**1.1. (1 point)**

$f(x_1, x_2) = \theta x_1^{\alpha} x_2^{\beta}$

$f(tx_1, tx_2) = \theta(tx_1)^{\alpha}(tx_2)^{\beta} = t^{\alpha+\beta}\theta x_1^{\alpha} x_2^{\beta} = t^{\alpha+\beta} f(x_1, x_2)$

Hence, with $t > 1$, we can conclude that:

If $(\alpha + \beta) < 1$, then $f(tx_1, tx_2) < tf(x_1, x_2)$, hence DRTS.

If $(\alpha + \beta) = 1$, then $f(tx_1, tx_2) = tf(x_1, x_2)$, hence CRTS.

If $(\alpha + \beta) > 1$, then $f(tx_1, tx_2) > tf(x_1, x_2)$, hence IRTS.

**1.2. (1 point)**

$$TRS = -\frac{\frac{df}{dx_1}}{\frac{df}{dx_2}} = -\frac{\alpha\theta x_1^{\alpha-1} x_2^{\beta}}{\beta\theta x_1^{\alpha} x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1} = -\frac{x_2}{x_1}$$

**1.3. (1 point)**

From the TRS one can conclude that:

(i) If $x_2$ is large and $x_1$is small, the TRS is strongly negative. This implies that if one increases $x_1$ from a small value, one can decrease $x_2$ by a lot and keep producing the same amount.

In contrast, (ii) if $x_2$ is small and $x_1$is large, the TRS is close to zero. This implies that if one increases $x_2$ from a small value, one can decrease $x_1$ by a lot and keep producing the same amount.

The economic intuition is that: if we already use a lot of $x_2$ ($x_1$) using more of $x_2$ ($x_1$) is not that productive. Indeed, if we already use a lot of $x_2$ ($x_1$), we can decrease $x_2$ ($x_1$) by a lot and only increase $x_1$ ($x_2$) by a little and keep producing the same. One can write that this reflects a preference for a “balanced” input bundle over an “extreme” input bundle.

**2.1. (1 point)**

The formula for the WACM is that:

$$\boldsymbol{w}^t \boldsymbol{x}^t \leq \boldsymbol{w}^t \boldsymbol{x}^s , \qquad \forall s, t \;\; with\, y^s \geq y^t.$$

We observe $\boldsymbol{w}$, $\boldsymbol{x}$, and $y$ so we can test WACM.

Let t=1 and s=2. Since $y^1 = y^2$, it both needs to hold that:

$$\boldsymbol{w}^1 \boldsymbol{x}^1 \leq \boldsymbol{w}^1 \boldsymbol{x}^2$$

And

$$\boldsymbol{w}^2 \boldsymbol{x}^2 \leq \boldsymbol{w}^2 \boldsymbol{x}^1$$

The first holds:

$\boldsymbol{w}^1 \boldsymbol{x}^1 = 10 * 4 + 20 * 2 = 80$

$\boldsymbol{w}^1 \boldsymbol{x}^2 = 10 * 2 + 20 * 4 = 100$

Since $\boldsymbol{w}^1 \boldsymbol{x}^1 < \boldsymbol{w}^1 \boldsymbol{x}^2$.

The second also holds:

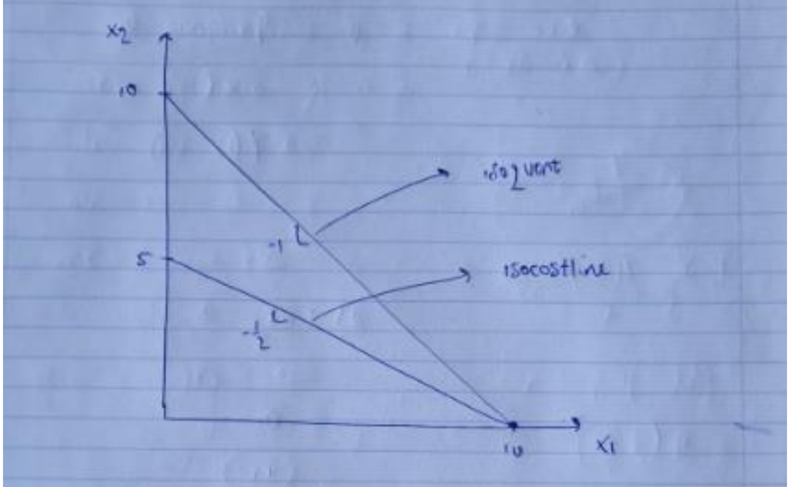
$\boldsymbol{w}^2 \boldsymbol{x}^2 = 20 * 2 + 10 * 4 = 80$

$\boldsymbol{w}^2 \boldsymbol{x}^1 = 20 * 4 + 10 * 2 = 100$

Since $\boldsymbol{w}^2 \boldsymbol{x}^2 < \boldsymbol{w}^2 \boldsymbol{x}^1$.

