Master in Mathematical Finance

Interest Rate and Credit Risk Models

Exam – 10 January 2018

Time: 2h

1. Please consider the following information on the Euro area yield curve for the 5th January 2018 (figures in %):

Maturities	Euro Money Market	German Government Debt
Overnight	-0,413	
1 week	-0,410	
1month	-0,423	
3 months	-0,385	
6 months	-0,320	
1 year	-0,223	
2 years		-0,605
3 years		-0.484
5 years		-0,203
10 years		0,438

1.1. Compute the price of a futures contract for the 3-month Euribor, with expiry date in April 2018. (2,0/20)

Futures price = 100 – implied interest rate (%)

Implied interest rate = ${}_3f_3 = [(1+s_{m+n})^{m+n}/(1+s_m)^m]^{1/n} = [(1+s_{0,5})^{0,5}/(1+s_{0,25})^{0,25}]^{1/0,25}$ = -0,15% => Futures price =100,15

1.2. Considering that the 5 and 10 year maturities of the Government debt are represented by bonds paying annual coupons, with a redemption value of 100 Euros and coupon rates of 2% and 3%, respectively, compute the number of 10-year bonds to use in a duration hedging strategy of a portfolio comprised by 100 bonds representative of the 5-year maturity. (2,5/20)

q = - (P x Dp) / (H x Dh)

Portfolio value = 100 5y bonds

Price of 5 year bond = 2/(1-0,00203) + 2/(1-0,00203)^2 + 2/(1-0,00203)^3 + 2/(1-0,00203)^4 + 102/(1-0,00203)^5 = 100,5534 => Portfolio value (P) = 111,0824 x 100 = 11108,24

5y bond Duration = 4,82

Price of 10 year bond = 3/(1+0,0438)+ 3/(1+0,0438)^2 + … + 3/(1+0,0438)^9 + 103/(1+0,0438)^{10} = 125.0135

10y Bond Duration = 8,94

q = - (P x Dp) / (H x Dh) = - (11108,24 x 4,82)/ (125,0135 x 8,94) = -47,9

1.3. Assuming that most shifts in this yield curve are parallel or just involve changes in the slope and the volatility of interest rates is constant, present an adequate affine model to characterize the curve, by specifying the main equations (e.g. the spot, the one-period forward, the volatility and the term premium curves). (3,0/20)

$$y_{n,t} = \frac{1}{n}\left(A_n + B_{1,n} z_{1t} + B_{2,n} z_{2t}\right)$$

$$f_{n,t} = \delta + \frac{1}{2}\sum_{i=1}^{2}\left[\lambda_i^2 \sigma_i^2 - \left(\lambda_i \sigma_i + \frac{1-\varphi_i^n}{1-\varphi_i}\sigma_i\right)^2\right] + \sum_{i=1}^{2}\left[\varphi_i^n z_{it}\right]$$

$$\Lambda_{n,t} = E_t p_{n,t+1} - p_{n+1,t} - y_{1,t} = \frac{1}{2}\sum_{i=1}^{k}\left[\lambda_i^2 \sigma_i^2 - \left(\lambda_i \sigma_i + \frac{1-\varphi_i^n}{1-\varphi_i}\sigma_i\right)^2\right]$$

$$= \sum_{i=1}^{k}\left[-\lambda_i \sigma_i^{\,2} B_{i,n} - \frac{B_{i,n}^{\;\;2}\sigma_i^{\,2}}{2}\right]$$

$$Var_t\left(y_{n,t+1}\right) = \frac{1}{n^2}\sum_{i=1}^{k}\left(B_{i,n}^2\sigma_i^2\right)$$

1.4. Considering the model presented in the previous question, please show how can one conclude that a financial asset is riskier when its pay-off is negatively correlated to the stochastic discount factor. (2,5/20)

$$\Lambda_{n,t} = -COV_t\left(i_{n,t+1}, m_{t+1}\right) - Var_t\left(i_{n,t+1}\right)/2$$

- Risk premium determined by the covar. of the asset's rate of return with the stochastic discount factor => the lower the covar., the higher the risk premium is.
- Rationale behind this.

2. Please consider the marginal probabilities of default for the company EFG (in addition to the interest rate information on the Government Debt provided in question 1):

Maturity (years)	Prob.Default
1	0,53%
2	1,05%

2.1. Compute the premium of a credit default swap with the following features: (2,5/20)

Maturity = 2 years

Notional = € 10.000.000

Payment in case of default = 80% of the notional

E_1[*pay-off*(2)|ND] = *pay-off* in case of no default *(1-PD) + *pay-off* in default *PD

=0 x (1– 0.0105) + 0.8 x 0.0105 = 0.0084

E_1[*pay-off*(2)|D] = 0.8

$E_0[X(2)] = E_1[pay\text{-}off(2)|ND]*(1\text{-}PD) + E_1[pay\text{-}off(2)|D] *PD = 0.0084 \times (1 - 0.0053) + 0.8 \times 0.0053 = 0,01259548$

$V(0,2) = F(0,2)\ E_0[X(2)]$ *nocional = 1,0122107 * 0.01259548*10000000 = €127492,8.

If the premium is paid on an annual basis, we'll have:

127492,8 = 1.002234984x p +1,0122107 x p +

p = 63289,27

2.2. Please explain how would you assess the credit risk of EFG by using a structural model, presenting the theoretical framework and the information required. (2,5/20)

- Merton model
  - Rationale
  - Assumptions
  - Equation system
  - Problems => Moody's KMV

2.3. Assuming that the price of a bond issued by EFG one year ahead will be 105 or 103, respectively if the rating is A or B at that time and the recovery rate is 40%, please compute the 1-year Credit VaR of a portfolio comprised by 1000 bonds issued by this company, for an adequate percentile. (3,0/20)

Assuming the current rating A and the following transition probabilities, in addition to the data provided:

Ratings	Prob.Transition (%) (1)	Loan Value at year-end (2)	Difference to the mean (3)=(2)-μ	Contribution to the variance (4)=(1)x(3)^2
A	98,70%	105	0,3599	0,127844146
B	0,77%	103	-1,6401	0,020712446
D	0,53%	40	-64,6401	22,1452154
Mean		104,6401		
Variance		22,29377199		
St-dev		4,721628108		

1-year 99% Credit-Var = Mean-$P_{1,B}$ (as the probability of having 1 year after a rating not above B = P(B)+P(D)) = 0,77%+0,53%=1,3 ≈ 1% = 104,6261-103 = 1,6261.

2.4. What would be the main consequences of adding a second bond to this portfolio in what concerns to the calculation of the Credit VaR. (2,0/20)

- All combinations between the 2 bond prices for the several possible ratings would have to be considered
- joint migration probabilities
- correlation coefficient between the 2 bond prices