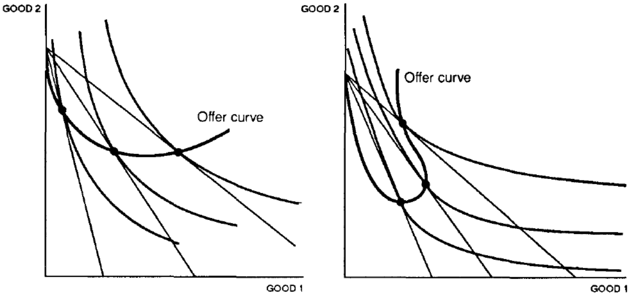
Price offer curve and Marshallian demand curve

How does demand change if we change the price, but keep income fixed?

Price offer curve: The points of utility-maximizing consumption bundles as the price varies while income is kept fixed.

Marshallian demand curves: The function that relates the utility-maximizing demand for each good to the price of that good.

Two type of goods

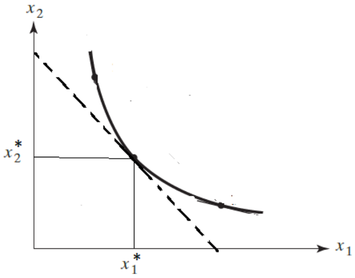


Ordinary good: higher price means less demand, so that the **Marshallian demand curve is downward sloping**. Good 1 in the left figure is ordinary.

Giffen good: higher price means more demand, so that the **Marshallian demand curve is upward sloping**. Good 1 in the right figure is giffen.

Exercise

The graph below indicates the optimal consumption bundle. Imagine price p_1 for good x_1 increases.



1. Draw the new budget constraint
2. Draw a new indifference curve such that x_1 is a giffen good.
3. Draw the Marshallian demand curve for x_1.

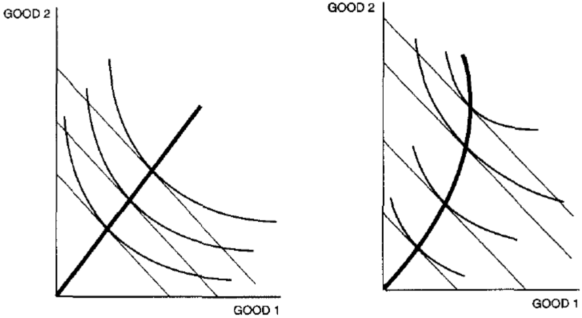
Income expansion path and Engel curve

How does demand change if we change income, but keep prices fixed?

Income expansion path: The points of utility-maximizing consumption bundles as income varies while prices are kept fixed.

Engel curves: The function that relates the utility-maximizing demand for each good to the income of the consumer.

Two type of goods

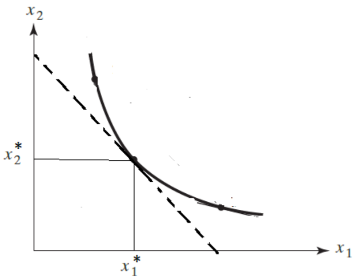


Normal good: higher income means more demand, so that **Engel curve is upward sloping**. Good 1 and 2 in the left figure are normal goods.

Inferior good: higher income means less demand, so that **Engel curve is downward sloping**. Good 1 in the right figure is an inferior good. The prototypical inferior good has preferred, yet more expensive, substitutes: once income increases, consumer switches towards more expensive options.

Exercise

The graph below indicates the optimal consumption bundle. Imagine income *m* decreases.



1. Draw the new budget constraint. Use the formula of the budget constraint to explain why this happens.
2. Draw a new indifference curve such that x_1 is an inferior good.
3. Draw the Engel curve for x_1.
4. What are the characteristics of an inferior good. Explain your reasoning via a decrease in *m*.

Decomposing changes in demand

We will next introduce the **Slutsky equation**, which decomposes changes in the Marshallian demand due to changes in the price into two effects.

This allows us to discuss the **law of demand**, which, among other things, clarifies that giffen goods must be inferior goods. This also means that giffen goods share the characteristics of inferior goods.

The Slutsky equation

The Hicksian demand function at utility level u for good i is $h_i(\mathbf{p}, u)$. From duality we know that:

$$h_i(\mathbf{p}, u) = x_i(\mathbf{p}, e(\mathbf{p}, u)),$$

where $x_i(\mathbf{p}, m)$ is the Marshallian demand function and $m = e(\mathbf{p}, u)$ is the income required to achieve utility u.

We can differentiate both the LHS and RHS with respect to p_i,

$$\frac{\partial h_i(\mathbf{p}, u)}{\partial p_i} = \frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} + \frac{\partial x_i(\mathbf{p}, m)}{\partial m} \frac{\partial e(\mathbf{p}, u)}{\partial p_i}.$$

The Slutsky equation

To slightly rewrite this equation, we can use Shephard's lemma that $\partial e(\mathbf{p}, u) / \partial p_i = h_i(\mathbf{p}, u)$, and that $h_i(\mathbf{p}, u) = x_i(\mathbf{p}, m)$ at $m = e(\mathbf{p}, u)$.

