Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 1 February 2017

Time: 2:15h1

Group I (8,0)

Please describe how to:

1. assess financial market participants' expectations from option prices. (2,5)

- CCAPM
- RND
- Cumulative Distribution

2. estimate a risk-neutral density function with non-parametric methodologies (also identifying the pros and cons of these methodologies). (3,0)

- price of the symmetric of a butterfly spread
- arithmetic approximation
    - Pros: Easier to calculate
    - Cons: very irregular RNDs
- price of the symmetric of a butterfly spread
- kernel smoothing
    - Pros: smoother RNDs
    - Cons: More complex to calculate

3. compute the probability of default for a company from its stock price. (2,5)

- Merton model:
    - Equations
    - Unknowns
    - Distribution
    - Open issues

Group II (6,0)

How can one argue that:

1. in a Wiener process, the uncertainty is proportional to time? (1,5)

- W(t) – W(s) ~ N (0, sqrt(t-s))
- charts

2. the Geometric Brownian Motion hypothesis is relevant for the distribution of a financial asset price? (2,5)

- GBM implies (through Itô's Lemma) that the asset price is log-normally distributed (rate of return normally distributed)

3. an asset is riskier when its pay-off is negatively correlated to the stochastic discount factor (2,0)

$$\Lambda_t = E_t\left[i_{t+1}\right] - i_{t+1}^{f} = -\rho_{M_{t+1},i_{t+1}} \frac{\sigma_{M_{t+1}} \sigma_{i_{t+1}}}{E_t\left[M_{t+1}\right]}$$

$$E_t\left[i_{t+1}\right] = i_{t+1}^{f} - \frac{Cov_t\left[M_{t+1}, i_{t+1}\right]}{E_t\left[M_{t+1}\right]}$$

The interest rate of an asset results from the risk-free rate, adjusted by a risk factor => the lower the covariance, the higher the risk and the interest rate.

Group III (6,0)

1. Please comment on the trade-off between CIR and Vasicek interest rate models (2,5)

- Presentation of models
- CIR offers models closer to reality than Vasicek (stochastic variance), but are more complex to estimate (imposing e.g. sign restrictions on the determinants of volatility)

2. Please describe a 2-factor Gaussian affine model for the term structure of interest rates, presenting the equations for the yield curve, the short-term rate, the volatility curve and the term premium (3,5)

$$-m_{t+1} = \xi + \gamma^T z_t + \lambda^T V(z_t)^{1/2} \varepsilon_{t+1}$$

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + V(z_t)^{1/2} \varepsilon_{t+1}$$

$$-p_{n,t} = A_n + B_n{}^T z_t$$

$$A_n = A_{n-1} + \xi + B_{n-1}{}^T (I - \Phi)\theta - \frac{1}{2}(\lambda + B_{n-1})^T \alpha(\lambda + B_{n-1})$$

$$B_n{}^T = \gamma^T + B_{n-1}{}^T \Phi - \frac{1}{2}(\lambda + B_{n-1})^T \beta^T z_t (\lambda + B_{n-1})$$

$$y_{n,t} = \frac{1}{n}\left(A_n + B_n{}^T z_t\right)$$

$$y_{1,t} = \xi - \frac{1}{2}\lambda^T \alpha \lambda + \left[\gamma^T - \frac{1}{2}\lambda^T \beta^T \lambda\right] z_t$$

$$Var_t\left(y_{n,t+1}\right) = \frac{1}{n^2} B_n{}^T V(z_t) B_n$$

$$\begin{aligned}\Lambda_{n,t} &= E_t p_{n,t+1} - p_{n+1,t} - y_{1,t} \\ &= -\sum_{i=1}^{k}\left[\lambda_i B_{i,n} + \frac{B_{i,n}{}^2}{2}\right]\alpha_i - \sum_{i=1}^{k}\left(\lambda_i B_{i,n} + \frac{B_{i,n}{}^2}{2}\right)\beta_i^T z_t\end{aligned}$$