Hence, WACM cannot be rejected.

**2.2. (1 point)**




isocostline: $x_1 + 2x_2 = \underline{c}$

$2x_2 = \underline{c} - x_1$

$x_2 = \underline{c}/2 - 1/2\, x_1$

if $\underline{c} = 10$, then

$x_2 = 5 - \frac{1}{2} x_1$ and

$(x_1, x_2) = (10, 0)$ is on the isocostline and allows firm to produce $y = 10$ against lowest cost since the intercept is lowest

$x_1 = 10$ and $x_2 = 0$

$\underline{c} = 10$

**2.3. (1 point)**

$x_1 = 0$ and $x_2 = 10$

$\underline{c} = 20$

**2.4. (1 point)**

For all values $w_1 = w_2$.

Explanation (1): In this case the isoquant and the isocost line overlap for all values of $x_1$ and $x_2$, so any value of $x_1$ and $x_2$ is cost minimizing.

Explanation (2): The production function implies that $x_1$ and $x_2$ are perfect substitutes. In question 2.2 and 2.3 we have shown that in this case the firm only uses the input that is cheapest to minimize costs. In turn, if both are equally expensive, the firm will be indifferent in her choice of inputs.

**3.1. (1 point)**

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 to reach:

$$x_1 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\alpha-1}}$$

$$x_2 = \frac{m}{p_2} - \left(\frac{p_1}{p_2}\right)^{\frac{\alpha}{\alpha-1}}$$

**3.2. (1 point)**

We defined the elasticity of demand as:

$$\epsilon(x_1) = \frac{\partial x_1}{\partial p_1} \times \frac{p_1}{x_1}$$

Using this formula and that $p_2 = 1$, we can write:

$$\epsilon(x_1) = \left(\frac{1}{\alpha - 1}\right) p_1^{\frac{1}{\alpha-1}-1} \times \frac{p_1}{\left(p_1^{\frac{1}{\alpha-1}}\right)} = \left(\frac{1}{\alpha - 1}\right)$$

An alternative route to answer this question is to note that an elasticity is equal to the logarithmic derivate. For the elasticity of demand, this implies:

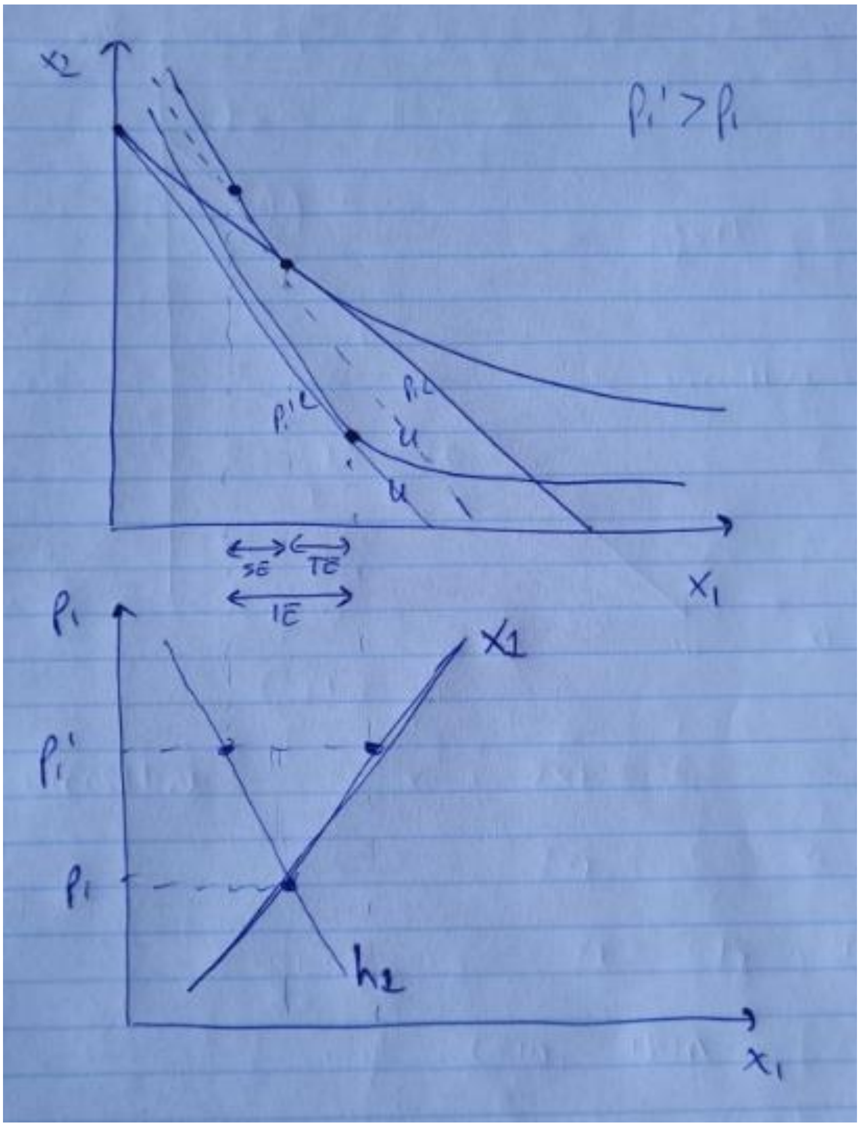
$$\epsilon(x_1) = \frac{\partial x_1}{\partial p_1} \times \frac{p_1}{x_1} = \frac{\partial \ln(x_1)}{\partial \ln(p_1)}$$