Recall that we introduced Shephard's lemma as $\partial c(\mathbf{w}, y) / \partial w_i = x_i(\mathbf{w}, y)$. The cost function $c(\mathbf{w}, y)$ and w_i in the CMP are the expenditure function $e(\mathbf{p}, u)$ and p_i in the EMP, respectively.

We can now define the **Slutsky equation** as follows:

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} = \frac{\partial h_i(\mathbf{p}, u)}{\partial p_i} - \frac{\partial x_i(\mathbf{p}, m)}{\partial m} x_i(\mathbf{p}, m),$$

where $h_i(\mathbf{p}, u)$ is the Hicksian demand function at utility level u and $x_i(\mathbf{p}, m)$ is the Marshallian demand function at income $m = e(\mathbf{p}, u)$.

Income and substitution effect

The Slutsky equation shows that the change in Marshallian demand due to a change in the price can be **decomposed** into two effects:

$$\underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i}}_{\text{TE}} = \underbrace{\frac{\partial h_i(\mathbf{p}, u)}{\partial p_i}}_{\text{SE}} - \underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial m} x_i(\mathbf{p}, m)}_{\text{IE}}.$$

Total effect (TE): When p_i increases, how does the Marshallian demand x_i change while income m is kept fixed?

This total effect can be decomposed into two effects:

Substitution effect (SE): When p_i increases, how does the Hicksian demand h_i change while utility u is kept fixed?

Income effect (IE): When p_i increases, the consumers' purchasing power decreases, and how does this affect Marshallian demand x_i?

Income and substitution effect

We know the following about *SE* and *IE*:

$$\underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i}}_{\text{TE}\lesseqgtr 0} = \underbrace{\frac{\partial h_i(\mathbf{p}, u)}{\partial p_i}}_{\text{SE}\leq 0} - \underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial m} x_i(\mathbf{p}, m)}_{\text{IE}\lesseqgtr 0}.$$

SE is always non-positive. That is, $\partial h_i(\mathbf{p}, u) / \partial p_i \leq 0$. This follows from the convexity assumption on preferences: When p_i increases, budget line gets steeper, and touches the indifference curve at lower x_i. Intuition: when p_i increases, consumer decreases x_i and increases x_j to keep the same utility.

IE is positive for a normal good and negative for an inferior good. That is, $\partial x_i(\mathbf{p}, m) / \partial m > 0$ for a normal good and $\partial x_i(\mathbf{p}, m) / \partial m < 0$ for an inferior good.

Law of demand

Therefore we can conclude the following about the **law of demand**:

$$\underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i}}_{\text{TE}\lesseqgtr 0} = \underbrace{\frac{\partial h_i(\mathbf{p}, u)}{\partial p_i}}_{\text{SE}\leq 0} - \underbrace{\frac{\partial x_i(\mathbf{p}, m)}{\partial m} x_i(\mathbf{p}, m)}_{\text{IE}\lesseqgtr 0}.$$

Ordinary good: an increase in p_i decreases x_i (TE$<$0). This implies that an ordinary good must be **normal** with $SE < 0 < IE$ or **inferior** with $SE < IE < 0$.

Giffen good: an increase in p_i increases x_i (TE$>$0). This implies that a giffen good must be **inferior** with $IE < SE < 0$.

Hence, a **giffen good** must be strongly inferior: it must have preferred, yet more expensive, substitutes, and it helps if the SE is small and/or if the good takes upon a large share of a consumer's income.

Law of demand

The table below summarizes how does x_i change if p_i changes, while income is kept fixed. Hence, it summarizes whether:

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_i} \lesseqgtr 0.$$

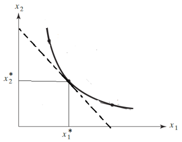
We can use the **law of demand** to distinguish between the types of goods:

Type of good	SE	IE	$\partial x_i(\mathbf{p}, m)/\partial p_i$
Ordinary good that is normal	< 0	> 0	< 0
Ordinary good that is inferior	< 0	< 0	$< 0^*$
Giffen good	< 0	< 0	$> 0^{**}$

* which implies $SE < IE < 0$.
** which implies $IE < SE < 0$.

Exercise

The graph below indicates the optimal consumption bundle. Imagine price p_1 for good x_1 increases.



1. Draw the new budget constraint

2. Draw a new indifference curve and a third budget constraint for the following four scenarios separately:

   2.1 $SE < 0 < IE$. How is this good called?

   2.2 $SE < IE < 0$. How is this good called?

   2.3 $IE < SE < 0$. How is this good called?

   2.4 $SE < 0$ and $IE = 0$.