Using that $p_2 = 1$, we can write $\ln(x_1) = \left(\frac{1}{\alpha-1}\right)\ln(p_1)$. Hence, it is immediate that:

$$\epsilon(x_1) = \frac{\partial \ln(x_1)}{\partial \ln(p_1)} = \left(\frac{1}{\alpha - 1}\right)$$

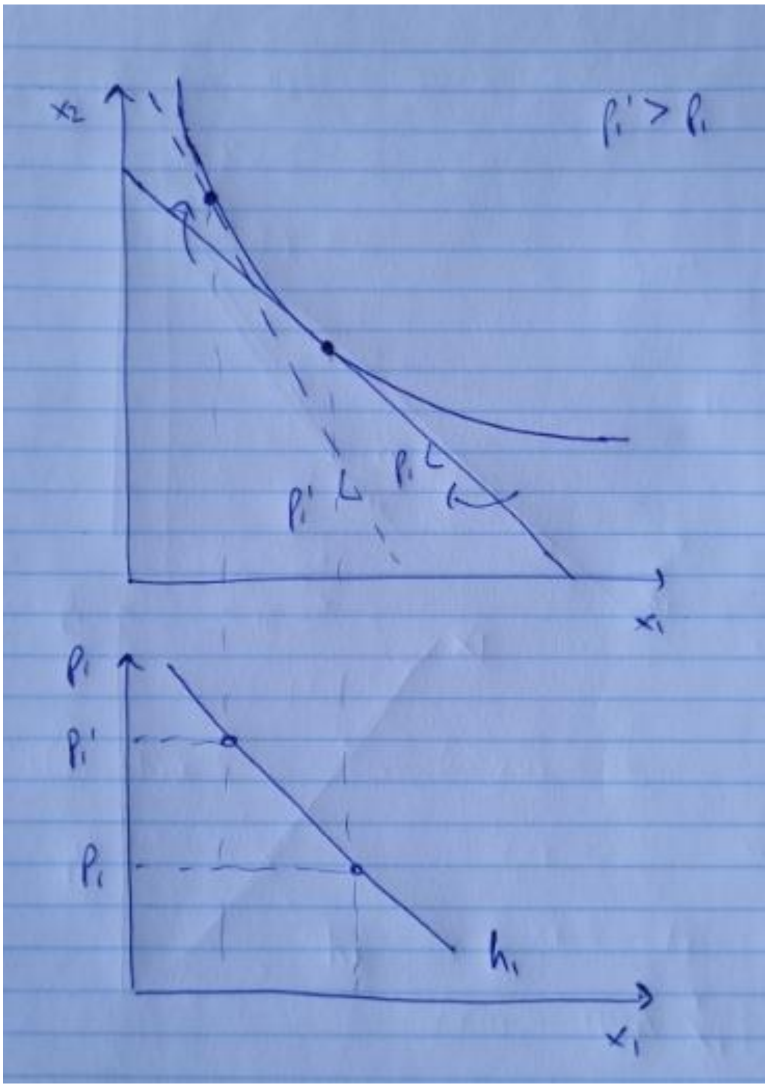
**3.3. (1 point)**

One can conclude that the income effect is smaller than zero *and* dominates the substitution effect.



**3.4. (1 point)**

The convexity assumption.




**3.5. (1 point)**

An income effect of zero.

The Slutsky equation is:

$$\frac{\partial x}{\partial p} = \frac{\partial h}{\partial p} - \frac{\partial x}{\partial m} x$$

If the income effect is zero, then $\frac{\partial x}{\partial m} = 0$, so that:

$$\frac{\partial x}{\partial p} = \frac{\partial h}{\partial p}$$

Hence the Marshallian and Hicksian demand curve have the same slope and so are equal at every price.

**4.1. (2 points)**

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 to reach:

$$x_1 = \left(\frac{p_1}{2\,p_2}\right)^{-3}$$

We know that $p_2 = 2$, so that:

$$x_1 = \left(\frac{p_1}{4}\right)^{-3} = 64p_1^{-3}$$

4. Now integrate x1 from 2 to 4

$$\int_2^4 64p_1^{-3}dp_1 = -64\frac{1}{2}\,p_1^{-2} = -32p_1^{-2}|_2^4 = -32 \times 4^{-2} - -32 \times 2^{-2} = 6$$

**4.2. (2 points)**

The change in consumer surplus is the area to the left of the Marshallian demand curve. For this utility function the income effect is zero, and so the Marshallian demand curve coincides with the Hicksian demand curves. This implies that for this utility function the change in consumer surplus is also equal to the area to the left of the Hicksian demand curves. The latter areas are referred to as the compensating and equivalent variation and are both exact measures of welfare.

**5.1. (2 points)**

There are two conditions for the long run equilibrium:

$Y(p) = X(p)$

$\pi_i = 0 \quad \forall\, i$

1. We derive the firms' supply function $y_i(p)$ for each firm i.

$mc_i(y) = \frac{dc_i(y)}{dy} = 8y,$

and since supply curve is $mc_i(y) = p$,

we have that $y_i(p) = \frac{1}{8}p$.

2. We derive market supply, which is the sum over all firms m.

$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} \frac{1}{8}p = \frac{m}{8}p.$

3. We use the first condition to find equilibrium price and firm supply in terms of number of firms m.

$$\frac{m}{8}p = 50 - 2p$$

$$p = \frac{400}{m+16}$$

$$y_i(p) = \frac{1}{8}p = \frac{1}{8}\frac{400}{m+16}$$

4. We use the second condition to find the number of firms m so that profits are zero.

$$\pi_i = py_i(p) - c_i(y) = 0$$

$$\pi_i = \frac{1}{8}\left(\frac{400}{m+16}\right)^2 - \frac{4}{8^2}\left(\frac{400}{m+16}\right)^2 - 16 = 0$$

$$\frac{4}{8^2}\left(\frac{400}{m+16}\right)^2 = 16$$

$$\left(\frac{400}{m+16}\right)^2 = 256$$

$$\frac{400}{m+16} = 16$$

$$m = 9$$

Hence, in the long run there will be 9 active firms in this perfect competitive market.

**6.1. (1 point)**

We can write the inverse demand curve as:

$$p = 50 - \frac{1}{2}y$$

Now we can write the profit function as:

$$\pi = TR(y) - TC(y) = \left(50 - \frac{1}{2}y\right)y - (50 + 20y + y^2)$$

Marginal revenue is equal to:

$$\frac{\partial TR}{\partial y} = MR = 50 - y$$

Marginal cost is equal to:

$$\frac{\partial TC}{\partial y} = MC = 20 + 2y$$

Setting MR equal to MC and solve for y, which gives us:

$$50 - y = 20 + 2y$$

$$y = 10$$

**6.2. (1 point)**

A monopolist with a demand curve that is downwards sloping faces a tradeoff: To generate more revenue it needs to sell more, but it can only do so by decreasing the price. When demand is inelastic, it can only sell more if it decreases the price by a lot. It turns out this is not worth it: If demand is inelastic, the monopolist *decreases* revenue by selling more as the required decrease in the price is too large. More precisely, one can show that for a monopolist MR<0 if demand is inelastic.