3. Draw the Marshallian and Hicksian demand curve for good x_1 for all four situations above.

Exercise

Consider a consumer with utility function $u(\mathbf{x}) = x_1^{\alpha} x_2^{1-\alpha}$ and budget constraint $p_1 x_1 + p_2 x_2 = m$. Before we have shown that:

$$
\begin{aligned}
x_1(\mathbf{p}, m) =& \frac{\alpha m}{p_1} \\
h_1(\mathbf{p}, u) =& \left(\left(\frac{1-\alpha}{\alpha} \right) \left(\frac{p_1}{p_2} \right) \right)^{\alpha - 1} u \\
v(\mathbf{p}, m) =& \alpha^{\alpha} (1-\alpha)^{1-\alpha} p_1^{-\alpha} p_2^{\alpha - 1} m
\end{aligned}
$$

Confirm Slutsky's equation using the solutions above.

Quasilinear utility function

A **quasilinear utility function** is linear in one of the goods:

$$u(\mathbf{x}) = \phi(x_1, ..., x_{n-1}) + x_n.$$

For instance, with two goods, the quasilinear utility can be linear in x_2:

$$u(x_1, x_2) = \phi(x_1) + x_2.$$

A quasilinear utility function is often used since its **income effect for x_1 is zero**. That is, $\partial x_1(\mathbf{p}, m)/\partial m = 0$.

This implies that all changes in the Marshallian demand due to changes in the price are driven by the *SE*, so that the Marshallian and Hicksian demand function are the same. Indeed, with $IE = 0$ we have that:

$$\underbrace{\frac{\partial x_1(\mathbf{p}, m)}{\partial p_1}}_{\text{TE} \leq 0} = \underbrace{\frac{\partial h_1(\mathbf{p}, u)}{\partial p_1}}_{\text{SE} \leq 0},$$

and by duality we already knew that $h_1(\mathbf{p}, u) = x_1(\mathbf{p}, m)$ at $m = e(\mathbf{p}, u)$.

Quasilinear utility function

We can show that with quasilinear utility $u = \phi(x_1) + x_2$ the income effect for x_1 is zero and that the Marshallian and Hicksian demand coincide.

In particular, this result follows from the optimally condition that the slopes of the indifference curve and the budgetline need to be equal: MRS $=$ ERS.

This optimally condition follows from the first two FOCs of the Lagrange method and is the same across UMP and EMP.

It turns out that with quasilinear utility you do not need the third FOC (i.e., the constraint) to solve for x_1. Hence, UMP and EMP give you the same x_1.

Quasilinear utility function

We can show that with quasilinear utility $u = \phi(x_1) + x_2$ the income effect for x_1 is zero and that the Marshallian and Hicksian demand coincide.

First, obtain the **Marshallian demand** via the UMP:

$$\mathcal{L} = \phi(x_1) + x_2 - \lambda(p_1 x_1 + p_2 x_2 - m).$$

First two FOCs are $\frac{\partial \mathcal{L}}{\partial x_1} = \phi'(x_1) - \lambda p_1 = 0$ and $\frac{\partial \mathcal{L}}{\partial x_2} = 1 - \lambda p_2 = 0$. We can substitute $\lambda = \frac{1}{p_2}$ in the first FOC and find that:

$$\phi'(x_1) = \frac{p_1}{p_2} \rightarrow x_1(\mathbf{p}) = \phi'^{-1}\left(\frac{p_1}{p_2}\right), \quad \text{so that } \frac{\partial x_1(\mathbf{p})}{\partial m} = 0.$$

Second, obtain the **Hicksian demand** via the EMP:

$$\mathcal{L} = p_1 x_1 + p_2 x_2 - \lambda(\phi(x_1) + x_2 - u)$$

First two FOCs are $\frac{\partial \mathcal{L}}{\partial x_1} = p_1 - \lambda\phi'(x_1) = 0$ and $\frac{\partial \mathcal{L}}{\partial x_2} = p_2 - \lambda = 0$. We can substitute $\lambda = p_2$ in the first FOC and find that:

$$\phi'(x_1) = \frac{p_1}{p_2} \rightarrow h_1(\mathbf{p}) = \phi'^{-1}\left(\frac{p_1}{p_2}\right), \quad \text{so that } h_1(\mathbf{p}) = x_1(\mathbf{p}).$$

Exercise

Consider the following UMP,

$$\max_{x_1, x_2} \sqrt{x_1} + x_2,$$

$$\text{such that } p_1 x_1 + p_2 x_2 = m.$$

1. Find the Marshallian demand function for x_1.

2. What do you conclude about the income effect for good x_1?

3. Find the Hicksian demand function for x_1, and conclude that $h_1(\mathbf{p}) = x_1(\mathbf{p})$.

4. Write down the formula for the indifference curve for the quasilinear utility function above. Draw a few indifference curves with varying levels of utility in a graph of x_2 against against x_1. What is special about the shape of these indifference curves?

5. Given the special shape of these indifference curves, graphically show that the income effect is zero.

Homework exercises

Exercises: 8.6 and exercises on the slides